States with values in the Łukasiewicz groupoid

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We consider certain groupoid-valued measures and their connections with quantum logic states. Let $*$ stand for the Łukasiewicz t-norm on $[0,1]^2$. Let us consider the operation $\diamond$ on $[0,1]$ by setting $x \diamond y = (x^\perp * y^\perp)^\perp * (x * y)^\perp$, where $x^\perp = 1 - x$. Let us call the triple $L = ([0,1], \diamond, 1)$ the Łukasiewicz groupoid. Let B be a Boolean algebra. Denote by $\mathcal{L}(B)$ the set of all L-valued measures (L-valued states). We show that the family $\mathcal{L}(B)$ consists precisely of the union of classical real states and Z_2-valued states. With the help of this result we characterize the L-valued states on orthomodular posets. Since the orthomodular posets are often understood as "quantum logics" in the logico-algebraic foundation of quantum mechanics, our approach based on a fuzzy-logic notion actually selects a special class of quantum states. As a matter of separate interest, we construct an orthomodular poset without any L-valued state.

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