Black holes, horizons and the universe: implications of an analogy

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June 19, 2020

Abstract

The similarity of the universe observable with a black hole, given by the common possession of a horizon, can be used both to obtain the values of the Planck units with a non-dimensional method, and to calculate some characteristic parameters of the universe (entropy, temperature, density) in the various stages of its evolution. In addition, it provides an independent estimate for the current number density of photons of the background radiation, and provides an explanation for the large difference between the value of the cosmological constant predicted by grand unified theories and that resulting from the observations. Finally, by the fact that the density of the universe, considered as a big black hole, is equal to the critical value (flat universe) it follows that it has zero total energy and entropy.

1. Through the mirror

Starting from the definition of the escape velocity as the speed necessary to escape the gravitational attraction of a body of mass M placed at a distance R, and move away to infinity

$$v_f = \sqrt{\frac{2GM}{R}} \tag{1.1}$$

we can imagine that this body is compressed in a radius R_S (called gravitational radius or Schwarzschild radius, named after the German physicist who developed this solution of the equations of general relativity in 1916) so small, that the escape velocity is equal to the speed of light:

$$R_S = \frac{2GM}{c^2} \tag{1.2}$$

This is precisely what happens in a black hole; the spherical surface of radius R_S surrounding the central singularity is called the event horizon.

In this paper we assume that there is a mirror symmetry between the effects produced by an event horizon outside a black hole and those produced within the universe observable by a particular type of cosmological horizon, the apparent horizon - defined as the extreme boundary of trapping surfaces [1] -, and from this hypothesis we calculate some characteristic parameters of the universe in different eras.

Our attempt is less daring than it seems: already in 1977, Gibbons and Hawking tried to extend the connection between horizons and thermodynamics to cosmological models [10], and in 1989 Roger Penrose, comparing the closed universe to a black hole, estimated that it should have an entropy of 10^{123} [19]; later Seth Lloyd and Y. Jack Ng, assuming that the universe is a kind of quantum computer which accumulates information on the surface of its horizon, obtained for the cosmic entropy a value of 10^{122} [15; 17], and more recently various authors came to affirm that temperature and entropy are general properties of all horizons, whatever the reference metric is [77; 14; 18]. There are also undeniable common properties between the horizon of a black hole and the apparent horizon:

a) the escape velocity (local) is equal for both to c, *and the signals coming from regions increasingly closer to the apparent horizon, and more and more distant from us, undergo a dilation of the wavelength according to the same law that regulates the redshift of light that climbs the gravitational field of a black hole;*

b) both black holes and the observable universe can be considered thermodynamically as quasi-static or adiabatic systems, that is, which do not exchange energy with the outside and change their properties very slowly; this allows in both cases to calculate their temperature and entropy.

It will be used as a model the Schwarzschild black hole, as it has no electric charge and angular momentum as appears to be the observable universe. The equations, unless otherwise indicated, will be written explicitly.

2. The universe is flat

In the cosmological model developed in 1922 by Alexander Friedmann (without cosmological constant) the curvature of the universe and its ultimate destiny depend univocally on the average mass-energy density: if it is less than a certain critical value the curvature is negative and the expansion lasts forever (hyperbolic or open universe); if it is higher than this value, the curvature is positive and the expansion is followed by a re-contraction phase which ends in a Big Crunch (spherical or closed universe); finally, if the cosmic density is exactly equal to the critical value, the curvature is zero and the expansion continues indefinitely as in the hyperbolic model, but more slowly (parabolic, flat or Einstein-de Sitter universe). Now, if the observable universe is analogous to a black hole, its density is exactly equal to this critical value; in other words, at least

[1] A traping surface is a two-dimensional spatial surface characterized by the fact that both the rays of light perpendicular to it directed inwards, and those directed outwards, end up converging in the origin.

the cosmic region in which we live has a Euclidean space-time geometry (flat universe) and therefore is destined to expand forever. In fact, we have:

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{M}{\frac{4}{3}\pi\left(\frac{2GM}{c^2}\right)^3} = \frac{3c^6}{32\pi G^3 M^2} = \frac{3c^2}{8\pi G R^2} = \rho_c; \qquad \Omega_0 = \frac{\rho}{\rho_c} = 1 \tag{2.1}$$

This first result has an important consequence: as pointed out in 2007 by Akbar and Cai [12], in the case of a flat universe the radius of the apparent horizon has the same value as the Hubble radius, defined as the inverse of the Hubble parameter:

$$R_A = R_H = c/H \tag{2.2}$$

3. The Planck units

To understand the properties of objects such as black holes, we must refer to extreme quantities, far removed not only from our common experience, but also from experiments that can be carried out with the most powerful particle accelerators. In 1899 Max Planck proposed a set of "natural" units of measurement based on three physical constants: the speed of light in vacuum c, the universal gravity constant G and the constant of electromagnetism h he discovered. In his honor they were called Planck or Planck-Wheeler units, named after the American physicist John A. Wheeler in the '50s sensed their profound meaning for the understanding of physical laws.

How can you calculate these fundamental units? The method commonly taught to students consists in properly combining h, c and G to obtain quantities having respectively the size of a time, a length, a mass, etc., but it risks making them appear as an artificial construct, devoid of real physical meaning. It seems more appropriate to obtain them through a thought experiment.

In order for a black hole to swallow a particle, this must have a reduced Compton wavelength (let's call it "Compton radius" for brevity) not greater than the radius of the hole itself, therefore $\hbar/mc \le 2GM_{BH}/c^2$. By equalizing the two members of the expression, Planck's mass, energy, time and length are obtained

$$M_P = \sqrt{\frac{\hbar c}{2G}} \simeq 1.54\times10^{-8}\ \text{Kg} \tag{3.1}$$

$$M_P c^2 = \sqrt{\frac{\hbar c^5}{2G}} \simeq 1.38\times10^{9}\ \text{J} \tag{3.2}$$

$$t_P = \frac{\hbar}{M_P c^2} = \sqrt{\frac{2\hbar G}{c^5}} \simeq 7.62 \times 10^{-44} \text{ s} \tag{3.3}$$

$$l_P = ct_P = \sqrt{\frac{2\hbar G}{c^3}} \simeq 2.3 \times 10^{-35} \text{ m} \tag{3.4}$$

to which must be added the Planck gravitational ray, that is the Schwarzschild radius of the universe at t_P

$$R_P = \frac{2GM_P}{c^2} = \sqrt{\frac{2\hbar G}{c^3}} \simeq 2.3 \times 10^{-35} \text{ m} \tag{3.5}$$

4. Black holes thermodynamics

The first to raise the issue of compatibility of blacks holes with thermodynamics was Wheeler who in the late 60s formulated the conjecture "no hair", according to which matter falling into a black hole loses all its distinguishing features except mass, angular momentum and electric charge (the name derives from the icastic affirmation "Black holes have no hair"). A significant consequence of that conjecture was that the macroscopic thermodynamic state of a black hole would result from a very large number of indifferent microscopic states, and therefore these objects should have a very high entropy.

In 1972 Jacob D. Bekenstein, at the University of Austin in Texas, developing an observation by Demetrious Christodoulou that event horizon area of a black hole always tends to increase with each interaction with the surrounding environment, proposed that the entropy of black holes is proportional to the ratio between this area and the square of Planck's length, and that consequently black holes also have a temperature inversely proportional to the mass [4; 5]. The belief that nothing could come out of the horizon of a black hole was however so deep-rooted that Stephen W. Hawking, Brandon Carter and James Bardeen, who also confirmed the results of Christodoulou, reiterated in a jointly signed paper the principle deriving from the General Relativity (GR) for which, being the black hole a perfect absorber of radiation, its temperature had to be equal to zero [3]. Later, however, Hawking himself discovered that black holes can actually emit radiation.

The gravitational field near the event horizon is in fact so strong to create pairs of virtual particles from the void from ephemeral life and not directly detectable [2]; for the energy conservation theorem one of them must have positive energy and the other negative energy. If the particle with negative energy travels the distance R_{BH} that separates it from the singularity in less time than that allowed by the "Heisenberg loan" it will be absorbed by the black hole, while the other, the one with positive energy, will become real and will escape away from the infinite (the inverse process is impossible because for GR particles with negative energy cannot exist in ordinary space-time): the net result is a decrease in the energy of the black hole and therefore of its mass, with a progressive increase in the temperature, until total evaporation. With a series of complex calculations related to the quantum field theory Hawking found the formulas for the temperature and entropy of black holes [11; 12], but we can get them more easily from the indeterminacy relationship $mc^2 \times R_{BH}/c \geq \hbar$: the temperature of the black hole will therefore be

$$T_{BH} = \frac{mc^2}{k_B} = \frac{\hbar c}{k_B R_{BH}} = \frac{\hbar c^3}{2k_B G M_{BH}} \tag{4.1}$$

and from the principle of conservation $dE = TdS$, by placing $S = S_0 + dS$ and $S_0 = 0$, we obtain entropy

$$S_{BH} = \frac{dE_{BH}}{T_{BH} k_B} = M_{BH} c^2 \times \frac{2GM_{BH}}{\hbar c^3} = \frac{2GM_{BH}^2}{\hbar c} = \frac{M_{BH}^2}{M_P^2} = \frac{R_{BH}^2}{R_P^2} = \frac{c^3 R_{BH}^2}{2\hbar G} \tag{4.2}$$

5. From black holes to the universe

Hawking's formulas reveal all their heuristic potential when applied to the observable universe. On their basis in fact you can say that at Planck's time - when, as mentioned above, the radius of the apparent horizon was equal to R_P - the cosmic entropy, for (4.2), was equal to 1 bit

$$S_P = \frac{R_P^2}{R_P^2} = \frac{t_P^2}{t_P^2} = 1 \tag{5.1}$$

while temperature and density had to be

$$T_{tp} = \frac{\hbar c^3}{2k_B G M_P} = \frac{\hbar c}{k_B R_P} = \frac{\hbar}{k_B t_P} \simeq 10^{32} \text{ Kelvin} \tag{5.2}$$

[2] But the existence of which is however not irrelevant, as is shown by the Casimir effect: between two flat plates separated by a few millimeters only virtual particles can be created having a wavelength not exceeding this distance, therefore less in number than in the external space; it generates a pressure difference that pushes the plates to move closer to one another.

(the apparent horizon temperature at Planck's time should not be confused with the "Planck temperature" $T_P \simeq 1.4 \times 10^{32}$ K obtained with purely dimensional considerations)

$$\rho_P = \frac{3c^2}{8\pi G R_P^2} = \frac{3}{32\pi G t_P^2} \simeq 3.1\times 10^{95}\ \text{Kg/m}^3 \qquad (5.3)$$

It also had $S_P \propto \pi / H_P^2$ and $T_{tp} \propto H_P / 2\pi$ (where $H_P = 1/2t_P$ is the Hubble parameter at Planck's time), as predicted by Hawking in 1996 in the hypothesis that the temperature and entropy of a de Sitter universe derive from the existence of a cosmological horizon [13].

Generalizing the result just obtained we can calculate the temperature and entropy of this universal black hole for each cosmic epoch [3]:

$$T_t = T_{tp} \times \frac{R_P}{R_t} = \frac{\hbar c}{k_B R_t} \propto \frac{H_t}{2\pi} \qquad (5.4)$$

$$S_t = \frac{dE_t}{k_B T_t} = \frac{c^4 R_t}{2G} \times \frac{R_t}{\hbar c} = \frac{c^3 R_t^2}{2\hbar G} = \frac{R_t^2}{R_P^2} \propto \frac{\pi}{H_t^2} \qquad (5.5)$$

$$\rho_t = \frac{3c^2}{8\pi G R_t^2} \qquad (5.6)$$

It can thus be considered to be established the relationship between the entropy of the universe and the number of quanta of fundamental area covering the apparent horizon surface, as well as the bond between the temperature and the entropy, on the one hand, and the Hubble constant (and thus, once again, the apparent horizon radius) on the other, according to the prediction of Hawking. Furthermore, it is proven that the observable universe, like black holes, satisfies the holographic principle, i.e. its horizon, like the event horizon of a black hole, contains the maximum amount of entropy physically possible for the mass-energy contained in the inside [15; 16; 17]. Of course, what has been said is not an irrefutable demonstration that the region of the universe in which we live is a huge black hole emitting Bekenstein-Hawking radiation, but certainly it is a very

[3] In reality, the increase in universal energy and entropy and the correlative decrease in Bekenstein-Hawking temperature with expansion can be considered an effect due to the cosmological redshift, that is, to the fact that the photons emitted from the horizon at the time of Planck are detected here-now [8]: in fact for $z_P + 1 = R_t / R_P$ the observable universe, seen from a reference system in solidarity with the horizon, it still has, and forever, $E_t / (z_P + 1) = E_P$, $\rho_t \times (z_P + 1)^2 = \rho_P$, $S_t / (z_P + 1)^2 = S_P$ and $T_t \times (z_P + 1) = T_{tp}$.

important indication in this sense.

It should be emphasized that the Hawking temperature of the cosmic horizon in the various eras is very different from the temperature of the background radiation: this decreases with the expansion proportionally to the inverse of the scale factor (and therefore for example, during the radiative phase, according to $1/t^{1/2}$), while the temperature of the horizon changes more quickly, according to $1/t$. The following table shows the respective values for some cosmic epochs:

Epoca	t (sec)	T CMB (K)	T Hawking (K)
Beginning of inflation	10^{-35}	8.7×10^{27}	7.6×10^{23}
End of the electroweak symmetry	10^{-10}	2.8×10^{15}	7.6×10^{-2}
Quark-antiquark annihilation	10^{-6}	2.8×10^{13}	7.6×10^{-6}
Quark confinement in protons	10^{-4}	2.8×10^{12}	7.6×10^{-8}
Neutrinos decay	1	2.8×10^{10}	7.6×10^{-12}
Electron-positon annihilation	4	1.4×10^{10}	1.9×10^{-12}
Nucleosyinthesis	180	2.1×10^{9}	4.2×10^{-14}

Applying (5.5) to the present time ($t_0 \simeq 13.8\times10^9$ years $\simeq 4.35\times10^{17}$ seconds; $R_0 = ct_0 \simeq 13.8\times10^9$ light-years $\simeq 1.3\times10^{26}$ meters) [11] we obtain the current entropy of the apparent horizon:

$$S_0 = \frac{R_0^2}{R_P^2} = \frac{t_0^2}{t_P^2} \simeq 3.25\times10^{121} \tag{5.7}$$

It is interesting to compare the entropy value described above with what would be obtained if all the energy contained within the horizon resides in the photons of the background radiation:

$$\frac{c^4\times(1.3\times10^{26})}{2Gk_B 2.725} \simeq 2.1\times10^{92} \tag{5.8}$$

It confirms that the current Hawking temperature of the universe is much lower than the CMB temperature; by (5.4) we find

$$T_0 = \frac{\hbar c}{k_B R_0} = \frac{\hbar}{k_B t_0} \simeq 1.76\times10^{-29}\ \mathrm{K} \tag{5.9}$$

and therefore the observable universe must be pervaded by a "freezing" background of particles - perhaps gravitons - with a Compton radius equal to the cosmic radius:

$$\lambda_0 = \frac{\hbar c}{k_B T_0} = R_0 \tag{5.10}$$

We also see that the relationship between the entropy of the horizon and the entropy of the background radiation gives us a measure of the extent of the non-adiabatic reheating processes, with the production of particles and entropy, which occurred at the end of the cosmic inflation, and therefore of the expansion rate undergone by the universe observable in that phase:

$$N_{e-foldings} = \ln\left(\frac{3.25\times10^{121}}{2.1\times10^{92}}\right) = \ln\left(1.55\times10^{29}\right) \simeq 67.21 \tag{5.11}$$

So in the inflationary phase, the observable universe, which at Planck's time was enclosed within a sphere of radius R_P, expanded, doubling its size for about $67.22/\ln(2) \simeq 96.97$ times, until it included $\left(1.55\times10^{29}\right)^3 \simeq 3.73\times10^{87}$ Planckian causal spheres. From here we can immediately calculate the current number of photons of the CMB and their density, which are in good agreement with the observational data [6]:

$$N_{ph0} \simeq 3.73\times10^{87} \tag{5.12}$$

$$N_{ph0}/cm^3 = \frac{N_{ph0}}{V_0} \simeq 402.33 \tag{5.13}$$

Finally, for (5.6) the current density of the observable universe is equal to

$$\rho_0 = \frac{3c^2}{8\pi G R_0^2} = \frac{3}{8\pi G t_0^2} \simeq 9.47\times10^{-27}\ \text{Kg/m}^3 \tag{5.14}$$

and the relationship between densities at the present time and at Planck's time is

$$\frac{\rho_0}{\rho_P} = \frac{R_P^2}{R_0^2} = \left(\frac{t_P}{t_0}\right)^2 \simeq 3.08\times10^{-122} \tag{5.15}$$

This could explain why the energy of the vacuum or "dark energy", which many identify with the cosmological constant hypothesized and then rejected by Einstein, currently has a value of about 121 orders of magnitude lower than that predicted by the quantum theories of great unification (GUT): in reality this incredibly small value would be the consequence of the fact that we observe here-now the incredibly great acceleration suffered by the horizon at the beginning of universal history, "reduced" by the cosmological red-shift according to forecasts of General Relativity. Indeed we have

$$\Lambda_t = \frac{3}{R_t^2} = \frac{3}{(z_P+1)^2 R_P^2} \tag{5.16}$$

which for $z_P + 1 = R_0/R_P = t_0/t_P \simeq 5.7\times10^{60}$ gives $\Lambda_0 \simeq 1.77\times10^{-52}$ m^{-2}, that is 9.23×10^{-122} in Planck units.

6. Evolution without entropy?

But an even more remarkable consequence emerges from our hypothesis. As we have seen, the density of a black hole universe is equal to the critical value, which means that the potential energy within the apparent horizon is equal to the kinetic energy with the changed sign (U = –Θ) (2.1); therefore the total energy of the observable universe, such as that of a spacecraft moving away from Earth at escape velocity, is E = U + Θ = 0. This Newtonian approach corresponds in RG to a particular solution of Friedmann's equation $\dot{R}_t + Kc^2 = 8\pi G\rho_t R_t^2/3$[4], that with curvature *K = 0* which corresponds to a total zero energy per unit of mass ($\dot{R}_t^2 - 2GM/R_t = 0$). In favor of this statement is also the fact that in a flat spacetime we have $\rho(t)\to 0$: its asymptotic state is the empty spacetime of Minkowski [9]. Now, since entropy is given by the ratio *dS = dE/T,*, the positive entropy combined with the energy of the cosmic matter must correspond to a negative gravitational entropy equal in absolute value, but with the opposite sign; therefore also the total entropy of the observable universe must be equal to zero.

(Of course this does not mean, as many naively say, that the universe arose out of nothing: "nothing", whatever this term means, is very different from a universe filled with billions of galaxies, stars, planets and amateur cosmologists, in which the null value of energy and entropy is given by the elision of two terms - one positive linked to the material constituents, the other negative deriving from the gravitational attraction between them - of order of 10^{69} Joules and 10^{121} dimensionless units respectively)

Perhaps the explanation of how a universe devoid of net energy and entropy can evolve can be found by replacing the concept of "singularity" with that of "instability" as proposed by I. Prigogine [20]: the universe would not have been born from a point of infinite density, but from an empty space-time, therefore with net energy equal to zero, in which quantum fluctuations would have led to the formation of micro-black holes with Planckian mass which would have evaporated in 7×10-44 seconds producing a radiation bath at very high temperature, subsequently organized into elementary particles and then into increasingly complex structures, and resulting in a progressive, symmetrical and

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[4] The dot indicates the derivative with respect to time.

opposite growth of the material and gravitational components of energy and entropy. A process perhaps destined to repeat itself every time that the cosmic density is reduced to values so low as to allow new fluctuations to take over.

Will these "necessary demonstrations" be verified by "sensible experiences"? For now, the answer is beyond the horizon.

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