Continuity and axioms of probability of fuzzy events

Mirko Navara
Center for Machine Perception, Department of Cybernetics,
Faculty of Electrical Engineering,
Czech Technical University in Prague,
Technická 2, 166 27 Prague
Czech Republic
navara@cmp.felk.cvut.cz

Abstract

Soon after the introduction of fuzzy sets, Zadeh suggested to define probability on them by integrals. However, his approach does not deal with the domain and other attributes needed to define the integral correctly. D. Butnariu and E. P. Klement presented an axiomatic approach based on *tribes* of fuzzy sets as a fuzzification of the notion of σ-algebra. They proved that in the most important cases each state (=probability measure) is a convex combination of an *integral* state (in the sense of Zadeh) and a *support* state which depends only on the support of a fuzzy set. Support states do not seem to be well motivated; they do not distinguish between various nonzero membership degrees of a fuzzy set. Hans Weber, together with G. Barbieri, suggested to add a condition of *σ-order continuity*, which overcomes this difficulty and excludes support states. We bring arguments that this condition is natural and fits well to the intentions of the axiomatic system of D. Butnariu and E. P. Klement.

We also discuss further extensions of these ideas, leading to "point-free" generalization of the notion of tribe. We give motivation for such a generalization and possible formulation of its principles. Finally, we summurize perspectives of further investigations in this direction.