SPACETIME FOAM, HOLOGRAPHIC PRINCIPLE, AND BLACK HOLE QUANTUM COMPUTERS

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Spacetime foam, also known as quantum foam, has its origin in quantum fluctuations of spacetime. Arguably it is the source of the holographic principle, which severely limits how densely information can be packed in space. Its physics is also intimately linked to that of black holes and computation. In particular, the same underlying physics is shown to govern the computational power of black hole quantum computers.

1. Introduction

Early last century, Einstein's general relativity promoted spacetime from a passive and static arena to an active and dynamical entity. Nowadays many physicists also believe that spacetime, like all matter and energy, undergoes quantum fluctuations. These quantum fluctuations make spacetime foamy on small spacetime scales. (For a discussion of the relevant phenomenology and for a more complete list of references, see Ref. 1.)

But how large are the fluctuations? How foamy is spacetime? Is there any theoretical evidence of quantum foam? In what follows, we address these questions. By analysing a gedanken experiment for spacetime measurement, we show, in section 2, that spacetime fluctuations scale as the cube root of distances or time durations. Then we argue that this cube root dependence is consistent with the holographic principle. In section 3, we discuss how quantum foam affects the physics of clocks (accuracy and lifetime) and computers (computational rate and memory space). We also show that the physics of spacetime foam is intimately connected to that of black holes, giving a poor man's derivation of the Hawking black hole lifetime and the area law of black hole entropy. Lastly a black hole computer is shown to compute at a rate linearly proportional to its mass.

2. Quantum Fluctuations of Spacetime

If spacetime indeed undergoes quantum fluctuations, the fluctuations will show up when we measure a distance (or a time duration), in the form of uncertainties in the measurement. Conversely, if in any distance (or time duration) measurement, we cannot measure the distance (or time duration) precisely, we interpret this intrinsic limitation to spacetime measurements as resulting from fluctuations of spacetime.

The question is: does spacetime undergo quantum fluctuations? And if so, how large are the fluctuations? To quantify the problem, let us consider measuring a distance l. The question now is: how accurately can we measure this distance? Let us denote by δl the accuracy with which we can measure l. We will also refer to δl as the uncertainty or fluctuation of the distance l for reasons that will become obvious shortly. We will show that δl has a lower bound and will use two ways to calculate it. Neither method is rigorous, but the fact that the two very different methods yield the same result bodes well for the robustness of the conclusion. (Furthermore, the result is also consistent with well-known semi-classical black hole physics. See section 3.)

Gedanken Experiment. In the first method, we conduct a thought experiment to measure l. The importance of carrying out spacetime measurements to find the quantum fluctuations in the fabric of spacetime cannot be over-emphasized. According to general relativity, coordinates do not have any intrinsic meaning independent of observations; a coordinate system is defined only by explicitly carrying out spacetime distance measurements. Let us measure the distance between two points. Following Wigner[2], we put a clock at one point and a mirror at the other. Then the distance l that we want to measure is given by the distance between the clock and the mirror. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance l. However, quantum uncertainties in the positions of the clock and the mirror introduce an inaccuracy δl in the distance measurement. We expect the clock and the mirror to contribute comparable uncertainties to the measurement. Let us concentrate on the clock and denote its mass by m. Wigner argued that if it has a linear spread δl when the light signal leaves the clock, then its position spread grows to $\delta l + \hbar l (mc\delta l)^{-1}$ when the light signal returns to the clock, with the minimum at $\delta l = (\hbar l/mc)^{1/2}$. Hence one concludes that

$$\delta l^2 \gtrsim \frac{\hbar l}{mc}. \tag{1}$$

General relativity provides a complementary bound. To see this, let the clock be a light-clock consisting of a spherical cavity of diameter d, surrounded by a mirror wall of mass m, between which bounces a beam of light. For the uncertainty in distance measurement not to be greater than δl, the clock must tick off time fast enough that $d/c \lesssim \delta l/c$. But d, the size of the clock, must be larger than the Schwarzschild radius $r_S \equiv 2Gm/c^2$ of the mirror, for otherwise one cannot read the time registered on the clock. From these two requirements, it follows that

$$\delta l \gtrsim \frac{Gm}{c^2}. \tag{2}$$

The product of Eq. (2) with Eq. (1) yields

$$\delta l \gtrsim (l l_P^2)^{1/3} = l_P \left(\frac{l}{l_P}\right)^{1/3}, \tag{3}$$

where $l_P = (\hbar G/c^3)^{1/2}$ is the Planck length. (Note that the result is independent of the mass of the clock and, hence, one would hope, of the properties of the specific clock used in the measurement.) The end result is as simple as it is strange and appears to be universal: the uncertainty δl in the measurement of the distance l cannot be smaller than the cube root of $l l_P^2$.[3] Obviously the accuracy of the distance measurement is intrinsically limited by this amount of uncertainty or quantum fluctuation. We conclude that there is a limit to the accuracy with which one can measure a distance; in other words, we can never know the distance l to a better accuracy than the cube root of $l l_P^2$. Similarly one can show that we can never know a time duration τ to a better accuracy than the cube root of τt_P^2, where $t_P \equiv l_P/c$ is the Planck time. Because the Planck length is so inconceivably short, the uncertainty or intrinsic limitation to the accuracy in the measurement of any distance, though much larger than the Planck length, is still very small. For example, in the measurement of a distance of one kilometer, the uncertainty in the distance is to an atom as an atom is to a human being.

The Holographic Principle. Alternatively we can estimate δl by applying the holographic principle.[4,5] In essence, the holographic principle[6] says that although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram. To be more precise, let us consider a spatial region measuring l by l by l. According to the holographic principle, the number of degrees of freedom that this cubic region can contain is bounded by the surface area of the region in Planck units, i.e., l^2/l_P^2, instead of by the volume of the region as one may naively expect. This principle is counterintuitive, but is supported by black hole physics in conjunction with the laws of thermodynamics, and it is embraced by both string theory and loop quantum gravity. So strange as it may be, let us now apply the holographic principle to deduce the accuracy with which one can measure a distance.

First, imagine partitioning the big cube into small cubes. The small cubes so constructed should be as small as physical laws allow so that we can associate one degree of freedom with each small cube. In other words, the number of degrees of freedom that the region can hold is given by the number of small cubes that can be put inside that region. But how small can such cubes be? A moment's thought tells us that each side of a small cube cannot be smaller than the accuracy δl with which we can measure each side l of the big cube. This can be easily shown by applying the method of contradiction: assume that we can construct small cubes each of which has sides less than δl. Then by lining up a row of such small cubes along a side of the big cube from end to end, and by counting the number of such small cubes, we would be able to measure that side (of length l) of the big cube to a better accuracy than δl. But, by definition, δl is the best accuracy with which we can measure l. The ensuing contradiction is evaded by the realization that each of the smallest cubes (that can be put inside the big cube) measures δl by δl by δl. Thus, the number of degrees of freedom in the region (measuring l by l by l) is given by $l^3/\delta l^3$, which, according to the holographic principle, is no more than l^2/l_p^2. It follows that δl is bounded (from below) by the cube root of $l l_P^2$, the same result as found above in the gedanken experiment argument. Thus, to the extent that the holographic principle is correct, spacetime indeed fluctuates, forming foams of size δl on the scale of l. Actually, considering the fundamental nature of spacetime and the ubiquity of quantum fluctuations, we should reverse the argument and then we will come to the conclusion that the "strange" holographic principle has its origin in quantum fluctuations of spacetime.

3. From Spacetime Foam to Black Hole Computers

So far there is no experimental evidence of spacetime foam. In view of this lack of experimental evidence, we should at least look for theoretical corroborations (aside from the "derivation" of the holographic principle discussed above). Fortunately such corroborations do exist — in the sector of black hole physics. To show that, we have to make a small detour to consider clocks and computers[7,8] first.

Clocks. Consider a clock (technically, a simple and "elementary" clock, not composed of smaller clocks that can be used to read time separately or sequentially), capable of resolving time to an accuracy of t, for a period of T (the running time or lifetime of the clock). Then bounds on the resolution time and the lifetime of the clock can be derived by following an argument very similar to that used above in the analysis of the gedanken experiment to measure distances. The two arguments are very similar; one obtains[7]

$$t^2 \gtrsim \frac{\hbar T}{mc^2}, \qquad t \gtrsim \frac{Gm}{c^3}, \tag{4}$$

the analogs of Eq. (1) and Eq. (2) respectively. One can also combine these two equations to give[7]

$$T/t^3 \lesssim t_P^{-2} = \frac{c^5}{\hbar G}, \tag{5}$$

the analog of Eq. (3), which relates clock precision to its lifetime. (For example, for a femtosecond (10^{-15} sec) precision, the bound on the lifetime of a simple clock is 10^{34} years.)

Computers. We can easily translate the above relations for clocks into useful relations for a simple computer (technically, it refers to a computer designed to perform highly serial computations, i.e., one that is not divided into subsystems computing in parallel). Since the resolution time t for clocks is the smallest time interval relevant in the problem, the fastest possible processing frequency is given by its reciprocal, i.e., $1/t$. Thus if ν denotes the clock rate of the computer, i.e., the number of operations per bit per unit time, then it is natural to identify ν with $1/t$. To identify the number I of bits of information in the memory space of a simple computer, we recall that the running time T is the longest time interval relevant in the problem. Thus, the maximum number of steps of information processing is given by the running time divided by the resolution time, i.e., T/t. It follows that one can identify the number I of bits of the computer with T/t. (One can think of a tape of length cT as the memory space, partitioned into bits each of length ct.) In other words, the translations from the case of clocks to the case of computers consist of substituting the clock rate of computation for the reciprocal of the resolution time, and substituting the number of bits for the running time divided by the resolution time. The bounds on the precision and lifetime of a clock given by Eq. (4) and Eq. (5) are now translated into bounds on the rate of computation and number of bits in the computer, yielding respectively

$$I\nu \lesssim \frac{mc^2}{\hbar}, \qquad \nu \lesssim \frac{c^3}{Gm}, \qquad I\nu^2 \lesssim \frac{c^5}{\hbar G} \sim 10^{86}/sec^2. \tag{6}$$

The first inequality shows that the speed of computation is bounded by the energy of the computer divided by Planck's constant, in agreement with the result found by Margolus and Levitin[9], and by Lloyd[10] (for the ultimate limits to computation). The last bound is perhaps even more intriguing: it requires the product of the number of bits and the square of the computation rate for *any* simple computer to be less than the square of the reciprocal of Planck time,[7] which depends on relativistic quantum gravity (involving c, $\hbar$, and G). This universal relation links together our concepts of information/computation, relativity, gravity, and quantum uncertainty. Numerically, the computation bound is about seventy-six orders of magnitude above what is available for a current lap-top computer performing ten billion operations per second on ten billion bits, for which $I\nu^2 \sim 10^{10}/s^2$.

Black Holes. Now we can apply what we have learned about clocks and computers to black holes.[7,8] Let us consider using a black hole to measure time. It is reasonable to use the light travel time around the black hole's horizon as the resolution time of the clock, i.e., $t \sim \frac{Gm}{c^3} \equiv t_{BH}$, then from the first equation in Eq. (4), one immediately finds that

$$T \sim \frac{G^2 m^3}{\hbar c^4} \equiv T_{BH}. \tag{7}$$

Thus, if we had not known of the Hawking lifetime (T_{BH}) for black hole evaporation, this remarkable result would have implied that there is a maximum lifetime for a black hole!

Finally, let us consider using a black hole to do computations. This may sound like a ridiculous proposition. But if we believe that black holes evolve according to quantum mechanical laws, it is possible[10], at least in principle, to program black holes to perform computations that can be read out of the fluctuations in the Hawking black hole radiation. How large is the memory space of a black hole computer, and how fast can it compute? Applying the results for computation derived above, we readily find the number of bits in the memory space of a black hole computer, given by the lifetime of the black hole divided by its resolution time as a clock, to be

$$I = \frac{T_{BH}}{t_{BH}} \sim \frac{m^2}{m_P^2} \sim \frac{r_S^2}{l_P^2}, \tag{8}$$

where $m_P = \hbar/(t_P c^2)$ is the Planck mass, m and r_S^2 denote the mass and event horizon area of the black hole respectively. This gives the number of bits I as the event horizon area in Planck units, in agreement with the identification of a black hole entropy. Furthermore, the number of operations per unit time for a black hole computer is given by

$$I\nu = \frac{T_{BH}}{t_{BH}} \times \frac{1}{t_{BH}} \sim \frac{mc^2}{\hbar}, \tag{9}$$

its energy divided by Planck's constant, as first found by Lloyd[10]. Note that all the bounds on computation discussed above are saturated by black hole computers. Thus one can even say that once they are programmed to do computations, black holes are the ultimate simple computers.

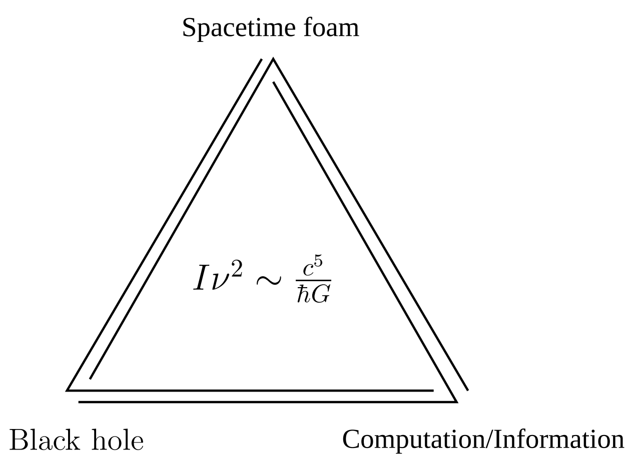

Figure 1. The *universal* relation for black hole computers, $I\nu^2 \sim c^5/\hbar G$, is a combined product of the physics behind spacetime foam, black holes, and computation/information.

All these results reinforce the conceptual interconnections of the physics underlying spacetime foam, black holes, and computation. It is intersting that these three subjects share such intimate bonds and are brought together here [see Fig. 1]. The internal consistency of the physics we have uncovered also vindicates the simple (some would say overly simple) arguments we present in section 2, in the derivation of the limits to spacetime measurements and the elucidation of the structure of spacetime foam.

4. Summary

We have analyzed a gedanken experiment for spacetime measurements to show that spacetime fluctuations scale as the cube root of distances or time durations. This cube root dependence is strange, but has been shown to be consistent with the holographic principle and with semi-classical black hole physics in general. (To us, this result for spacetime fluctuations is as beautiful as it is strange. Dare we agree with Francis Bacon in his observation: There is no excellent beauty that hath not some strangeness in the proportion.) We have also shown that the physics of spacetime foam is intimately connected to computation. The unity of physics, underlying spacetime foam, the holographic principle, black holes, and quantum computers, is hereby adequately (if not overwhelmingly) demonstrated.

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