Quasi-triangular functions and continuous embedding

Paola Cavaliere, Paolo de Lucia, Anna De Simone*

* Department of Mathematics and Applications
University "Federico II" of Naples – Italy

Abstract

We start by defining "quasi-triangular" functions. This can be done for functions on any domain in which an orthogonality relation is given, the only further requirement being that the domain contains the supremum of any pair of orthogonal elements. Even if quasi-triangular functions mimic some properties of the norms, no vector structure, even no addition, is assumed on the domain.

We consider a measure space (S, Σ, μ), and the corresponding Riesz space $L^0(\mu)$. Any solid subspace X of $L^0(\mu)$ is used as the domain of the functions we work with. Quasi-monotone functions, quasi-additive functions and quasi-norms defined on vector subspaces of $L^0(\mu)$ are remarkable examples of quasi-triangular functions.

It turns out that the topology τ_μ of the convergence in measure can be described in terms of the vector lattice structure of the space only, with no re- ference to the structure of the underlying measure space. This is the reason why it is natural in this context to work with quasi-triangular functions.

To each quasi-triangular function Φ it is possible to associate in a standard way a topology τ_Φ. We obtain, in case the measure space (S, Σ, μ) is σ-finite, a continuous embedding

$$(X, \tau_\Phi) \hookrightarrow (L^0(\mu), \tau_\mu).$$