The limit set of Henstock–Kurzweil integral sums of vector-valued functions

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Abstract

Let X be a Banach space and $f\colon [0,1] \to X$ be a given, not necessarily measurable, function. Then for an integration process $\mathcal{I}$, defined by means of Riemann or Riemann-type sums (like Riemann integral, Riemann–Lebesgue integral or Henstock-Kurzweil integral), one can consider the set $I_{\mathcal{I}}(f)$ of all limit points of integral sums of f. For a non-integrable function f the set $I_{\mathcal{I}}(f)$ is a substitute for the ordinary $\mathcal{I}$-integral.

We consider the notion of limit set for Henstock–Kurzweil integral sums. We describe the limit set $I_{HK}(f)$ of Henstock–Kurzweil integral sums of f when f is a real-valued function which has both a Henstock–Kurzweil integrable minorant and majorant. In the case of infinite-dimensional X, we show that the limit set $I_{HK}(f)$ is a convex non-empty set when X is separable and the function $\|f(\cdot)\|$ has a Lebesgue integrable majorant. Under the same hypotheses, we give a complete description of $I_{HK}(f)$ in terms of the Aumann integral of a suitable multifunction.

References

[CDPK] D.C., L. Di Piazza, V. Kadets. Description of the limit set of Henstock-Kurzweil integral sums of vector-valued functions. *J. Math. Anal. Appl.* 421 (2015), 2, 1151-1162.