Dark energy from vacuum entanglement

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We suggest that vacuum entanglement energy associated with the entanglement entropy of the universe is the origin of dark energy. The observed properties of dark energy can be explained by using the nature of entanglement energy without modification of gravity or exotic matter. From the number of degrees of freedom in the standard model, we obtain the equation of state parameter $\omega_\Lambda^0 \simeq -0.93$ and $d \simeq 0.95$ for the holographic dark energy, which are consistent with current observational data at the 95% confidence level.

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The cosmological constant problem is one of the most important unsolved puzzles in modern physics [1]. There is strong evidence from Type Ia supernova (SN Ia) observations [2] that the universe is expanding at an accelerating rate. A simple explanation for this acceleration is the existence of negative pressure fluids, called the dark energy, whose pressure p_Λ and density ρ_Λ satisfy $\omega_\Lambda \equiv p_\Lambda/\rho_\Lambda < -1/3$. (See Eq. (16)). Although, there are various dark energy models rely on materials such as quintessence [3], k-essence [4], phantom [5], and Chaplygin gas [6] among many, the identity of this dark energy remains a mystery. These models usually require fine tuning of potentials or unnatural characteristics of the materials. On the other hand, entanglement (a nonlocal quantum correlation) [7] is now treated as an important physical quantity. The possibility of exploiting entanglement in quantum information processing applications such as quantum key distribution and quantum teleportation has led to intense study of this quantity by the quantum information community. Recently, there has been renewed interest [8, 9, 10] in studying black hole entropy using entanglement entropy [11, 12] in the context of the AdS/CFT correspondence [13]. In this paper, we suggest that there is an unexpected relation between dark energy and entanglement which are the two most puzzling entities in modern physics.

It is well known [14] that a simple combination of the Planck scale and IR cutoff L (of order of inverse of the Hubble parameter H) gives an energy density comparable to the observed cosmological constant or dark energy. This can be understood in terms of the holographic principle proposed by 't Hooft and Susskind [15], which is a conjecture claiming that all of the information in a volume can be described by the physics at the boundary of the volume and that the maximum entropy in a volume is proportional to its surface area. Cohen et al [16] proposed a relation between the UV cut-off l of an effective theory and L by considering that the total energy in a region of size L can not be larger than the mass of a black hole of that size. Thus, for $L = H^{-1}$, the zero-point vacuum energy density is bounded as

$$\rho_\Lambda = l^{-4} \lesssim \frac{M_P^2}{L^2} = M_P^2 H^2. \tag{1}$$

Interestingly, saturating the bound gives ρ_Λ comparable to the observed dark energy density $\sim 10^{-10} eV^4$ for $H = H_0 \sim 10^{-33}\ eV$, the present Hubble parameter. The success of this estimation over the naive estimate $\rho_\Lambda = O(M_P^4)$ can be attributed to the fact that quantum field theory over-counts the independent physical degrees of freedom inside the volume. Thus, dark energy models based on the holographic principle have an advantage over other models in that they do not need an *ad hoc* mechanism to cancel the $O(M_P^4)$ zero-point energy of the vacuum. This simple holographic dark energy model is suggestive, but not without problems of its own. Hsu [17] pointed out that for $L = H^{-1}$, the Friedmann equation $\rho = 3M_P^2 H^2$ makes the dark energy behave like matter rather than a negative pressure fluid, and prohibits accelerating expansion of the universe. Later, Li [18] suggested that holographic dark

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energy of the form

$$\rho_\Lambda = \frac{3d^2 M_P^2}{R_h^2}, \quad (2)$$

would give an accelerating universe, where the future event horizon (R_h) is used instead of the Hubble horizon as the IR cutoff L. Here d is an $O(1)$ constant. However this use of R_h has yet to be adequately justified. Attempts [19, 20, 21, 22] have been made to overcome this IR cutoff problem in other ways, for example, by using non-minimal coupling to a scalar field [20, 21] or interaction between dark energy and dark matter [22]. Despite some success, the holographic dark energy models usually lack either an explanation for the microscopic origin of the dark energy or an explanation for why d, the constant that determines the characteristics of the dark energy, is approximately one.

In this paper we propose that these problems can be overcome in a natural manner by identifying dark energy as entanglement energy associated with the entanglement entropy S_{Ent} of the universe. Our model also suggests a way to derive d and ω_Λ from the standard model of particle physics.

From the Reeh-Schlieder theorem [23] it is known that the vacuum for general quantum fields violates Bell inequality and has entanglement [24, 25] when there are causally disconnected regions. Entanglements of Bose [26] and Fermi [27] states have been studied using a thermal Green's function approach. In Ref. [28] it was suggested that the Hadamard Green's function representing quantum fluctuation of the vacuum is useful for the study of entanglement in a scalar field vacuum. These relations between vacuum quantum fluctuations and entanglement are reminiscent of the vacuum fluctuation model of dark energy [29].

There are two natural physics related to the event horizon; black hole physics and entanglement physics. However, identifying R_h as a black hole horizon is problematic, because dark energy should not include ordinary matter energy, while black hole energy includes all the energy inside the horizon. In quantum information theory, the event horizon plays a role of an information barrier and this leads to modification of energy of subsystem inside the horizon, which is the entanglement energy. Therefore, the vacuum entanglement energy is a remaining plausible candidate for holographic dark energy. The entanglement entropy is the von Neumann entropy $S_{Ent} = -Tr(\rho_A log \rho_A)$ associated with the reduced density matrix $\rho_A \equiv Tr_B \rho_{AB}$ of a bipartite system AB described by a density matrix ρ_{AB} [7]. For pure states such as the quantum fields vacuum, S_{Ent} is a good measure of entanglement. When there is an event horizon, a natural choice is to divide the system into two subsystems - inside and outside the event horizon - and to trace over one of these subsystems to calculate the entanglement, because the event horizon represents the global causal structure [30]. Thus, S_{Ent} is intrinsically related to the event horizon rather than the particle horizon or the Hubble horizon. The future event horizon is given by

$$R_h \equiv R(t) \int_t^\infty \frac{dR(t')}{H(t')R(t')^2}, \quad (3)$$

which can be used as a typical length scale of the system with the horizon. Here we consider the flat ($k = 0$) Friedmann universe which is favored by observations [31] and inflationary theory [32] and described by the metric

$$ds^2 = -dt^2 + R^2(t)d\Omega^2, \quad (4)$$

where $R(t)$ is the scale factor as usual. The entanglement entropy of the quantum field vacuum with a horizon is generally expressed in the form

$$S_{Ent} = \frac{\beta R_h^2}{a^2}, \quad (5)$$

where β is an $O(1)$ constant that depends on the nature of the field. Here, a is the UV cut-off of quantum gravity and different from l which is the UV cut-off of a low energy effective theory [16] (see below for details). S_{Ent} has a form consistent with the holographic principle, although it is derived from quantum field theory without using the principle. Entanglement entropy for a single massless scalar field in the Friedmann universe is calculated in Ref. [33, 34]. By performing numerical calculations on a sphere lattice, they obtained $\beta = 0.30$. If there are N_{dof} spin degrees of freedom of quantum fields in R_h, due to the additivity of the entanglement entropy [7], we can add up the contributions from all of the individual fields to S_{Ent} [33], that is, $S_{Ent} = N_{dof} \beta R_h^2/a^2$, where for simplicity we assume the same β for all fields.

In [35] the entanglement energy E_{Ent} is defined as disturbed vacuum energy due to the presence of a boundary. There, entanglement energy proportional to the radius of the spherical volume was derived from quantum field theory. Thus, for the event horizon, the entanglement energy is generally given by

$$E_{Ent} = \alpha R_h, \quad (6)$$

where α is a constant depending on the exact mathematical definition of E_{Ent}. We suggest that this entanglement energy is the origin of dark energy. Once we obtain ρ_Λ from E_{Ent}, the negative pressure p_Λ can be derived from the conservation of energy momentum tensor,

$$p_\Lambda = \frac{d(R^3\rho_\Lambda)}{dR(-3R^2)} \tag{7}$$

as usually done in holographic dark energy models (see Eq. (6) of Ref. [18]). Recall that this equation can be derived from the Freedmann equation with perfect fluid having a energy momentum tensor of the form

$$T_{\mu\nu} = (\rho_\Lambda + p_\Lambda)U_\mu U_\nu - p_\Lambda g_{\mu\nu}, \tag{8}$$

where $U^\mu U_\mu = 1$. Eq. (7) indicates that perfect fluid with increasing energy as the universe expands has a negative pressure. Then, it is straight forward to obtain $\omega_\Lambda = p_\Lambda/\rho_\Lambda$ (see Eq. (13)). We will show below that our theory gives the desired form of holographic dark energy. Thus, we can use the all known formalism of typical holographic dark energy models for our model.

Now let us determine the coefficient α in Eq. (6). Although the mathematical definition of entanglement energy is not well-established, there are several reasonable conjectures for E_{Ent} in Ref. [35, 36]. Inspired by the holographic principle, we adopt the following definition among them:

$$dE_{Ent} \equiv T_{Ent} dS_{Ent}. \tag{9}$$

Note that this is *not* the first law of thermodynamics for E_{Ent} which needs a pressure term *but* a mere definition of E_{Ent} we choose in this paper. In [35], it was shown that this definition for E_{Ent} is good for black holes. Our entanglement energy in Eq. (9) is this modified vacuum energy and hence "internal" energy which looks like some "thermal energy" related to entanglement entropy. To calculate E_{Ent}, the most natural choice for the "temperature" related to the event horizon is the Gibbons-Hawking temperature $T_{Ent} = 1/(2\pi R_h)$ [37, 38, 39]. By integrating dE_{Ent} we obtain

$$E_{Ent} = \frac{\beta N_{dof} R_h}{\pi a^2}. \tag{10}$$

From Eq. (6) and Eq. (10), we see $\alpha = \beta N_{dof}/\pi a^2$. Then, the entanglement energy density within the event horizon is given by

$$\rho_\Lambda = \frac{3E_{Ent}}{4\pi R_h^3} = \frac{3\beta N_{dof}}{4\pi^2 a^2 R_h^2} \equiv \frac{3d^2 M_P^2}{R_h^2}, \tag{11}$$

which has the form (Eq. (2)) for the holographic dark energy. From the above equation we immediately obtain a formula for the constant

$$d = \frac{\sqrt{\beta N_{dof}}}{2\pi a M_P} \tag{12}$$

for the first time. Although the constant d determines the characteristic of the dark energy and the final fate of the universe, it has been constrained only by observations so far.

Interestingly, our model can be easily verified by current observations. The equation of state for dark energy of the form in Eq. (1) is as follows [30, 40]

$$\omega_\Lambda = -\frac{1}{3}\left(1 + \frac{2\sqrt{\Omega_\Lambda}}{d}\right), \tag{13}$$

where Ω_Λ is the density parameter of the dark energy. Now, by inserting the expression for d in Eq. (12) into the above equation, we obtain ω_Λ directly from the number of spin degrees of freedom N_{dof} in the standard model(SM):

$$\omega_\Lambda = -\frac{1}{3}\left(1 + \frac{4\pi a M_P\sqrt{\Omega_\Lambda}}{\sqrt{\beta N_{dof}}}\right). \tag{14}$$

Since $aM_P \simeq 1$, $\beta \simeq 1$, and $N_{dof} = O(10^2)$, Eqs. (12) and (13) gives us $d \simeq 1$ and $\omega_\Lambda \simeq -1$. Thus, the above calculation explains why $d \simeq 1$ from a particle physics view point. More precisely, we choose natural values $a = 1/M_P$, $\beta = 0.3$, the dark energy density parameter for the present $\Omega_\Lambda = 0.73$ and the matter density parameter

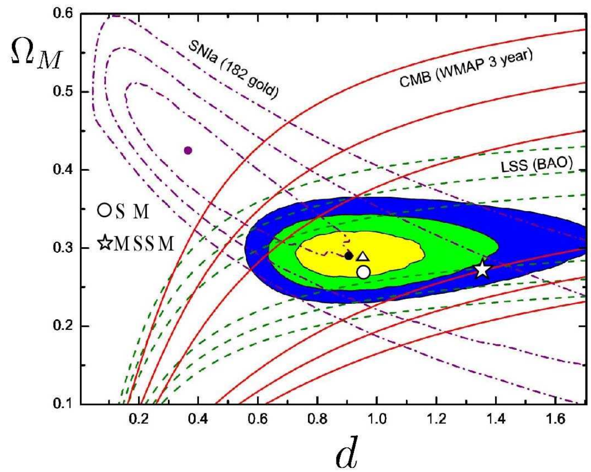

FIG. 1: Allowed parameter region for d and Ω_M from SNIa+CMB+SDSS joint analysis done by Zhang and Wu (Fig. 2 of [42]). The bright region at the center corresponds to the region within the 1 σ confidence level contour. The black dot denotes the best-fit point from the observations. The white dot represents our theoretical prediction for the SM and the star for the MSSM with $\Omega_M = 0.27$. The triangle denotes our prediction with $\Omega_M = 0.29$ for the SM. (Courtesy of F. Wu)

$\Omega_M = 0.27$ favored by recent observations [41]. Using $N_{dof} = 118$ for the SM, we obtain $d \simeq 0.95$ and $\omega^0_\Lambda \simeq -0.93$ for the present. Remarkably, this theoretical value for ω^0_Λ is consistent with current observational data from SN Ia, the cosmic microwave background (CMB), and the Sloan Digital Sky Survey (SDSS) [40, 42, 43, 44, 45, 46, 47] (see Fig. 9 in [43] and Fig. 15 in [41]) at the 95% confidence level. For example, the combination of 3-year WMAP data and the Supernova Legacy Survey data [41] yields $\omega^0_\Lambda = -0.97^{+0.07}_{-0.09}$, which is in agreement with our prediction, although ω_Λ is assumed to be independent of time in that paper. Very recently, Zhang and Wu [42] perform a joint analysis of constraints on d with the latest observational data including the gold sample of SN Ia, the shift parameter of CMB and the baryon acoustic oscillation (BAO) from the SDSS. This gives $d = 0.91^{+0.26}_{-0.18}$, which contains our value $d \simeq 0.95$ within the 1 σ region (see Fig. 1). Thus, our model well explains observed properties of dark energy. For the minimal supersymmetric standard model (MSSM), $N_{dof} = 244$; this value of N_{dof} gives $d \simeq 1.36$ and $\omega^0_\Lambda \simeq -0.75$, which slightly violates the constraint $\omega^0_\Lambda < -0.76$ from SN Ia data [2].

This result indicates that, for our model, SM degrees of freedom is good for N_{dof} and the Planck length scale is good for the UV cut-off a. The reasons behind this might be as follows. The origin of our entanglement energy is different from the energy of an low energy effective theory considered by Cohen et al's proposal [16] which motivates the usual holographic dark energy models. The entanglement energy is related to quantum information loss at the horizon and to the vacuum quantum fluctuation in quantum gravity theory, which is usually believed as the origin of dark energy or the holographic principle. Thus, the natural UV cut-off of our model is the Planck length as in many related literatures [35, 48]. What can we say about N_{dof}? Considering the Planck scale UV cut-off, it is desirable to use also the degree of freedoms at the Planck scale. This value depends on the model of the unification theory, which varies from $O(10^2)$ to $O(10^3)$ and makes the explicit value of d vary approximately from 1 to 3. However, SM is the only model that is verified by various experiments so far. Therefore, it is still plausible that the degrees of freedom at the Planck scale could be similar to that of SM and we can use SM degrees of freedom for N_{dof}. Even in the case that a larger unification theory (such as string theory) is the true theory, contributions from non-SM fields to vacuum fluctuation might be negligible due to symmetry breaking of those sectors. Although our theory still has some ambiguity to be resolved in these parameters, it is interesting that our theory predicts the observed d value with the Planck scale and SM degrees of the freedom.

To obtain a more precise value of ω^0_Λ, it is essential to calculate the exact value of $\beta = \beta_i$ for every field i in the SM or MSSM. Then β_i and the number of degrees of freedom of the i-th field, N^i_{dof}, should satisfy the relation $\sum_i \beta_i N^i_{dof} = 4\pi^2 d^2$, derived by the same arguments as those leading to Eq. (12). An interesting question here is whether the above equation gives $d = 1$ for the SM. Using Eq. (28) of Ref. [30] one can also obtain the time dependency of the equation of state;

$$\begin{aligned} \omega_\Lambda &= \left(1 + \frac{2\sqrt{\Omega_\Lambda}}{d}\right)\left(-\frac{1}{3} + z\frac{\sqrt{\Omega_\Lambda}(1-\Omega_\Lambda)}{6d}\right) \\ &= -0.93 + 0.11z \end{aligned} \tag{15}$$

for SM, where z is the red shift parameter.

In general, holographic dark energy models including ours tacitly assume the presence of the accelerating expansion of the universe. If not, the holographic dark energy could not be finite. Since the accelerating universe is an observational fact, this assumption is plausible. Alternatively, if we first assume finite S_{Ent} at any finite time, then the accelerating universe is a natural consequence. From Eq. (5), finite S_{Ent} implies a finite R_h. From Eq. (3), it is easy to see that the event horizon exists only when $\int_t^\infty dt'/R(t')$ converges, that is, the universe should accelerate ($R(t) \sim t^n,\ n > 1$) as $t \to \infty$ for R_h to be finite (unless the universe is oscillating) [49]. The accelerating universe satisfies

$$\frac{\ddot{R}}{R} = -\frac{4\pi(\rho + 3p)}{3} = n(n-1)t^{-2} > 0, \tag{16}$$

hence, $\rho + 3p < 0$, i.e., $\omega \equiv p/\rho < -1/3$. Here the dot denotes a time derivative. Thus, the finiteness of S_{Ent} demands a finite R_h and requires that the universe should accelerate and dark energy should dominate as $t \to \infty$.

There are already many scenarios that explain the cosmic coincidence problem in the context of holographic dark energy. For example, in [50] to solve the coincidence problem, an interaction between dark matter [51] and dark energy was introduced. Li suggested inflation at the GUT scale with the minimal number of e-folds of expansion $N \simeq 60$ as a solution [30].

In summary, we suggest a model in which dark energy is identified as the entanglement energy of the universe. This model could explain many observed properties of dark energy without modification of gravity, exotic fields or particles. Using only standard model fields, the holographic principle, and entanglement theory, our model predicts the equation of state and the constant d of dark energy which are well consistent with observations. Our analysis also indicates that the holographic principle and the entanglement theory can play a fundamental role not only in the physics of black holes or string theory but also in cosmology [52, 53, 54].

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