Iterations of Steiner symmetrizations

Aljoša Volčič
Università della Calabria
volcic@unical.it

In the attempt of solving the isoperimetric problem (open since Zenodorus who lived in the second century BC) Steiner missed the important point of the existence of the solution. To fill the gap, W. Gross (1916) constructed, given a convex body K, a sequence of directions $\{u_n\}$ such that iterated Steiner symmetrals, which decrease the perimeter, converge, in the Hausdorff distance, to the ball K^* centered at the origin and having the same volume as K.

Peter Mani (1986) was the first to understand that the convergence of the successive Steiner symmetrizations of a **convex body** K to K^* holds *almost surely* with respect to uniform probability on the set of all sequences of directions.

He conjectured that this happens also if we consider successive Steiner symmetrizations of a **compact set**. The conjecture was confirmed in 2006 by van Schaftingen. We provided in 2013 a different proof.

Recently several papers appeared concerning convergence of iterations of Steiner symmetrizations.

Bianchi, Klain, Lutwak, Yang and Zhang proved in 2011 the following result.

Theorem *If K is a* **convex body** *and U is a countable set of directions, then it can be ordered in a sequence $\{u_n\}$ such that the successive Steiner iterations of K in that directions converge to K^*.*

We improved recently this result in two directions. On one hand the seed K of the iteration is allowed to be **compact** rather than convex, and on the other hand we proved that there exists a universal ordering of U which works for every seed.

The first step of the proof consists in the analogous statement for measurable sets (with L_1 convergence), then the result is extended to L_1 functions (and L_1 convergence), then to continuous functions with compact support (and uniform convergence) and finally to compact sets (with convergence in Hausdorff distance).