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Limit theorems for measures with values in abstract structures

ABSTRACT

We consider the basic notions, tools and properties useful in order to prove the main convergence theorems. First, we deal with some fundamental properties of filters/ideals and we recall the classical concept of densities, matrix methods, filter/ideal convergence and its fundamental properties.We deal with some basic notions on lattice groups and vector lattices, and in particular we point out some important mathematical tools in these structures, which will be useful in the sequel. We consider the fundamental properties of order convergence and (D)-convergence in lattice groups, which replace the so-called ε-technique. We deal with the Fremlin lemma and the Maeda-Ogasawara-Vulikh representation theorem for Archimedean lattice groups. We investigate also filter/ideal convergence/divergence in the (ℓ)-group setting, and its main properties. We present the main topics on lattice group-valued measures. We relate finite additivity to countable additivity of (ℓ)-group-valued measures, using a Drewnowski-type approach, to find (global) σ-additive restrictions of (global) (s)-bounded measures on a suitable σ-algebra, and by means of the Stone Isomorphism technique, which allows to construct some (global) countably additive extensions of (global) (s)-bounded lattice group-valued measures.

Furthermore, we consider some recent developments of limit theorems in the setting of filter convergence. In general, when one treats filter convergence with respect to a given free filter of $\mathbb{N}$, it is impossible to obtain results analogous to the classical Brooks-Jewett, Vitali-Hahn-Saks, Nikodým convergence, Nikodým boundedness and Dieudonné-type theorems when $\Phi \neq \Phi_{\text{cofin}}$, where Φ_{cofin} is the filter of the cofinite subsets of $\mathbb{N}$. However, under suitable hypotheses on the involved filter, it is possible to get still some results, concerning the behavior of a subsequence of the given sequence of measures, indexed by an element of the filter. In this spirit, different kinds of limit theorems have been proved with respect to filter convergence. First we deal with Schur-type theorems, and successively we investigate some classes of filters, for which some versions of Schur-type theorem hold, for measures with values in lattice groups. We use some sliding hump-type techniques and prove some Schur-type theorems. As consequences we give some Nikodým convergence, Nikodým boundedness, Vitali-Hahn-Saks and Dieudonné-type theorems for lattice group-valued measures. We give also some versions of these theorems, whose it is possible to give a direct proof without using the Schur theorem. We note that, in the particular case of positive measures, it is possible to prove several filter limit theorems by requiring weaker hypotheses on the involved filter. In this framework, we investigate filter exhaustiveness. We give some conditions, which in general, when $\Phi \neq \Phi_{\text{cofin}}$, cannot be dropped. These theorems are formulated when σ-additivity and related concepts are formulated in the classical like setting or with respect to a single (O)-sequence. Similar equivalence results were given by L. Drewnowski in the classical case for topological group-valued measures. In particular, when it is proved that the Nikodým convergence theorem implies the Brooks-Jewett theorem, we consider countably additive restrictions of finitely additive (s)-bounded topological group-valued measures, defined on suitable σ-algebras However in the lattice group setting, in order to relate finitely and countably additive measures, it is not advisable to use an approach of this kind. Indeed, in topological groups, the involved convergences fulfil some suitable properties, which are not always satisfied by order convergence in (ℓ)-groups, because in general it does not have a topological nature. So, to prove our results, we use the Stone Isomorphism technique by means of which it is possible to construct a σ-additive extension of a finitely additive (s)-bounded measure, and to study the properties of the starting measures in connection with the corresponding ones of the considered extensions. We give also some versions of basic matrix theorems. Note that in general these kinds of theorems, in their ideal/filter formulation, do not give immediate results like in the classical case, since in lattice groups the nature of order convergence is in general not topological, and because filter convergence is not inherited by subsequences.