

# A measure of gravitational entropy and structure formation

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Increasing inhomogeneity due to gravitational clumping reflects increasing gravitational entropy in a time evolving universe. Starting from an ensemble of uniformly distributed particles it is demonstrated that gravitational clustering is subject to a specific quantization rule for the amount of increase of gravitational entropy during the formation of inhomogeneities. The gain of gravitational entropy at each higher order merging process within the system is shown to result as a natural consequence from an extremal condition involved. The resulting discrete spectrum of nested, bound structures of specific mass and radius, ranging from the particle physics scale to galaxies and super clusters, provides a unified view of fundamental inhomogeneity scales in the universe from gravitational entropy considerations. Consequently, also the gravitational arrow of time points in the direction of stepwise increasing entropy or inhomogeneity, respectively.

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# 1 Introduction

The question why we observe in the universe discrete structure scales as elementary particles, stellar systems, globular clusters or galaxies, but nothing between, was originally addressed by Chandrasekar [1]. Today we argue that a suitable concept for the gravitational entropy of a discrete matter distribution is required to provide an understanding of the formation of structure scales in an expanding universe [2]. In contrast to thermodynamic systems driven to a uniform distribution, the components of gravitating systems tend to clump, thus implying a gravitational arrow of time, which points in the direction of growing inhomogeneity. The universe acts as a self-organizing system evolving spontaneously into increasingly complex structures due to the long range nature of gravitational interaction. Contrary to concepts of black hole entropy [3], and references therein, only a few attempts have been made to quantify the apparently contradictory behavior of thermodynamic and gravitating systems with regard to a consistent measure of gravitational entropy. Penrose [4] introduced the Weyl curvature hypothesis in view of a measure of local anisotropy and a phase space approach was suggested where gravitational entropy is interpretable as lack of knowledge of the field configuration considered [5]. Presently, no unique definition of a gravitational entropy for ordinary bound systems is available.

The building blocks of matter are atoms which themselves are made up by nucleons where those in turn are subject to quarks and gluons as constituents. As pointed out by Bekenstein [3] there is a variety of states and entropy available and obviously, the deeper we look into matter the more degrees of freedom are accessible and the higher is the entropy. Let me generalize this view from levels far above of condensed matter and start with superclusters made up by clusters of galaxies. These structures are found to have globular clusters and stellar systems as substructures and we ask if there is a link between all structure scales available, a quantized gravitational entropy formalism resulting in a sequence of fundamental inhomogeneities [6].

On observational grounds we can argue that any concept evaluating gravitational entropy from a monotonically increasing function must fail since the emergence of specific inhomogeneity scales due to perturbations at a specific cosmic epoch is a discrete scenario. In view of hierarchically nested discrete structure scales (galaxies, clusters of galaxies) gravitational entropy as measure of the degree of inhomogeneity in the universe is supposed to increase stepwise at each higher order merging process. In addition, we require that a suitable gravitational entropy representation should correspond in the black hole limit to the general second law (GSL) [7, 8] for the total entropy of the system. A master arrow of time is expected to point in the direction of discrete increasing gravitational entropy evolution as manifestation of irreversibility.

## 2 Black hole entropy and bound systems

The GSL of black hole dynamics provides the total entropy for systems containing black holes  $S^{tot}$  as balance between the black hole entropy  $S^{int}$  proportional to the area of the event horizon  $A$  and the entropy of the surrounding matter  $S^{ext}$  as

$$S^{tot} = S^{int} + S^{ext} = \frac{A}{l_P^2} + S^{ext} \quad (1)$$

where  $l_P$  is Planck's length and the requirement  $\delta S^{tot} \geq 0$  for all processes is widely believed (we stress the functional dependence and neglect factors of the order of unity, where appropriate; entropy is measured in natural units). By dumping matter into a black hole any loss of information or decrease of degrees of freedom in the outside world is stored in the event horizon. The existence of an upper bound for the entropy or information capacity of any object of total energy  $E$  and maximal size  $r$  was suggested by Bekenstein [9, 8]

$$S_B = \frac{Er}{\hbar c} = \frac{r_g r}{l_P^2} \quad (2)$$

where  $r_g$  is the gravitational radius. Weaker bounds were proposed from the holographic principle [10, 11], which suggests that the degrees of freedom of a spatial region reside in the boundary, and were illuminated in view of quantum information theory [3] and cosmological implications [12, 13, 14]. Estimates of the maximal gravitational entropy for the visible universe within a Hubble radius are found from the area of the horizon  $A_H$  as  $S \sim (A_H/l_P)^2 \sim 10^{122}$  [12, 15, 16] and we note that in de Sitter space also the cosmological constant  $\Lambda$  is quantized in terms of  $l_P$  as  $S = N = (l_P^2 \Lambda)^{-1} \sim 10^{122}$ , constraint by the Friedmann-Robertson-Walker cosmology [17].

We rewrite the GSL for an ensemble of  $N$  equal black holes of mass  $m$  localized in a universe of mass  $M$  and size  $R$  subject to critical density with  $M \sim R$  as

$$S^{tot} = NS^{int} + S^{ext} = N\left(\frac{r_g}{l_P}\right)^2 + \left(\frac{R}{r_g}\right)^2 = Nn^2 + N^2 \quad (3)$$

Equation (3) provides the link to ordinary bound systems as outlined below and the entropy bound (2) is recovered for  $r_g \rightarrow l_P$  or  $r_g \rightarrow R$  with  $(R/l_P)^2 \gg 1$ . Here consistency requires that the external entropy contribution is measured in terms of black hole units, the scale available in the system  $R$ , rather than in Planck units. If spacing of  $R$  would be performed in terms of  $l_P$  then the maximum information capacity  $(R/l_P)^2$  of the system would be stored in  $S^{ext}$  only, a contradiction to the information capacity of the individual black holes and the GSL. We argue therefore that the area spacing of closed systems must be performed with respect to available scales of closed subsystems.

The maximum information a system can hold knowing in detail it's configuration corresponds to the maximum entropy when knowing nothing about it's internal states

[3]. Hence, gravitational entropy of a closed system can be understood in view of a surface hiding the information content of the internal region of spacetime where it's value measures the missing information. We generalize this interpretation normally attributed to black hole entropy to ordinary gravitationally bound systems. For instance, the exact configuration of a globular cluster with respect to it's gravitating quanta, the stars as next lower order subsystems, can be identified from the outside world - an observer located in another cluster - as lack of knowledge of the internal configuration. According to the correspondence between information and entropy we adopt this as the gravitational entropy contribution of the inhomogeneity cluster with regard to the subsystems stars. On the same level we may refer to entropy in a space region as to a quantity representing the degrees of freedom within this region [18] where a gravitationally bound  $N$ -body system is subject to  $N(N-1)/2$  constraints represented by the links. 'Gravitational information' flows along all links between any pair of constituents of the cluster and by knowing all links we would account also for the missing information within the system. In particular, this view reflects the causal set context where links are the building blocks of the system and knowledge of the links between the elements of the causet is equivalent to the knowledge of the whole causal set [19, 20, 21]. Counting links with respect to the horizon was found to be proportional to the horizon area in the black hole context [21]. But what are closed systems in a gravitational sense if we do not refer to black holes? We define a gravitationally bound system only by the number of links representing the interaction between any pair of subsystems, equivalent to the number of constraints characterizing the degrees of freedom of the system. Tracing closed systems hierarchically by links between substructures was recognized by Leubner [6, 22] to result in a unique configuration when applying naturally involved extremal conditions constraining the gravitational entropy evolution due to formation of structure in the universe [23].

Consider now an ensemble of  $N_i$  uniformly distributed structures of species ( $i$ ) and mass  $m_i$  in a universe, stars or galaxies for instance, subject to gravitational interaction only. Let the system be rearranged due to density perturbations into  $N_{i+1}$  clusters of mass  $m_{i+1}$ , globular star clusters or clusters of galaxies denoted as species ( $i+1$ ) of equal richness  $n_i$  in view of their next lower order substructures ( $i$ ). Since each member of a gravitationally bound system is affected by all other constituents I propose the number of links  $n_i(n_i-1)/2$  between all members of a cluster to serve as measure of the gravitational entropy of the bound  $n_i$ - body system. The redistribution into  $N_{i+1}$  clusters causes a next higher order gravitational interaction between these systems as new units represented by  $N_{i+1}(N_{i+1}-1)/2$  links. Consequently, the entire system is subject to a natural separation between internal and external degrees of freedom. This is just what nature demonstrates since mutual interaction of substructures of spatially separated bound macrosystems is not realized. In this context the total gravitational entropy contribution  $\delta S_{i+1}$  as measure of the gain of inhomogeneity within the two level system is provided by

$$\delta S_{i+1} = N_{i+1}n_i(n_i - 1)/2 + N_{i+1}(N_{i+1} - 1)/2 \quad (4)$$

Applying large occupation numbers by  $n_i, N_{i+1} \gg 1$  and introducing a mass  $M$  of the entire system under consideration yields with  $m_{i+1} = n_i m_i$

$$\delta S_{i+1} \sim N_{i+1} n_i^2 + N_{i+1}^2 \sim N_{i+1} (m_{i+1}/m_i)^2 + (M/m_{i+1})^2 \quad (5)$$

a quantity proportional to the mass squared as required with regard to the black hole limit where equation (3) is recovered.

### 3 Hierarchy of structure scales

Upon generalization a hierarchically growing clustering process [24], constraint by elementary grouping principles, can be formulated in a universe  $\{G_0\}$ , filled initially by a specific number of particles  $\{g_0\}$ , see Figure 1. Let a clustering procedure with regard to a certain structure level ( $i$ ) result in a sequence of hierarchically nested sets of higher order clusters ( $\dots \{G_{i-1}\}, \{G_i\}, \{G_{i+1}\}, \{G_{i+2}\}, \dots$ ) where the members  $G_i$  of any specific level admit equal richness, a constraint discussed below. Substructures  $G_i$  sequentially ( $n$ - times) merge into higher order systems  $G_{i+n}$  finally approaching the root, a universe identified as cluster with one element. The two fundamental questions to be answered are: (*i*) do clusters of a common structure level admit nearly equal richness with regard to their members, which raises the question of the emergence of equal structure scales at the same cosmic time; and if confirmed: (*ii*) how many substructures merge in average forming sets of nearly equal occupation numbers of a next higher order structure level, which raises the question of the richness of equal structure scales with regard to their building blocks.

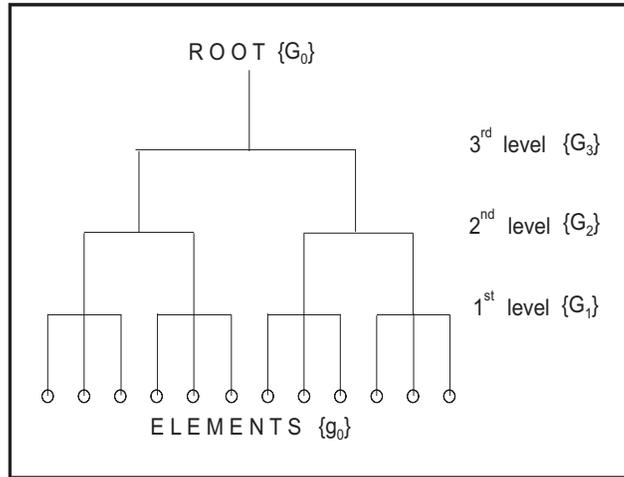


Figure 1: A hierarchical tree with 3 structure levels between the ground state and the root where the members of a common level admit equal richness.

In view of the unknown occupation numbers we verify the only feasible configuration where the gravitational entropy increase within any two levels of cluster formation ( $i \rightarrow i + 1$ ) approaches successively at each higher order merging process an extremum:

(i) the sum of internal contributions  $\delta S_{i+1}^{int} = N_{i+1} S_i^{int} \sim N_{i+1} n_i^2$  is subject to a *minimum* if all clusters  $\{G_{i+1}\}$  have the same occupation number  $n_i$  of elements  $\{G_i\}$ . This follows immediately from the quadratic appearance of  $n_i$  denoting the links between the elements together with particle conservation with regard to  $N_i$  and yields the equal richness condition for bound systems belonging to the same structure level as

$$N_i = N_{i+1} n_i \quad (6)$$

wherefore the assumption in Figure 1 is justified.

(ii) upon generalizing equation (4) in view of a nested hierarchy of structure levels the total entropy gain  $\delta S_{i+1}$  at any transition from some level ( $i$ ) to ( $i + 1$ ) is defined by the condition

$$\delta S_{i+1} = N_{i+1} n_i (n_i - 1)/2 + N_{i+1} (N_{i+1} - 1)/2 \implies \textit{extremum} \quad (7)$$

again naturally satisfying a *minimum*. Inserting from equation (6) for  $n_i$  where  $N_i$  characterizes the prearranged structure level ( $i$ ), and is therefore a frozen in constant of the system with regard to higher order merging processes, we can solve for  $N_{i+1}$  to arrive at a simple recurrence condition for the quantity of identical new inhomogeneities at level ( $i + 1$ ) as

$$N_{i+1} = (N_i^2/2)^{1/3} \quad (8)$$

where the limit of large occupation numbers  $N_{i+1}^3 \gg N_{i+1}^2$  was used. Relation (8) can be written with regard to (6) also in terms of the individual cluster occupation number as  $n_{i+1} = n_i^{2/3}$ .

As natural ingredient of the entropy concept condition (7) *minimizes* the total *increase* in gravitational entropy due to a merging process from any level  $i$  to level  $i + 1$ . In other words, as consequence of the separation this condition minimizes the sum of all internal constraints with respect to the microsystems (building blocks) of individual clusters and the external constraints between all clusters, driving the entire environment into a state of highest degree of autonomy. Any other configuration with respect to both directions (many new systems with few members / few new systems with many members where the limit of only one system where all particles are linked reproduces to the original system) results in a distribution of increased gravitational coupling between equal members, equivalent to enhanced gravitational entropy contribution due to clumping of structure from level  $i$  to level  $i + 1$ . The entropy gain is successively reduced at each higher order gravitational merging process and saturates. These are the the key issues of conditions (6), (7) and (8), respectively.

Let me introduce now physical observables and define an upper mass bound by  $M = N_i m_i$ , equivalently applicable at any structure level  $i + n$ . Upon substitution into equation (8) with regard to the proper indices, the recurrence relation for the mass scales of inhomogeneities permitted by the entropy constraint reads

$$m_{i+1} = (2m_i^2 M)^{1/3} \quad (9)$$

Approximate radii  $r_i$  of structure scales may be found from a basic principle of statistics for the mean error of spatial uncertainties of an ensemble of  $n_i$  particles within a volume of dimension  $r_{i+1}$  from  $r_i = r_{i+1}/\sqrt{n_i}$ , a relation interpretable also as area quantization in view of the holographic principle. This yields an invariant effective for any fundamental structure scale as

$$\frac{m_i}{r_i^2} = \frac{m_{i+1}}{r_{i+1}^2} = \dots = \frac{M}{R^2} = \Sigma_0 = \text{const.} \quad (10)$$

and we note already here that significant support for a functional dependence  $m \propto r^2$ , a constant surface density  $\Sigma_0$  for astrophysical objects was found on observational grounds, see section 4. After combining relations (9) and (10) the appropriate condition for the spatial dimensions of inhomogeneities subject to the entropy constraint reads

$$r_{i+1} = (\sqrt{2}r_i^2 R)^{1/3} \quad (11)$$

Finally, in view of astrophysical cluster analysis [25] we characterize by means of the richness  $n_i$  a cluster halo by the mean separation of neighbor clusters in recurrence notation as

$$d_i = \frac{2r_{i+1}}{n_i^{1/3}} \quad (12)$$

## 4 Observational test and discussion

A simple set of recurrence relations determines a global sequence of inhomogeneity scales in a predefined Hubble volume from gravitational entropy restrictions. Type Ia supernovae and microwave background observations support presently a flat, accelerating universe of critical density where the density parameter  $\Omega$  splits as  $\Omega = 0.7\Omega_q + 0.3\Omega_m = 1$  into a dark energy (quintessence) component and a matter contribution where a Hubble parameter of  $H_0 = 70 \text{ km sec}^{-1} \text{ Mpc}^{-1}$  is typically favored [26]. For an Einstein de Sitter universe the set of equations determining the sequential growth of structure scales can be solved by introducing Hubble's parameter only. This is a consequence of a natural ingredient of the proposed entropy concept since Planck's length, defining as lowest spatial bound the limit of information capacity at  $r_0 = l_P$ , provides a starting value, which generates a sequence that accurately reproduces observations on astrophysical scales. The

constant surface density on the right hand side of equation (10) is of the order of unity and available from the critical density wherefore also the starting value of the mass sequence is found as  $m_0 = l_P^2 \Sigma_0$ . With regard to equation (10) the quantity  $S_0 = N_0 = M/m_0 = (R/l_P)^2 = c^5/(\hbar H_0^2 G) = 6.7 \times 10^{121}$  is obtained as initial condition for equation (8) and corresponds to the current value of the Bekenstein-Hawking entropy of the universe inside a Hubble radius, identical to the quantization condition for the cosmological constant  $\Lambda$ , introduced in section 2. Furthermore, with the use of Plank's mass  $m_P$  a recently proposed bound on  $H_0$  from the largest geometric entropy per Hubble volume is reproduced as  $H_0 \leq m_P c^2 / \hbar \sqrt{N_0}$  [27]. According to equation (7) the proportionality  $S \sim N^2$  appears to generate a contradiction to the maximum entropy content within a Hubble volume with regard to the first two structure levels. This is resolved naturally by applying the term 'gravitational entropy' only for gravitational interaction with the restriction to mass scales above Planck's mass  $m > m_P$ , or  $N < \sqrt{N_0}$  and we keep below the required proportionality of radiation entropy  $S \sim N$ , which reflects also the transition from a radiation dominated to a matter dominated universe.

Table 1: Scaling properties of fundamental structures

<b>class</b>	<b><math>m_i</math>[g]</b>	<b><math>r_i</math>[cm]</b>	<b><math>d_i</math>[cm]</b>	<b><math>N_i</math></b>	<b>object</b>
$g_0$	$2.7 \times 10^{-66}$	$1.6 \times 10^{-33}$	$2.0 \times 10^{-26}$	$6.7 \times 10^{121}$	Planck scale
$G_1$	$1.3 \times 10^{-25}$	$3.6 \times 10^{-13}$	$2.4 \times 10^{-8}$	$1.3 \times 10^{81}$	hadronic matter
$G_2$	$1.9 \times 10^2$	$1.4 \times 10^1$	$2.8 \times 10^4$	$9.5 \times 10^{53}$	condensed matter
$G_3$	$2.3 \times 10^{20}$	$1.5 \times 10^{10}$	$3.1 \times 10^{12}$	$7.7 \times 10^{35}$	planetesimals
$G_4$	$2.7 \times 10^{32}$	$1.6 \times 10^{16}$	$7.1 \times 10^{17}$	$6.6 \times 10^{23}$	stellar systems
$G_5$	$2.9 \times 10^{40}$	$1.7 \times 10^{20}$	$2.7 \times 10^{21}$	$6.0 \times 10^{15}$	globular clusters
$G_6$	$6.7 \times 10^{45}$	$8.1 \times 10^{22}$	$6.4 \times 10^{23}$	$2.6 \times 10^{10}$	galaxies
$G_7$	$2.5 \times 10^{49}$	$5.0 \times 10^{24}$	$2.5 \times 10^{25}$	$7.0 \times 10^6$	galaxy clusters
$G_8$	$6.1 \times 10^{51}$	$7.7 \times 10^{25}$	$2.8 \times 10^{26}$	$2.9 \times 10^4$	superclusters
–	–	–	–	–	–
$G_0$	$1.8 \times 10^{56}$	$1.3 \times 10^{28}$	$1.3 \times 10^{28}$	1	the universe

Based on  $H_0$  some instructive parameters of the predicted global sequence of structure scales are presented in Table 1 and illuminated in Fig. 2. When testing a global structure quantization subject to ten discrete inhomogeneity scales within 120 orders of magnitude the question of mixing up objects with and without dark matter seems to be of no relevance. On the other hand, comparing the derived astrophysical mass scales with observations infers that the provided values must be identified as total mass of the specific inhomogeneity scale, including dark matter contributions on astrophysical scales. In this respect it is also possible to deduce from the results of the gravitational entropy concept and the knowledge of luminous matter the amount of dark matter mass contributions of

a certain astrophysical structure scale where table 1 also suggests to identify  $r_i$  with the luminous matter distribution and  $d_i$  with the halo dimensions. Moreover, the solution should be regarded as representative at timescales directly after relaxation and virialization of a specific structure disregarding subsequent evolutionary aspects as discussed recently in view of star clusters [28]. Only a brief outline with basic referencing for readers not familiar with the status regarding all different structure scales can be provided here to illuminate the proposed entropy concept in view of theoretical and observational evidence.

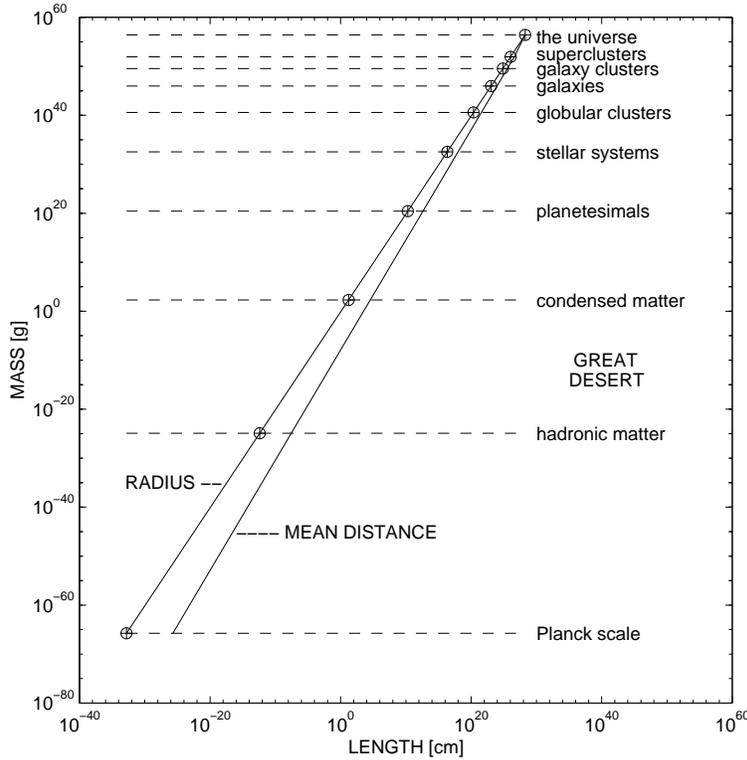


Figure 2: A schematic illustration of the hierarchy of structure scales. Radius and mean distance are obtained from equations (11) and (12).

It is reasonable to assign the **ground state**  $\mathbf{g}_0$  to a gravitational background [29] to which any bound system is coupled. Constraints on the mass are available from gravitational wave observations [30] and arguments were provided that gravity can appear due to polarization of instantons in the  $SO(4)$  gauge theory where their radius is comparable to Planck's length  $l_P = r_0$  in table 1 [31]. Recent attempts to explain the non-vanishing small value of the vacuum energy density favor a quintessence background as alternative to a cosmological constant [32, 23, 26]. This is supported by the starting value of the mass sequence  $m_0$ , which coincides with the acquired field mass necessary to dominate the current energy density of the universe [33]. The transition from the ground state to

hadronic matter reflects the ratio of strong to gravitational interaction where ground state hadrons as representatives of the **elementary particle physics scale**  $G_1$  are subject to a quark confinement length  $r_1$ . Introducing by  $\lambda_n = r_1$  the neutron Compton wavelength, Dirac's hypothesis for the particle number in the universe  $N_1 = (R/\lambda_n)^2$  [34] is a consequence of the proposed approach and also the popular Weinberg coincidence [35] for the pion mass  $m_\pi^3 \simeq \hbar^2 H_0/cG$  turns out as implicit content. Hence, the mystery of large numbers appears as simple content of the proposed global quantization concept. Proceeding through a 'great desert', predicted by force unification theories, to the structure scale  $G_2$  a mass density of the order of unity is predicted. In view of an assignment to a specific bound system evidently a high degree of structure variety is today available for 'clusters of atoms' denoted here as **condensed matter**. Interestingly, upon calculating the mean separation of the constituents of  $G_1$ , protons for instance, with respect to the characteristic cluster domain  $r_2$ , from (12) a value  $d_1 \simeq 10^{-8}cm$  of the order of Bohr's radius is found. This implies in conjunction with the entropy constraint the existence of a representative structure scale with a mass density of the order of unity, bound on atomic dimensions where prestellar grain may serve as possible candidate in view of the evolutionary history of the universe [36, 37].

On intermediate scales **planetesimals**  $G_3$ , comets and asteroids play a key role in theories of the evolution of the solar system [36, 37]. Within the formation of a massive extended protoplanetary disk, the development of a large number of comets and planetesimals with individual masses of the order of  $m_3 \simeq 10^{20}g$  provides the link between the phase of condensed matter  $G_2$  and stellar systems  $G_4$ . A protoplanetary disk of solar mass  $m_4 \simeq 10^{33}g$  and a radius of  $10^{14} - 10^{15}cm$  can contain a number of  $n_3 \simeq 10^{13}$  planetesimals. Referring to evolutionary theories, a marginally unstable cloud of solar mass  $m_\odot \simeq 10^{33}g$ , destined to form a **stellar system**  $G_4$  has a radius of the order of  $10^{17}cm$ , where the disk has a limiting lower mass of typically  $m \simeq 0.05m_\odot$ . Table 1 reflects all values sufficiently and is supported observationally also by the orbital characteristics of solar system comets and Kuiper populations defining the edge of the bound solar system [36, 38]. Next, from the entropy constraint a structure scale of the dimension of **globular clusters**  $G_5$  is predicted where observed radii can be averaged at about  $35pc \simeq 1 \times 10^{20}cm$  reaching up to  $100pc$  surrounded by large dark matter haloes [39]. Remarkably, old globular clusters were well fitted by a functional form  $m \sim r^2$  for their upper mass where a discrepancy with respect to young clusters was suggested to indicate different formation histories or later evolutionary effects [28]. This recent observational development where a cutoff mass  $m_c$  was found in the range  $10^6 M_\odot \leq m_c \leq 5 \times 10^6 M_\odot$  clearly supports the constant surface density (10) as limiting condition, indicating also that the presented results are representative for timescales just after the formation of a specific structure, disregarding evolutionary changes. The outermost halo-globulars of the galaxy are found at about  $100kpc$  from the galactic center indicating the galactic boundary, a value provided also from dark matter halo studies for galaxies [40].

At large astrophysical scales, the optically visible cores of representative members of **galaxies**  $G_6$  have mean radii  $r_6$  in the range of 20 kpc with a dominant dark matter mass of  $2 \times 10^{12} m_\odot$ . In average the halo mass distribution extends to about 200 kpc [40, 41], wherefore galaxies can be identified safely as structure level 6 of the hierarchy. **Clusters of galaxies**  $G_7$  are the largest virialized inhomogeneities in the universe [25] with a total mass estimated up to  $10^{16} m_\odot \simeq 10^{49} g$  and the dominant dark matter is distributed within a halo of about  $5 Mpc \simeq 10^{25} cm$ . Galaxy cluster formation is dated after the evolution of galaxies supporting the hierarchical scenario [42] where already early studies have indicated a constant surface density from N-body simulations [43, 44]. Observations of the distribution of **superclusters**  $G_8$  identify a network structure of scales up to  $(100 - 150) Mpc \simeq (3 - 4.5) \times 10^{26} cm$ , irregular in shape since not fully virialized. The supercluster network is surrounded by low density regions of similar scale, suggesting a cellular structure of the universe at such dimensions. Superclusters contain a fraction of 20 – 100 clusters with a mean mass estimated in the range of  $m_8 \simeq 5 \times 10^{50} g$  [42] and a separation scale of the order of their proper size implying that the predictions of the gravitational entropy approach are sufficiently supported by observations also on largest scales yet observed.

In further steps, the sequence converges to the **universe**  $G_0$  as  $N_i \Rightarrow 1$ , where equation (7) must be solved exactly and where the definition of a virialized bound system as unique cluster is not applicable anymore. The gravitational entropy constraint predicts the emergence of ten fundamental inhomogeneity scales in the universe including the groundstate  $(m_0, r_0)$  and a universe  $(M, R)$  as bounds, a result determined between  $10^{122} \geq N_i \geq 1$ .

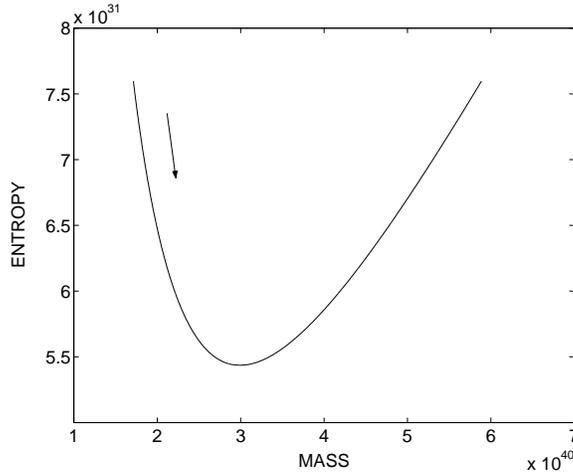


Figure 3: The mass evolution for globular clusters in view of the minimum gravitational entropy condition.

The proposed gravitational entropy concept extracts the only configuration of a system where the increase in entropy or information capacity due to higher order clumping

of inhomogeneities is minimized. Fig. 3 demonstrates the functional entropy-mass dependence, as obtained from (7) after substituting for the mass, with regard to the evolution of the globular cluster scale as example. Due to clumping of structure the entropy of the preceding level drops down as indicated by the arrow reaching the minimum in a relaxed state at a cluster cutoff mass  $m_5 = 2.9 \times 10^{40} g$ , supported by observations, where further mass aggregation would result again in a state of increased entropy. A minimum growth of entropy guarantees a configuration of maximum possible inhomogeneity at each structure level. Adding all entropy contributions of all degrees up to level  $i + 1$  yields

$$S^{tot} = \sum_i [N_{i+1}n_i(n_i - 1) + N_{i+1}(N_{i+1} - 1)], \quad \delta S^{tot} > 0 \quad (13)$$

indicating that gravitational entropy is increasing in a quantized manner at each higher order inhomogeneity level and defines thermodynamically consistent a gravitational master arrow of time. This one-way character allows spacetime to develop into states of increasingly nested higher order structure scales.

Increasing inhomogeneity due to gravitational clumping reflects growth of gravitational entropy in an evolving universe. In this respect a measure of gravitational entropy was introduced reproducing the GSL of black hole dynamics in the limit. The gain of gravitational entropy at each higher order merging process is suggested to result from an extremal condition, requiring a minimum increase of entropy. The proposed approach reproduces the observed global inhomogeneity scales where astrophysical scales, subject to gravitational interaction, are linked to the particle physics and Planck's scale within one unique concept. Thermodynamic equilibrium at the big bang requires the entropy to approach about it's maximum value  $S_{\max}$ . Since any gravitational contribution at some level  $m_i > m_P$  is subject to  $S_i \ll S_{\max}$  the entropy paradox appears to be resolved. Consistent with the thermodynamic view a gravitational master arrow of time can be defined that points in the direction of increasing entropy or inhomogeneity, generating a one-way character of the future. The underlying gravitational entropy concept implies in view of fundamental structure scales that we live in a universe of maximum autonomy.

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