On vector sublattices generated by finite sets

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Abstract. For a subset S of an (Archimedean) vector lattice E, we let $\mathrm{vlt}\,\mathrm{S}$ denote the vector sublattice of E generated by S, that is, the smallest vector sublattice of E containing S. There are known some explicit descriptions of $\mathrm{vlt}\,\mathrm{S}$. However they reflect in no way any special structure that the set S may have. Inspired by the result of Brown, Huijsmans and de Pagter that if $x, y \in E_+$, then $\mathrm{vlt}(\{\mathrm{x}, \mathrm{y}\}) = \mathrm{lin}(\mathrm{lin}\{\mathrm{x}, \mathrm{y}\})^+$, where "lin" states for the "linear span", we show a kind of "distributivity law" for the operator vlt: $\mathrm{vlt}(\mathrm{X} \cup \mathrm{Y} \cup \mathrm{Z}) = \mathrm{vlt}(\mathrm{X} \cup \mathrm{Y}) + \mathrm{vlt}(\mathrm{X} \cup \mathrm{Z})$ if Y and Z are orthogonal (we discuss also some consequences of this law). The talk is devoted to the question how "big" can be the sublattice $\mathrm{vlt}\,\mathrm{S}$ (for finite S), where "big" is meant in the sense of algebraic dimension, or in a topological sense. We give a characterization of those sets S in general function vector lattices E for which $\mathrm{vlt}\,\mathrm{S}$ is of a given finite dimension. Moreover, in general case of a vector lattice E, we describe a form of elements $x, y \in E_+$ for which $\dim \mathrm{vlt}(\{\mathrm{x}, \mathrm{y}\}) = \mathrm{n}$. We also introduce the class of finitely discrete elements $e \in E_+$, i..e, such that $\dim \mathrm{vlt}(\{\mathrm{x}, \mathrm{e}\}) < \infty$ for every $x \in E_+$. On the other hand we show that if E is a uniformly complete vector lattice, then every principal ideal in E of infinite dimension contains pairs of positive elements that generate vector sublattices of dimensions $\aleph_0$ as well as continuum. The last part of the talk concerns the question of the existence and minimality of a finite set S in a Banach lattice E with $\mathrm{vlt}\,\mathrm{S}$ dense in E.