ON THE FUNDAMENTAL THEOREM OF CALCULUS FOR FRACTAL SETS

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Abstract

The standard methods of ordinary calculus are usually inapplicable to fractal sets (see for example [3]), for this reason some mathematitions (see [5], [6]) introduced, in an analogous fashion as the classical Riemann integral, an integration process for functions defined on a closed fractal subset E of the real line of positive s-Hausdorff measure $\mathcal{H}^s$ (brief. *s-set*), with $0 < s < 1$. They proved that the usual elementary properties of the classical Riemann integral are still valid for such an integral moreover they provided also some natural reformulations of the Fundamental Theorem of Calculus, in which the notion of s-derivative (see [2]) is used. However, it is well known that, in the real line, the Riemann integral is inadequate to the Fundamental Theorem of Calculus since the function

$$F(x) = \begin{cases} x^2 \sin(1/x^2) & x \in (0,\, 1]; \\ 0 & x = 0; \end{cases} \tag{1}$$

is differentiable everywhere on $[0, 1]$, but F' is not Riemann integrable since it is unbounded. Moreover, a more detailed exam revels that F' is neither Lebesgue integrable. Therefore the best formulation for the Fundamental Theorem of Calculus is given by the the Henstock-Kurzweil (see [4]).

For this reason, in order to give the best formulation of the Fundamental Theorem of Calculus on fractal sets, in this talk, we will extend to real functions defined on a closed s-set E of the real line, with $0 < s < 1$, the Henstok-Kurzweil integration process. (see [1]).

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