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Cosmological event horizons, thermodynamics, and particle creation

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It is shown that the close connection between event horizons and thermodynamics which has been found in the case of black holes can be extended to cosmological models with a repulsive cosmological constant. An observer in these models will have an event horizon whose area can be interpreted as the entropy or lack of information of the observer about the regions which he cannot see. Associated with the event horizon is a surface gravity κ which enters a classical "first law of event horizons" in a manner similar to that in which temperature occurs in the first law of thermodynamics. It is shown that this similarity is more than an analogy: An observer with a particle detector will indeed observe a background of thermal radiation coming apparently from the cosmological event horizon. If the observer absorbs some of this radiation, he will gain energy and entropy at the expense of the region beyond his ken and the event horizon will shrink. The derivation of these results involves abandoning the idea that particles should be defined in an observer-independent manner. They also suggest that one has to use something like the Everett-Wheeler interpretation of quantum mechanics because the back reaction and hence the spacetime metric itself appear to be observer-dependent, if one assumes, as seems reasonable, that the detection of a particle is accompanied by a change in the gravitational field.

I. INTRODUCTION

The aim of this paper is to extend to cosmological event horizons some of the ideas of thermodynamics and particle creation which have recently been successfully applied to black-hole event horizons. In a black hole the inward-directed gravitational field produced by a collapsing body is so strong that light emitted from the body is dragged back and does not reach an observer at a large distance. There is thus a region of spacetime which is not visible to an external observer. The boundary of the region is called the event horizon of the black hole. Event horizons of a different kind occur in cosmological models with a repulsive Λ term. The effect of this term is to cause the universe to expand so rapidly that for each observer there are regions from which light can never reach him. We shall call the boundary of this region the cosmological event horizon of the observer.

The "no hair" theorems (Israel,[1] Muller zum Hagen *et al.*,[2] Carter,[3] Hawking,[4] Robinson[5,6]) imply that a black hole formed in a gravitational collapse will rapidly settle down to a quasistationary state characterized by only three parameters, the mass M_H, the angular momentum J_H, and the charge Q_H. A black hole of a given M_H, J_H, Q_H therefore has a large number of possible unobservable internal configurations which reflect the different possible initial configurations of the body that collapsed to produce the hole. In purely classical theory this number of internal configurations would be infinite because one could make a given black hole out of an infinitely large number of particles of indefinitely small mass. However, when quantum mechanics is taken into account, one would expect that in order to obtain gravitational collapse the energies of the particle would have to be restricted by the requirement that their wavelength be less than the size of the black hole. It would therefore seem reasonable to postulate that the number of internal configurations is finite. In this case one could associate with the black hole an entropy S_H which would be the logarithm of this number of possible internal configurations.[7,8,9] For this to be consistent the black hole would have to emit thermal radiation like a body with a temperature

$$T_H = G^2\left[\left(\frac{\partial S}{\partial M}\right)_{J,Q}\right]^{-1}.$$

The mechanism by which this thermal radiation arises can be understood in terms of pair creation in the gravitational potential well of the black hole. Inside the black hole there are particle states which have negative energy with respect to an external stationary observer. It is therefore energetically possible for a pair of particles to be spontaneously created near the event horizon. One particle has positive energy and escapes to infinity, the other particle has negative energy and falls into the black hole, thereby reducing its mass. The existence of the event horizon would prevent this happening classically but it is possible quantum-mechanically because one or other of the particles can tunnel through the event horizon. An equivalent way of looking at the pair creation is to regard the positive- and negative-energy particles as being the same particle which tunnels out from the black hole on a spacelike or past-directed timelike world line and is scattered onto a future-directed world line (Hartle and Hawking[10]). When one calculates the rate of particle emission by this process it turns out to be exactly what one would expect from a body with a temperature $T_H = \hbar(2\pi kc)^{-1}\kappa_H$, where κ_H is the surface gravity of the black hole and is related to M_H, J_H, and Q_H by the formulas

$$\kappa_H = (r_+ - r_-)c^2 r_0^{-2}\,,$$

$$r_\pm = c^{-2}[GM \pm (G^2M^2 - J^2M^{-2}c^2 - GQ^2)^{1/2}]\,,$$

$$r_0^2 = r_+^2 + G^{-2}J^2M^{-2}c^2\,,$$

$$A_H = 4\pi r_0^2\,.$$

A_H is the area of the event horizon of the black hole.

Combining this quantum-mechanical argument with the thermodynamic argument above, one finds that the total number of internal configurations is indeed finite and that the entropy is given by

$$S_H = (4G\hbar)^{-1}kc^3 A_H\,.$$

Cosmological models with a repulsive Λ term which expand forever approach de Sitter space asymptotically at large times. In de Sitter space future infinity is spacelike.[11,12] This means that for each observer moving on a timelike world line there is an event horizon separating the region of spacetime which the observer can never see from the region that he can see if he waits long enough. In other words, the event horizon is the boundary of the past of the observer's world line. Such a cosmological event horizon has many formal similarities with a black-hole event horizon. As we shall show in Sec. III it obeys laws very similar to the zeroth, first, and second laws of black-hole mechanics in the classical theory.[13] It also bounds the region in which particles can have negative energy with respect to the observer. One might therefore expect that particle creation with a thermal spectrum would also occur in these cosmological models. In Secs. IV and V we shall show that this is indeed the case: An observer will detect thermal radiation with a characteristic wavelength of the order of the Hubble radius. This would correspond to a temperature of less than 10^{-28} °K so that it is not of much practical significance. It is, however, important conceptually because it shows that thermodynamic arguments can be applied to the universe as a whole and that the close relationship between event horizons, gravitational fields, and thermodynamics that was found for black holes has a wider validity.

One can regard the area of the cosmological event horizon as a measure of one's lack of knowledge about the rest of the universe beyond one's ken. If one absorbs the thermal radiation, one gains energy and entropy at the expense of this region and so, by the first law mentioned above, the area of the horizon will go down. As the area decreases, the temperature of the cosmological radiation goes down (unlike the black-hole case), so the cosmological event horizon is stable. On the other hand, if the observer chooses not to absorb any radiation, there is no change in area of the horizon. This is another illustration of the fact that the concept of particle production and the back reaction associated with it seem not to be uniquely defined but to be dependent upon the measurements that one wishes to consider.[14-16]

The plan of the paper is as follows. In Sec. II we describe the black-hole asymptotically de Sitter solutions found by Carter.[20] In Sec. III we derive the classical laws governing both cosmological and black-hole event horizons. In Sec. IV we discuss particle creation in de Sitter space. We abandon the concept of particles as being observer-independent and consider instead what an observer moving on a timelike geodesic and equipped with a particle detector would actually measure. We find that he would detect an isotropic background of thermal radiation with a temperature $(2\pi)^{-1}\kappa_C$ where $\kappa_C = \Lambda^{1/2}3^{-1/2}$ is the surface gravity of the cosmological event horizon of the observer. Any other observer moving on a timelike geodesic will also see isotropic radiation with the same temperature even though he is moving relative to the first observer. This shows that they are not observing the same particles: Particles are observer-dependent. In Sec. V we extend these results to asymptotically de Sitter spaces containing black holes. The implications are considered in Sec. VI. It seems necessary to adopt something like the Everett-Wheeler interpretation of quantum mechanics because the back reaction and hence the spacetime metric will be observer-dependent, if one assumes, as seems reasonable, that the detection of a particle is accompanied by a change in the gravitational field.

We shall adopt units in which $G = \hbar = k = c = 1$. We shall use a metric with signature +2 and our conventions for the Riemann and the Ricci tensors are

$$v_{a;[b;c]} = \tfrac{1}{2} R^d{}_{abc}\, v_d\,,$$

$$R_{ab} = R_a{}^c{}_{bc}\,.$$

II. EXACT SOLUTIONS WITH COSMOLOGICAL EVENT HORIZONS

In this section we shall give some examples of event horizons in exact solutions of the Einstein equations

$$R_{ab}-\tfrac{1}{2}g_{ab}R+\Lambda g_{ab}=8\pi T_{ab}\ . \tag{2.1}$$

We shall consider only the case of Λ positive (corresponding to repulsion). Models with negative Λ do not, in general, have event horizons.

The simplest example is de Sitter space which is a solution of the field equations with $T_{ab}=0$. One can write the metric in the static form

$$ds^2=-(1-\Lambda r^2 3^{-1})dt^2+dr^2(1-\Lambda r^2 3^{-1})^{-1}+r^2(d\theta^2+\sin^2\theta\, d\phi^2)\ . \tag{2.2}$$

This metric has an apparent singularity at $r=3^{1/2}\Lambda^{-1/2}$. This singularity caused considerable discussion when the metric was first discovered.[17,18] However, it was soon realized that it arose simply from a bad choice of coordinates and that there are other coordinate systems in which the metric can be analytically extended to a geodesically complete space of constant curvature with topology $R^1\times S^3$. For a detailed description of these coordinate systems the reader is referred to Refs. 12 and 19. For our purposes it will be convenient to express the de Sitter metric in "Kruskal coordinates":

$$ds^2=3\Lambda^{-1}(UV-1)^{-2}\times[-4dU\,dV+(UV+1)^2(d\theta^2+\sin^2\theta\, d\phi^2)] \tag{2.3}$$

where

$$r=3^{1/2}\Lambda^{-1/2}(UV+1)(1-UV)\ , \tag{2.4}$$

$$\exp(2\Lambda^{1/2}3^{-1/2}t)=-VU^{-1}\ . \tag{2.5}$$

The structure of this space is shown in Fig. 1. In this diagram radial null geodesics are at $\pm 45°$ to the vertical. The dashed curves $UV=-1$ are timelike and represent the origin of polar coordinates and the antipodal point on a three-sphere. The solid curves $UV=+1$ are spacelike and represent past and future infinity $\mathscr{I}^-$ and $\mathscr{I}^+$, respectively.

In region I ($U<0$, $V>0$, $UV>-1$) the Killing vector $K=\partial/\partial t$ is timelike and future-directed. However, in region IV ($U>0$, $V<0$, $UV>-1$, K is still timelike but past-directed, while in regions II and III ($0<UV<1$) K is spacelike. The Killing vector K is null on the two surfaces $U=0$, $V=0$. These are respectively the future and past event horizons for any observer whose world line remains in region I; in particular for any observer moving along a curve of constant r in region I.

By applying a suitable conformal transformation one can make the Kruskal diagram finite and convert it to the Penrose-Carter form (Fig. 2). Radial null geodesics are still $\pm 45°$ to the vertical but the freedom of the conformal factor has been used

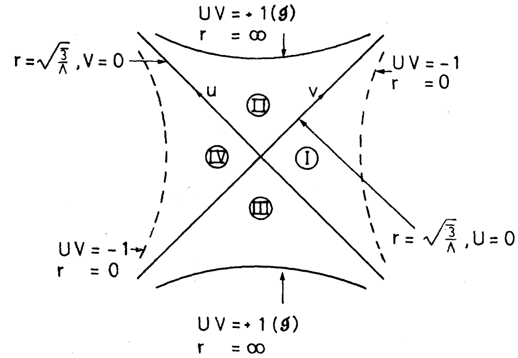

FIG. 1. Kruskal diagram of the (r,t) plane of de Sitter space. In this figure null geodesics are at $\pm 45°$ to the vertical. The dashed curves $r=0$ are the antipodal origins of polar coordinates on a three-sphere. The solid curves $r=\infty$ are past and future infinity $\mathscr{I}^-$ and $\mathscr{I}^+$, respectively. The lines $r=3^{1/2}\Lambda^{-1/2}$ are the past and future event horizons of observers at the origin.

to make the origin of polar coordinates, $r=0$, and future and past infinity, $\mathscr{I}^+$ and $\mathscr{I}^-$, straight lines. Also shown are some orbits of the Killing vector $K=\partial/\partial t$. Because de Sitter space is invariant under the ten-parameter de Sitter group, SO(4, 1), K will not be unique. Any timelike geodesic can be chosen as the origin of polar coordinates and the surfaces $U=0$ and $V=0$ in such coordinates will be the past and future event horizons of an observer moving on this geodesic. If one normalizes K to have unit magnitude at the origin, one can define a "surface gravity" for the horizon by

$$K_{a;b}K^b=\kappa_C K_a \tag{2.6}$$

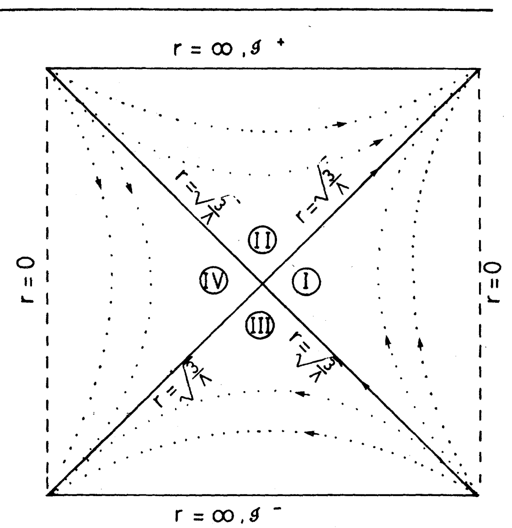

FIG. 2. The Penrose-Carter diagram of de Sitter space. The dotted curves are orbits of the Killing vector.

on the horizon. This gives

$$\kappa_C = \Lambda^{1/2} 3^{-1/2} . \tag{2.7}$$

The area of the cosmological horizon is

$$A_C = 12\pi\Lambda^{-1} . \tag{2.8}$$

One can also construct solutions which generalize the Kerr-Newman family to the case when Λ is nonzero.[20,21] The simplest of these is the Schwarzschild–de Sitter metric. When $\Lambda = 0$ the unique spherically symmetric vacuum spacetime is the Schwarzschild solution. The metric of this can be written in static form:

$$ds^2 = -(1 - 2Mr^{-1})dt^2 + dr^2(1 - 2Mr^{-1})^{-1} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) . \tag{2.9}$$

As is now well known, the apparent singularities at $r = 2M$ correspond to a horizon and can be removed by changing to Kruskal coordinates in which the metric has the form

$$ds^2 = -32M^3 r^{-1} \exp(-2^{-1}M^{-1}r) dU\, dV + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) , \tag{2.10}$$

where

$$UV = (1 - 2^{-1}M^{-1}r) \exp(2^{-1}M^{-1}r) \tag{2.11}$$

and

$$UV^{-1} = -\exp(-2^{-1}M^{-1}t) . \tag{2.12}$$

The Penrose-Carter diagram of the Schwarzschild solution is shown in Fig. 3. The wavy lines marked $r = 0$ are the past and future singularities. Region I is asymptotically flat and is bounded on the right by past and future null infinity $\mathscr{I}^-$ and $\mathscr{I}^+$. It is bounded on the left by the surfaces $U = 0$ and $V = 0$, $r = 2M$. These are future and past event horizons for observers who remain outside $r = 2M$. On the left-hand side of the diagram there is another a asymptotically flat region IV. The Killing vector $K = \partial/\partial t$ is now uniquely defined by the condition that it be timelike and of unit magnitude near $\mathscr{I}^+$ and $\mathscr{I}^-$. It is timelike and future-directed in region I, timelike and past-directed in region IV, and spacelike in regions II and III. The Killing vector K is null on the horizons which have area $A_H = 16\pi M^2$. The surface gravity, defined by (2.6), is $\kappa_H = (4M)^{-1}$.

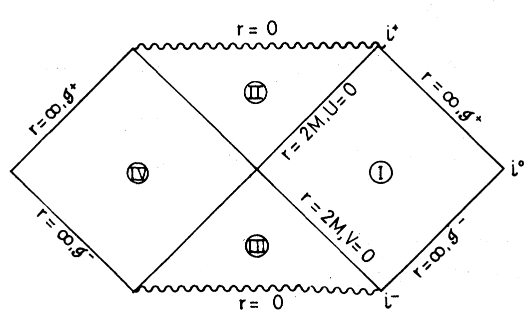

FIG. 3. The Penrose-Carter diagram of the Schwarzschild solution. The wavy lines and the top and bottom are the future and past singularities. The diagonal lines bounding the diagram on the right-hand side are the past and future null infinity of asymptotically flat space. The region IV on the left-hand-side is another asymptotically flat space.

The Schwarzschild solution is usually interpreted as a black hole of mass M in an asymptotically flat space. There is a straightforward generalization to the case of nonzero Λ which represents a black hole in asymptotically de Sitter space. The metric can be written in the static form

$$ds^2 = -(1 - 2Mr^{-1} - \Lambda r^2 3^{-1})dt^2 + dr^2(1 - 2Mr^{-1} - \Lambda r^2 3^{-1})^{-1} + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) . \tag{2.13}$$

If $\Lambda > 0$ and $9\Lambda M^2 < 1$, the factor $(1 - 2Mr^{-1} - \Lambda r^2 3^{-1})$ is zero at two positive values of r. The smaller of of these values, which we shall denote by r_+, can be regarded as the position of the black-hole event horizon, while the larger value r_{++} represents the position of the cosmological event horizon for observers on world lines of constant r between r_+ and r_{++}. By using Kruskal coordinates as above one can remove the apparent singularities in the metric at r_+ and r_{++}. One has to employ separate coordinate patches at r_+ and r_{++}. We shall not give the expressions in full because they are rather messy; however, the general structure can be seen from the Penrose-Carter diagram shown in Fig. 4. Instead of having two regions (I and IV) in which the Killing vector $K = \partial/\partial t$ is timelike, there are now an infinite sequence of such regions, also labeled I and IV depending upon whether K is future- or past-directed. There are also infinite sequences of $r = 0$ singularities and spacelike infinities $\mathscr{I}^+$ and $\mathscr{I}^-$. The surfaces $r = r_+$ and $r = r_{++}$ are black-hole and cosmological event horizons for observers moving on world lines of constant

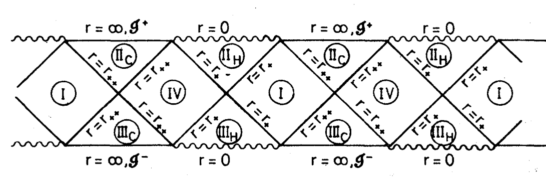

FIG. 4. The Penrose-Carter diagram for Schwarzschild–de Sitter space. There is an infinite sequence of singularities $r = 0$ and spacelike infinities $r = \infty$. The Killing vector $K = \partial/\partial t$ is timelike and future-directed in regions I, timelike and past-directed in regions IV and spacelike in the others.

r between r_+ and r_{++}.

The Killing vector $K = \partial/\partial t$ is uniquely defined by the conditions that it be null on both the black-hole and the cosmological horizons and that its magnitude should tend to $\Lambda^{1/2}3^{-1/2}r$ as r tends to infinity. One can define black-hole and cosmological surface gravities κ_H and κ_C by

$$K_{a;b}K^b = \kappa K_a \tag{2.14}$$

on the horizons. These are given by

$$\kappa_H = \Lambda 6^{-1} r_+^{-1}(r_{++} - r_+)(r_+ - r_{--}) , \tag{2.15a}$$

$$\kappa_C = \Lambda 6^{-1} r_{++}^{-1}(r_{++} - r_+)(r_{++} - r_{--}) , \tag{2.15b}$$

where $r = r_{--}$ is the negative root of

$$3r - 6M - \Lambda r^3 = 0 . \tag{2.16}$$

The areas of the two horizons are

$$A_H = 4\pi r_+^2 \tag{2.17}$$

and

$$A_C = 4\pi r_{++}^2 . \tag{2.18}$$

If one keeps Λ constant and increases M, r_+ will increase and r_{++} will decrease. One can understand this in the following way. When $M = 0$ the gravitational potential $g(\partial/\partial t, \partial/\partial t)$ is $1 - \Lambda r^2 3^{-1}$. The introduction of a mass M at the origin produces an additional potential of $-2Mr^{-1}$. Horizons occur at the two values of r at which $g(\partial/\partial t, \partial/\partial t)$ vanishes. Thus as M increases, the black-hole horizon r_+ increases and the cosmological horizon r_{++} decreases. When $9\Lambda M^2 = 1$ the two horizons coincide. The surface gravity K can be thought of as the gravitational field or gradient of the potential at the horizons. As M increases both κ_H and κ_C decrease.

The Kerr–Newman–de Sitter space can be expressed in Boyer-Lindquist-type coordinates as[20,21]

$$\begin{aligned} ds^2 = {} & \rho^2(\Delta_r^{-1}dr^2 + \Delta_\theta^{-1}d\theta^2) \\ & + \rho^{-2}\Xi^{-2}\Delta_\theta[a\,dt - (r^2 + a^2)d\phi]^2 \\ & - \Delta_r \Xi^{-2}\rho^{-2}(dt - a\sin^2\theta\, d\phi)^2 , \end{aligned} \tag{2.19}$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta , \tag{2.20}$$

$$\Delta_r = (r^2 + a^2)(1 - \Lambda r^2 3^{-1}) - 2Mr + Q^2 , \tag{2.21}$$

$$\Delta_\theta = 1 + \Lambda a^2 3^{-1}\cos^2\theta , \tag{2.22}$$

$$\Xi = 1 + \Lambda a^2 3^{-1} . \tag{2.23}$$

The electromagnetic vector potential A_a is given by

$$A_a = Qr\rho^{-2}\Xi^{-1}(\delta_a^{+} - a\sin^2\theta\,\delta_a^{\phi}) . \tag{2.24}$$

Note that our Λ has the opposite sign to that in Ref. 21.

There are apparent singularities in the metric at the values of r for which $\Delta_r = 0$. As before, these correspond to horizons and can be removed by using appropriate coordinate patches. The Penrose-Carter diagram of the symmetry axis ($\theta = 0$) of these spaces is shown in Fig. 5 for the case that Δ_+ has 4 distinct roots: r_{--}, r_-, r_+, and r_{++}. As before, r_{++} and r_+ can be regarded as the cosmological and black-hole event horizons, respectively. In addition, however, there is now an inner black-hole horizon at $r = r_-$. Passing through this, one comes to the ring singularity at $r = 0$, on the other side of which there is another cosmological horizon at $r = r_{--}$ and another infinity. The diagram shown is the simplest one to draw but it is not simply connected; one can take covering spaces. Alternatively one can identify regions in this diagram.

The Killing vector $\tilde{K} = \partial/\partial\phi$ is uniquely defined by the condition that its orbits should be closed curves with parameter length 2π. The other Kill-

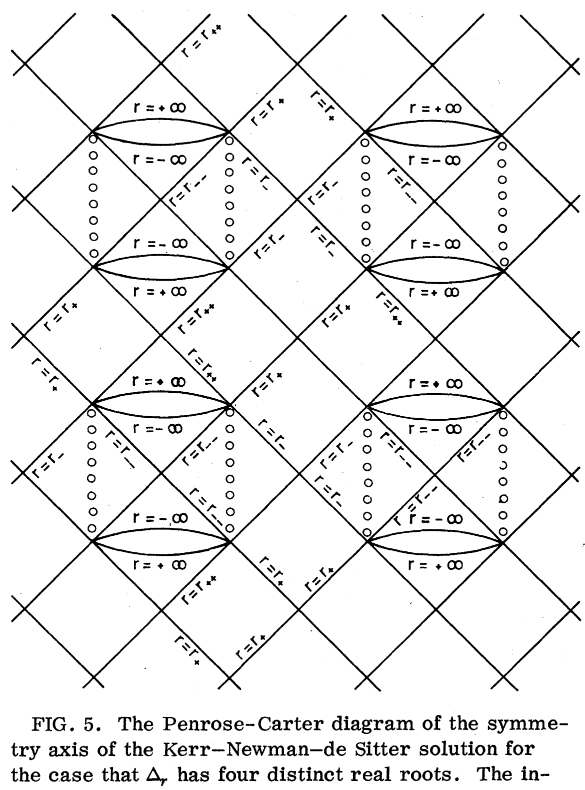

FIG. 5. The Penrose-Carter diagram of the symmetry axis of the Kerr–Newman–de Sitter solution for the case that Δ_r has four distinct real roots. The infinities $r = +\infty$ and $r = -\infty$ are not joined together. The external cosmological horizon occurs at $r = r_{++}$ the exterior black-hole horizon at $r = r_+$, the inner black-hole horizon at $r = r_-$. The open circles mark where the ring singularity occurs, although this is not on the symmetry axis. On the other side of the ring at negative values of r there is another cosmological horizon at $r = r_{--}$ and another infinity.

ing vector $K=\partial/\partial t$ is not so specially picked out. One can add different constants multiples of $\tilde{K}$ to K to obtain Killing vectors which are null on the different horizons and one can then define surface gravities as before. We shall be interested only in those for the r_+, r_{++} horizons. They are

$$\kappa_H=\Lambda 6^{-1}\Xi^{-1}(r_+-r_{--})(r_+-r_-)(r_{++}-r_+)(r_+{}^2+a^2)^{-1}, \tag{2.25}$$

$$\kappa_C=\Lambda 6^{-1}\Xi^{-1}(r_{++}-r_+)(r_{++}-r_-)(r_{++}-r_-)(r_+{}^2+a^2)^{-1}. \tag{2.26}$$

The areas of these horizons are

$$A_H=4\pi(r_+{}^2+a^2), \tag{2.27}$$

$$A_C=4\pi(r_{++}{}^2+a^2). \tag{2.28}$$

III. CLASSICAL PROPERTIES OF EVENT HORIZONS

In this section we shall generalize a number of results about black-hole event horizons in the classical theory to spacetimes which are not asymptotically flat and may have a nonzero cosmological constant, and to event horizons which are not black-hole horizons. The event horizon of a black hole in asymptotically flat spacetimes is normally defined as the boundary of the region from which one can reach future null infinity, $\mathscr{I}^+$, along a future-directed timelike or null curve. In other words it is $J^-(\mathscr{I}^+)$ [or equivalently $\dot{I}^-(\mathscr{I}^+)$], where an overdot indicates the boundary and J^- is the causal past (I^- is the chronological past). However, one can also define the black-hole horizon as $\dot{I}^-(\lambda)$, the boundary of the past of a timelike curve λ which has a future end point at future timelike infinity, i^+ in Fig. 3. One can think of λ as the world line of an observer who remains outside the black hole and who does not accelerate away to infinity. The event horizon is the boundary of the region of spacetime that he can see if he waits long enough. It is this definition of event horizon that we shall extend to more general spacetimes which are not asymptotically flat.

Let λ be a future inextensible timelike curve representing an observer's world line. For our considerations of particle creation in the next section we shall require that the observer have an indefinitely long time in which to detect particles. We shall therefore assume that λ has infinite proper length in the future direction. This means that it does not run into a singularity. The past of λ, $I^-(\lambda)$, is a terminal indecomposable past set, or TIP in the language of Geroch, Kronheimer, and Penrose.[22] It represents all the events that the observer can ever see. We shall assume that what the observer sees at late times can be predicted (classically at least) from a spacelike surface $\mathcal{S}$, i.e., $I^-(\lambda)\cap J^+(\mathcal{S})$ is contained in the future Cauchy development $D^+(\mathcal{S})$.[12] We shall also assume that $\dot{I}^-(\lambda)\cap J^+(\mathcal{S})$, the portion of the event horizon to the future of $\mathcal{S}$, is contained in $D^+(\mathcal{S})$. Such an event horizon will be said to be predictable. The event horizon will be generated by null geodesic segments which have no future end points but which have past end points if and where they intersect other generators.[12] In another paper[23] it is shown that the generators of a predictable event horizon cannot be converging if the Einstein equations hold (with or without cosmological constant), provided that the energy-momentum tensor satisfies the strong energy condition $T_{ab}u^a u^b \geq \frac{1}{2}T^a_a u^b u_b$ for any timelike vector u_a, i.e., provided that $\mu+P_i\geq 0$, $\mu+\sum_{i=1}^{i=3}P_i\geq 0$, where μ is the energy density and P_i are the principal pressures. This gives immediately the following result, which, because of the very suggestive analogy with thermodynamics, we call:

The second law of event horizons: The area of any connected two-surface in a predictable event horizon cannot decrease with time. The area may be infinite if the two-dimensional cross section is not compact. However, in the examples in Sec. II, the natural two-sections are compact and have constant area.

In the case of gravitational collapse in asymptotically flat spacetimes one expects the spacetime eventually to settle down to a quasistationary state because all the available energy will either fall through the event horizon of the black hole (thereby increasing its area) or be radiated away to infinity. In a similar way one would expect that where the intersection of $I^-(\lambda)$ with a spacelike surface $\mathcal{S}$ had compact closure (which we shall assume henceforth), there would only be a finite amount of energy available to be radiated through the cosmological event horizon of the observer and that therefore this spacetime would eventually approach a stationary state. One is thus lead to consider solutions in which there is a Killing vector K which is timelike in at least some region of $I^-(\lambda)\cap J^+(\mathcal{S})$. Such solutions would represent the asymptotic future limit of general spacetimes with predictable event horizons.

Several results about stationary empty asymptotically flat black-hole solutions can be generalized to stationary solutions of the Einstein equations, with cosmological constant, which contain predictable event horizons. The first such theorem is that the null geodesic generators of each connected component of the event horizon must coincide with orbits of some Killing vector.[24,21]

These Killing vectors may not coincide with the original Killing vector K and may be different for different components of the horizon. In either of these two cases there are at least two Killing vectors. One can chose a linear combination $\tilde{K}$ whose orbits are spacelike closed curves in $I^-(\lambda) \cap J^+(\mathcal{S})$. One could interpret this as implying that the solution is axisymmetric as well as being stationary, though we have not been able to prove that there is necessarily any axis on which $\tilde{K}$ vanishes.

Let $\hat{K}$ be the Killing vector which coincides with the generators of one component of the event horizon. If $\hat{K}$ is not hypersurface orthogonal and if then space is empty or contains only an electromagentic field, one can apply a generalized Lichnerowicz theorem[12,21] to show that $\hat{K}$ must be spacelike in some "ergoregion" of $I^-(\lambda)$. One can then apply energy extraction arguments[24,12] or the results of Hajicek[25] to show that this ergoregion contains another component of the event horizon whose generators do not coincide with the orbits of $\hat{K}$. It therefore follows that either $\hat{K}$ is hypersurface orthogonal (in which case the solution is static) or that there are at least two Killing vectors (in which case the solution is axissymmetric as well as stationary). If there is only a cosmological horizon and no black-hole horizon, then the solution is necessarily static.

One would expect that in the static vacuum case one could generalize Israel's theorem[1,2] to prove that the space was spherically symmetric. One could then generalize Birkhoff's theorem to include a cosmological constant and show that the space was necessarily the Schwarzschild–de Sitter space described in Sec. II. In the case that there was only a cosmological event horizon, it would be de Sitter space. In the stationary axisymmetric case one would expect that one could generalize and extend the results of Carter and Robinson[3,5] to show that vacuum solutions were members of the Kerr–de Sitter family described in Sec. II. If there is matter present it will distort the spacetime from the Schwarzschild–de Sitter or Kerr–de Sitter solution just as matter around a black hole in asymptotically flat space will distort the spacetime away from the Schwarzschild or Kerr solution.

The proof given in Ref. 13 of the zeroth law of black holes can be generalized immediately to the case of nonzero cosmological constant. One thus has:

The zeroth law of event horizons: The surface gravity of a connected component of the event horizon $\dot{I}^-(\lambda)$ is constant over that component. This is analogous to the zeroth law of nonrelativistic thermodynamics which states that the temperature is constant over a body in thermal equilibrium. We shall show in Secs. IV and V that quantum effects cause each component of the event horizon to radiate thermally with a temperature proportional to its surface gravity.

One can also generalize the first law of black holes. We shall do this for stationary axisymmetric solutions with no electromagnetic field and where $\dot{I}^-(\lambda) \cap J^+(\mathcal{S})$ consists of two components, a black-hole event horizon and a cosmological event horizon. Let K be the Killing vector which is null on the cosmological event horizon. The orbits of K will constitute the stationary frame which appears to be nonrotating with respect to distant objects near the cosmological event horizon. In the general case the normalization of K is somewhat arbitrary but we shall assume that some particular normalization has been chosen. The Killing vector $\hat{K}$ which coincides with the generators of the black-hole horizon can be expressed in the form

$$\hat{K} = K + \Omega_H \tilde{K}\,, \tag{3.1}$$

where Ω_H is the angular velocity of the black-hole horizon relative to the cosmological horizon in the units of time defined by the normalization of K and $\tilde{K}$ is the uniquely defined axial Killing vector whose orbits are closed curves with parameter length 2π.

For any Killing vector field ξ^a one has

$$\xi^{a;b}{}_{;b} = R^a{}_b\, \xi^b\,. \tag{3.2}$$

Choose a three-surface $\mathcal{S}$ which is tangent to $\tilde{K}$, and integrate (3.2) over it with $\xi = \tilde{K}$. On using Einstein's equations this gives

$$(8\pi)^{-1}\int_H \tilde{K}^{a;b} d\Sigma_{ab} + (8\pi)^{-1}\int_C \tilde{K}^{a;b} d\Sigma_{ab} = \int T^{ab}\tilde{K}_a d\Sigma_b\,, \tag{3.3}$$

where the three-surface integral on the right-hand side is taken over the portions of $\mathcal{S}$ between the black-hole and cosmological horizons and the two-surface integrals marked H and C are taken over the intersections of $\mathcal{S}$ with the respective horizons, the orientation being given by the direction out of $I^-(\lambda)$. One can interpret the right-hand side of (3.3) as the angular momentum of the matter between the two horizons. One can therefore regard the second term on the left-hand side of (3.3) as being the total angular momentum, J_C, contained in the cosmological horizon, and the first on the left-hand side term as the negative of the angular momentum of the black hole, J_H.

One can also apply Eq. (3.2) to the Killing vector K to obtain

$$(4\pi)^{-1}\int K^{a;b} d\Sigma_{ab} + (4\pi)^{-1}\int K^{a;b} d\Sigma_{ab} = \int 2(T_{ab} - \tfrac{1}{2} T^c_c g_{ab}) K^a d\Sigma_b + \int \Lambda (4\pi)^{-1} K_a d\Sigma^a\,. \tag{3.4}$$

One can regard the terms on the right-hand side of Eq. (3.4) as representing respectively the (positive) contribution of the matter and the (negative) contribution of the Λ term to the mass within the cosmological horizon. One can therefore regard the second term on the left-hand side as the (negative) mass M_C within the cosmological horizon and the first term on the left-hand side as the negative of the (positive) mass M_H of the black hole. As in Ref. 13, one can express M_H and M_C as

$$M_H = \kappa_H A_H (4\pi)^{-1} + 2\Omega_H J_H , \tag{3.5}$$

$$M_C = -\kappa_C A_C (4\pi)^{-1} . \tag{3.6}$$

One therefore has the Smarr-type[26] formulas

$$\begin{aligned} M_C &= -\kappa_C A_C (4\pi)^{-1} \\ &= \kappa_H A_H (4\pi)^{-1} + 2\Omega_H J_H + \int 2(T_{ab} - T^c_c g_{ab}) K^b d\Sigma^a \\ &\quad + (4\pi)^{-1} \int \Lambda K_a d\Sigma^a . \end{aligned} \tag{3.7}$$

One can take the differential of the mass formula in a manner similar to that in Ref. 13. One obtains:

The first law of event horizons.

$$\int \delta T_{ab} K^a d\Sigma^b = -\kappa_c \delta A_C (8\pi)^{-1} - \kappa_H \delta A_H (8\pi)^{-1} - \Omega_H \delta J_H , \tag{3.8}$$

where δT_{ab} is the variation in the matter energy-momentum tensor between the horizons in a gauge in which $\delta K^a = \delta \tilde{K}^a = 0$.

From this law one sees that if one regards the area of a horizon as being proportional to the entropy beyond that horizon, then the corresponding surface gravity is proportional to the effective temperature of that horizon, that is, the temperature at which that horizon would be in thermal equilibrium and therefore the temperature at which that horizon radiates. In the next section we shall show that the factor of proportionality between temperature and surface gravity is $(2\pi)^{-1}$. This means that the entropy is $\frac{1}{4}$ the area. In the case of the cosmological horizon in de Sitter space the entropy is $3\pi\Lambda^{-1} \geq 10^{120}$ because $\Lambda < 10^{-120}$.

IV. PARTICLE CREATION IN DE SITTER SPACE

In this section we shall calculate particle creation in solutions of the Einstein equations with positive cosmological constant. The simplest example is de Sitter space and particle production in this situation has been studied by Nachtmann,[27] Tagirov,[28] Candelas and Raine,[29] and Dowker and Critchley,[30] among others. They all used definitions of particles that were observer-independent and invariant under the de Sitter group. Under these conditions only two answers are possible for the rate of particle creation per unit volume, zero or infinity, because if there is nonzero production of particles with a certain energy, then by de Sitter group invariance there must be the same rate of creation of particles with all other energies. It is therefore not surprising that the authors mentioned above chose their definitions of particles to get the zero answer.

An observer-independent definition of particles is, however, not relevant to what a given observer would measure with a particle detector. This depends not only on the spacetime and the quantum state of the system, but also on the observer's world line. For example, Unruh[14] has shown that in Minkowski space in the normal vacuum state accelerated observers can detect and absorb particles. To a nonaccelerating observer such an absorption will appear to be emission from the accelerated observer's detector. In a similar manner, an observer at a constant distance from a black hole will detect a steady flux of particles coming out from the hole with a thermal spectrum while an observer who falls into the hole will not see many particles.

A feature common to the examples of a uniformly accelerated observer in Minkowski space and an observer at constant distance from the black hole is that both observers have event horizons which prevent them from seeing the whole of the spacetime and from measuring the complete quantum state of the system. It is this loss of information about the quantum state which is responsible for the thermal radiation that the observers see. Because any observer in de Sitter space also has an event horizon, one would expect that such an observer would also detect thermal radiation. We shall show that this is indeed the case. This can be done either by the frequency-mixing method in which the thermal radiation from black holes was first derived,[31,32] or by the path-integral method of Hartle and Hawking.[10] We shall adopt the latter approach because it is more elegant and gives a clearer intuitive picture of what is happening. The same results can, however, be obtained by the former method.

As in the method of Hartle and Hawking,[10] we construct the propagator for a scalar field of mass m by the path integral

$$G(x, x') = \lim_{\epsilon \to 0} \int_0^\infty dW F(W, x, x') \exp[-(im^2 W + \epsilon W^{-1})] , \tag{4.1}$$

where

$$F(W, x, x') = \int \delta x[w] \exp\left[\frac{i}{4}\int_0^W g(\dot{x}, \dot{x})dw\right] \quad (4.2)$$

and the integral is taken over all paths $x(w)$ from x to x'.

As in the Hartle and Hawking paper,[10] this path integral can be given a well-defined meaning by anàltyically continuing the parameter W to negative imaginary values and analytically continuing the coordinates to a region where the metric is positive-definite. A convenient way of doing this is to embed de Sitter space as the hyperboloid

$$-T^2 + S^2 + X^2 + Y^2 + Z^2 = 3\Lambda^{-1} \quad (4.3)$$

in the five-dimensional space with a Lorentz metric:

$$ds^2 = -dT^2 + dS^2 + dX^2 + dY^2 + dZ^2 . \quad (4.4)$$

Taking T to be $i\tau$ (τ real), we obtain a sphere in five-dimensional Euclidean space. On this sphere the function F satisfies the diffusion equation

$$\frac{\partial F}{\partial \Omega} = \tilde{\Box}^2 F , \quad (4.5)$$

where $\Omega = iW$ and $\tilde{\Box}^2$ is the Laplacian on the four-sphere. Because the four-sphere is compact there is a unique solution of (4.5) for the initial condition

$$F(0, x, x') = \delta(x, x') , \quad (4.6)$$

where $\delta(x, x')$ is the Dirac δ function on the four-sphere. One can then define the propagator $G(x, x')$ from (4.1) by analytically continuing the solution for F back to real values of the parameter W and real coordinates x and x'. Because the function F is analytic for finite points x and x', any singularities which occur in $G(x, x')$ must come from the end points of the integration in (4.1). As shown in Ref. 10, there will be singularities in $G(x, x')$ when, and only when, x and x' can be joined by a null geodesic. This will be the case if and only if

$$(T - T')^2 = (S - S')^2 + (X - X')^2 + (Y - Y')^2 + (Z - Z')^2 . \quad (4.7)$$

The coordinates, T, S, X, Y, Z can be related to the static coordinates t, r, θ, ϕ used in Sec. II by

$$T = (\Lambda 3^{-1} - r^2)^{1/2} \sinh \Lambda^{1/2} 3^{-1/2} t , \quad (4.8)$$

$$S = (\Lambda 3^{-1} - r^2)^{1/2} \cosh \Lambda^{1/2} 3^{-1/2} t , \quad (4.9)$$

$$X = r \sin\theta \cos\phi , \quad (4.10)$$

$$Y = r \sin\theta \sin\phi , \quad (4.11)$$

$$Z = r \cos\theta . \quad (4.12)$$

The horizons $\Lambda r^2 = 3$ are the intersection of the hyperplanes $T = \pm S$ with the hyperboloid. As in Ref. 10 we define the complexified horizon by $\Lambda r^2 = 3$, θ, ϕ real. On the complexified horizon X, Y, and Z are real and either $T = S = \Lambda^{-1/2} 3^{1/2} V$, $U = 0$ or $T = -S = \Lambda^{-1/2} 3^{1/2} U$, $V = 0$. By Eq. (4.7) a complex null geodesic from a real point (T', S', X', Y', Z') on the hyperboloid can intersect the complex horizon only on the real sections $T = \pm S$ real. If the point (T', S', X', Y', Z') is in region I ($S > |T|$) the propagator $G(x', x)$ will have a singularity on the past horizon at the point where the past-directed null geodesic from x' intersects the horizon. As shown in Ref. 10, the ϵ convergence factor in (4.1) will displace the pole slightly below the real axis in the complex plane on the complexified past horizon. The propagator $G(x', x)$ is therefore analytic in the upper half U plane on the past horizon. Similarly, it will be analytic in the lower V plane on the future horizon.

The propagator $G(x', x)$ satisfies the wave equation

$$(\Box_x^2 - m^2) G(x', x) = -\delta(x, x') \quad (4.13)$$

Thus if x' is a fixed point in region I, the value $G(x', x)$ for a point in region II will be determined by the values of $G(x', x)$ on a characteristic Cauchy surface for region II consisting of the section of the $U = 0$ horizon for real $V \geq 0$ and the section of the $V = 0$ horizon for real $U \geq 0$. The coordinates r and t of the point x are related to U and V by

$$e^{2\kappa_C t} = VU^{-1} \quad (4.14)$$

$$r = (1 + UV)(1 - UV)^{-1} \kappa_C^{-1} \quad (4.15)$$

If one holds r fixed at a real value but lets $t = \tau + i\sigma$, then

$$U = |U| \exp(-i\sigma\kappa_C) , \quad (4.16)$$

$$V = |V| \exp(+i\sigma\kappa_C) . \quad (4.17)$$

For a fixed value of σ the metric (2.3) of de Sitter space remains real and unchanged. Thus the value of $G(x', x)$ at a complex coordinate t of the point x but real r, θ, ϕ can be obtained by solving the Klein-Gordon equation with real coefficients and with initial data on the Cauchy surface $V = 0$, $U = |U| \exp(-i\kappa_C \sigma)$ and $U = 0$, $V = |V| \exp(+i\kappa_C \sigma)$. Because $G(x', x)$ is analytic in the upper half U plane on $V = 0$ and the lower half V plane on $U = 0$, the data and hence the solution will be regular provided that

$$-\pi\kappa_C^{-1} \leq \sigma \leq 0 . \quad (4.18)$$

The operator

$$\left(\frac{\partial}{\partial t}\right)_r = \kappa_C \left(V \frac{\partial}{\partial V} - U \frac{\partial}{\partial U}\right) \quad (4.19)$$

commutes with the Klein-Gordon operator $\Box_x^2 - m^2$ and is zero when acting on the initial data for σ

satisfying (4.18). Thus the solution $G(x', x)$ determined by the initial data will be analytic in the coordinates t of the point x for σ satisfying Eq. (4.18).

This is the basic result which enables us to show that an observer moving on a timelike geodesic in de Sitter space will detect thermal radiation.

The propagator we have defined appears to be similar to that constructed by other authors.[28-30] However, our use of the propagator will be different: Instead of trying to obtain some observer-independent measure of particle creation, we shall be concerned with what an observer moving on a timelike geodesic in de Sitter space would measure with a particle detector which is confined to a small tube around his world line. Without loss of generality we can take the observer's world line to be at the origin of polar coordinates in region I. Within the world tube of the particle detector the spacetime can be taken as flat.

The results we shall obtain are independent of the detailed nature of the particle detector. However, for explicitness we shall consider a particle model of a detector similar to that discussed by Unruh[14] for uniformly accelerated observers in flat space. This will consist of some system such as an atom which can be described by a nonrelativistic Schrödinger equation

$$i\,\frac{\partial\Psi}{\partial t'} = H_0\Psi + g\phi\Psi\ ,$$

where t' is the proper time along the observer's world line, H_0 is the Hamiltonian of the undisturbed particle detector and $g\phi\Psi$ is a coupling term to the scalar field ϕ. The undisturbed particle detector will have energy levels E_i and wave functions $\Psi_i(\vec{R}')e^{-iE_i t}$, where $\vec{R}'$ represents the spatial position of a point in the detector.

By first-order perturbation theory the amplitude to excite the detector from energy level E_i to a higher-energy level E_j is proportional to

$$\int dt' \int d^3\vec{R}'\,\overline{\Psi}_j\, g\phi\Psi_i \exp[-i(E_i - E_j)t'] .$$

In other words, the detector responds to components of field ϕ which are positive frequency along the observer's world line with respect to his proper time. By superimposing detector levels with different energies one can obtain a detector response function of a form

$$f(t')h(\vec{R})\,,$$

where $f(t')$ is a purely positive-frequency function of the observer's proper time t' and h is zero outside some value of r' corresponding to the radius of the particle detector. Let $\mathcal{P}$ be a three-surface which completely surrounds the observer's world line. If the observer detects a particle, it must have crossed $\mathcal{P}$ in some mode k_j which is a solution of the Klein-Gordon equation with unit Klein-Gordon norm over the hypersurface $\mathcal{P}$. The amplitude for the observer to detect such a particle will be

$$\int\int fh(x')G(x', x)\overleftrightarrow{\partial}_a \overline{k}_j(x)dV'd\Sigma^a\,, \qquad (4.20)$$

where the volume integral in x' is taken over the volume of the particle detector and the surface integral in x is taken over $\mathcal{P}$.

The hypersurface $\mathcal{P}$ can be taken to be a spacelike surface of large constant r in the past in region III and a spacelike surface of large constant r in the future in region II. In the limit that r tends to infinity these surfaces tend to past infinity $\mathscr{I}^-$ and future infinity $\mathscr{I}^+$, respectively. We shall assume that there were no particles present on the surface in the distant past. Thus the only contribution to the amplitude (4.20) comes from the surface in the future. One can interpret this as the spontaneous creation of a pair of particles, one with positive and one with negative energy with respect to the Killing vector $K = \partial/\partial t$. The particle with positive energy propagates to the observer and is detected. The particle with negative energy crosses the event horizon into region II where K is spacelike. It can exist there as a real particle with timelike four-momentum. Equivalently, one can regard the world lines of the two particles as being the world line of a single particle which tunnels through the event horizon out of region II and is detected by the observer.

Suppose the detector is sensitive to particles of a certain energy E. In this case the positive-frequency-response function $f(t)$ will be proportional to e^{-iEt}. By the stationarity of the metric, the propagator $G(x', x)$ can depend on the coordinates t' and t only through their difference. This means that the amplitude (4.20) will be zero except for modes k_j of the form $\chi(r, \theta, \varphi)\, e^{-iEt}$. If one takes out a δ function which arises from the integral over $t - t'$, the amplitude for detection is proportional to

$$\mathcal{E}_E(\vec{R}', \vec{R}) = \int_{-\infty}^{+\infty} dt\, e^{-iEt} G(0, \vec{R}'; t, \vec{R})\,, \qquad (4.21)$$

where $\vec{R}'$ and $\vec{R}$ denote respectively (r', θ', ϕ') and (r, θ, φ) and the radial and angular integrals over the functions h and χ have been factored out. Using the result derived above that $G(x', x)$ is analytic in a strip of width $\pi\kappa_C^{-1}$ below the real t axis, one can displace the contour in (4.21) down $\pi\kappa_C^{-1}$ to obtain

$$\mathcal{E}_E(\vec{R}', \vec{R}) = \exp(-\pi E_C \kappa_C^{-1}) \int_{-\infty}^{+\infty} dt\, e^{-iEt} G(0, \vec{R}', t - i\pi\kappa_C^{-1}, \vec{R}) . \tag{4.22}$$

By Eqs. (4.16) and (4.17) the point $(t - i\pi\kappa_C^{-1}, r, \theta, \varphi)$ is the point in region III obtained by reflecting in the origin of the U, V plane. Thus

$$\begin{pmatrix}\text{amplitude for particle of energy } E \text{ to propagate} \\ \text{from region II and be absorbed by observer}\end{pmatrix} = \exp(-\pi E \kappa_C^{-1}) \begin{pmatrix}\text{amplitude for particle with energy} \\ E \text{ to propagate from region III and} \\ \text{be absorbed by observer}\end{pmatrix} . \tag{4.23}$$

By time-reversal invariance the latter amplitude is equal to the amplitude for the observer's detector in an excited state to emit a particle with energy E which travels to region II. Therefore

$$\begin{pmatrix}\text{probability for detector to absorb} \\ \text{a particle from region II}\end{pmatrix} = \exp(-2\pi E \kappa_C^{-1}) \begin{pmatrix}\text{probability for detector to emit} \\ \text{a particle to region II}\end{pmatrix} . \tag{4.24}$$

This is just the condition for the detector to be in thermal equilibrium at a temperature

$$T = (2\pi)^{-1}\kappa_C = (12)^{-1/2}\pi^{-1}\Lambda^{1/2} . \tag{4.25}$$

The observer will therefore measure an isotropic background of thermal radiation with the above temperature. Because all timelike geodesics are equivalent under the de Sitter group, any other observer will also see an isotropic background with the same temperature even though he is moving relative to the first observer. This is yet another illustration of the fact that different observers have different definitions of particles. It would seem that one cannot, as some authors have attempted, construct a unique observer-independent renormalized energy-momentum tensor which can be put on the right-hand side of the classical Einstein equations. This subject will be dealt with in another paper.[16]

Another way in which one can derive the result that a freely moving observer in de Sitter space will see thermal radiation is to note that the propagator $G(x, x')$ is an analytic function of the coordinates T, S, T', S', or alternatively U, V, U', V' except when x and x' can be joined by null geodesics. On the other hand, the static-time coordinate t is a multivalued function of T and S or U and V, being defined only up to an integral multiple of $2\pi i\kappa_C^{-1}$. Thus the propagator $G(x', x)$ is a periodic function of t with period $2\pi i\kappa_C^{-1}$. This behavior is characteristic of what are known as "thermal Green's functions."[33] These may be defined (for interacting fields as well as the noninteracting case considered here) as the expectation value of the time-ordered product of the field operators, where the expectation value is taken not in the vacuum state but over a grand canonical ensemble at some temperature $T = \beta^{-1}$. Thus

$$G_T(x', x) = i\,\mathrm{Tr}[e^{-\beta H}\mathcal{J}\,\phi(x')\phi(x)]/\mathrm{Tr}\,e^{-\beta H} , \tag{4.26}$$

where $\mathcal{J}$ denotes Wick time-ordering and H is the Hamiltonian in the observer's static frame. ϕ is the quantum field operator and Tr denotes the trace taken over a complete set of states of the system. Therefore

$$\begin{aligned} -iG_T(\vec{R}', t', \vec{R}, t) &= \mathrm{Tr}[e^{-\beta H}\mathcal{J}\,\phi(\vec{R}, t)\varphi(\vec{R}, t')]/\mathrm{Tr}\,e^{-\beta H} \\ &= \mathrm{Tr}[e^{-\beta H}\mathcal{J}\,\varphi(\vec{R}', t')e^{\beta H}e^{-\beta H}\phi(\vec{R}, t)]/\mathrm{Tr}\,e^{-\beta H} \\ &= \mathrm{Tr}[e^{-\beta H}\mathcal{J}\,\phi(\vec{R}', t' + i\beta)\phi(\vec{R}', t)]/\mathrm{Tr}\,e^{-\beta H} \\ &= -iG_T(\vec{R}', t' + i\beta; \vec{R}, t) . \end{aligned} \tag{4.27}$$

Since

$$\phi(\vec{R}, t) = e^{-\beta H}\phi(\vec{R}, t - i\beta)e^{\beta H} . \tag{4.28}$$

Thus the thermal propagator is periodic in $t - t'$ with period iT^{-1}. One would expect $G_T(x', x)$ to have singularities when x and x' can be connected by a null geodesic and these singularities would be repeated periodically in the complex $t' - t$ plane. It therefore seems that the propagator $G(x', x)$ that we have defined by a path integral is the same as the thermal propagator $G_T(x', x)$ for a grand canonical ensemble at temperature T $T = (2\pi)^{-1}\kappa_C$ in the observer's static frame. Thus to the observer it will seem as if he is in a bath of blackbody radiation at the above temperature. It is interesting to note that a similar result was found for two-dimensional de Sitter space by Figari, Hoegh-Krohn, and Nappi[34] although they did not appreciate its significance in terms of particle creation.

The correspondence between $G(x',x)$ and the thermal Green's function is the same as that which has been pointed out in the black-hole case by Gibbons and Perry.[35] As in their paper, one can argue that because the free-field propagator $G(x',x)$ is identical with the free-field thermal propagator $G_T(x',x)$, any n-point interacting Green's function $\hat{G}$ which can be constructed by perturbation theory from G in a renormalizable field theory will be identical to the n-point interacting thermal Green's function constructed from G_T in a similar manner. This means that the result that an observer will think himself to be immersed in blackbody radiation at temperature $T=\kappa_C(2\pi)^{-1}$ will be true not only in the free-field case that we have treated but also for fields with mutual interactions and self-interactions. In particular, one would expect it to be true for the gravitational field, though this is, of course, not renormalizable, at least in the ordinary sense.

It is more difficult to formulate the propagator for higher-spin fields in terms of a path integral. However, it seems reasonable to define the propagators for such fields as solutions of the relevant inhomogeneous wave equation with the boundary conditions that the propagator from a point x' in region I is an analytic function of x in the upper half U plane and lower half V plane on the complexified horizon. With this definition one obtains thermal radiation just as in the scalar case.

V. PARTICLE CREATION IN BLACK-HOLE DE SITTER SPACES

For the reasons given in Sec. III one would expect that a solution of Einstein's equations with positive cosmological constant which contained a black hole would settle down eventually to one of the Kerr–Newman–de Sitter solutions described in Sec. II. We shall therefore consider what would be seen by an observer in such a solution. Consider first the Schwarzschild–de Sitter solution. Suppose the observer moves along a world line λ of constant r, θ, and ϕ in region I of Fig. 4. The world line λ coincides with an orbit of the static Killing vector $K=\partial/\partial t$. Let $\varphi^2=g(K,K)$ on λ. One would expect that the observer would see thermal radiation with a temperature $T_C=(2\pi\psi)^{-1}\kappa_C$ coming from all directions except that of the black hole and thermal radiation of temperature $T_H=(2\pi\varphi)^{-1}\kappa_H$ coming from the black hole. The factor ψ appears in order to normalize the static Killing vector to have unit magnitude at the observer. The variation of ψ with r can be interpreted as the normal red-shifting of temperature.

There are, however, certain problems in showing that this is the case. These difficulties arise from the fact that when one has two or more sets of horizons with different surface gravities one has to introduce separate Kruskal-type coordinate patches to cover each set of horizons. The coordinates of one patch will be real analytic functions of the coordinates of the next patch in some overlap region between the horizons in the real manifold. However, branch cuts arise if one continues the coordinates to complex values. To see this, let U_1, V_1 be Kruskal coordinates in a patch covering a pair of intersecting horizons with a surface gravity κ_1 and let U_2, V_2 be a neighboring coordinate patch covering horizons with surface gravity κ_2. In the overlap region one has

$$V_1U_1^{-1}=-e^{2\kappa_2 t}\,, \tag{5.1}$$

$$V_2U_2^{-1}=-e^{2\kappa_2 t}\,. \tag{5.2}$$

Thus

$$-V_2U_2^{-1}=(-V_1)^P U_1^{-P}\,, \tag{5.3}$$

where $P=\kappa_2\kappa_1^{-1}$. There is thus a branch cut in the relation between the two coordinate patches if $\kappa_2\neq\kappa_1$.

One way of dealing with this problem would be to imagine perfectly reflecting walls between each black-hole horizon and each cosmological horizon. These walls would divide the manifold up into a number of separate regions each of which could be covered by a single Kruskal-coordinate patch. In each region one could construct a propagator as before but with perfectly reflecting boundary conditions at the walls. By arguments similar to those given in the previous section, these propagators will have the appropriate periodic and analytic properties to be thermal Green's functions with temperatures given by the surface gravities of the horizons contained within each region. Thus an observer on the black-hole side of a wall will see thermal radiation with the black-hole temperature, while an observer on the cosmological side of the wall will see radiation with the cosmological temperature. One would expect that, if the walls were removed, an observer would see a mixture of radiation as described above.

Another way of dealing with the problem would be to define the paopagator $G(x',x)$ to be a solution of the inhomogeneous wave equation on the real manifold which was such that if the point were extended to complex values of a Krushal-type-coordinate patch covering one set of intersecting horizons, it would be analytic on the complexified horizon in the upper half or lower half U or V plane depending on whether the point x was respectively to the future or the past of $V=0$ or $U=0$. Then, using a similar argument to that in the previous section about the dependence of the propagator on initial data on the complexified horizon, one can show that the propagator $G(x', x)$ between a point x' in region I and a point x in region II_C is analytic in a strip of width $\pi\kappa_C^{-1}$ below the real axis of the complex t plane. Similarly, the propagator $G(x', x)$ between a point x' in region I and a point x in region II_H will be analytic in a strip of width $\pi\kappa_H^{-1}$. Using these results one can show that

$$\begin{pmatrix}\text{probability of a particle of energy } E,\\ \text{relative to the observer, propagating}\\ \text{from } \mathscr{I}^+ \text{ to observer}\end{pmatrix} = \exp[-(E2\pi\psi\kappa_C^{-1})]\begin{pmatrix}\text{probability of a particle of energy } E,\\ \text{relative to the observer, propagating}\\ \text{from observer to } \mathscr{I}^+\end{pmatrix}, \tag{5.4}$$

and similarly the probability of propagating from the future singularity of the black hole will be related by the appropriate factor to the probability for a similar particle to propagate from the observer into the black hole. These results establish the picture described at the beginning of this section.

One can derive similar results for the Kerr-de Sitter spaces. There is an additional complication in this case because there is a relative angular velocity between the black hole and the cosmological horizon. An observer in region I who is at a constant distance r from the black hole and who is nonrotating with respect to distant stars will move on an orbit of the Killing vector K which is null on the cosmological horizon. For such an observer the probability of a particle of energy E, relative to the observer, propagating to him from beyond the future cosmological horizon will be $\exp[-(2\pi\psi E\kappa_C^{-1})]$ times the probability for a similar particle to propagate from the observer to beyond the cosmological horizon. The probabilities for emission and absorption by the black hole will be similarly related except that in this case the energy E will be replaced by $E-n\Omega_H$, where n is the aximuthal quantum number or angular momentum of the particle about the axis of rotation of the black hole and Ω_H is the angular velocity of the black-hole horizon relative to the cosmological horizon. As in the ordinary black-hole case, the black hole will exhibit superradiance for modes for which $E < n\Omega_H$. In the case that the observer is moving on the orbit of a Killing vector K which is rotating with respect to the cosmological horizon, one again gets similar results for the radiation from the cosmological and black-hole horizons with E replaced by $E-n\Omega_C$ and $E-n\Omega_H$, respectively. Where Ω_C and Ω_H are the angular velocities of the cosmological and black-hole horizons relative to the observers frame and are defined by the requirement that $K+\Omega_C\tilde{K}$ and $K+\Omega_H\tilde{K}$ should be null on the cosmological and black-hole horizons.

VI. IMPLICATIONS AND CONCLUSIONS

We have shown that the close connection between event horizons and thermodynamics has a wider validity than the ordinary black-hole situations in which it was first discovered. As observer in a cosmological model with a positive cosmological constant will have an event horizon whose area can be interpreted as the entropy or lack of information that the observer has about the regions of the universe that he cannot see. When the solution has settled down to a stationary state, the event horizon will have associated with it a surface gravity κ which plays a role similar to temperature in the classical first law of event horizons derived in Sec. III. As was shown in Sec. IV., this similarity is more than an analogy: The observer will detect an isotropic background of thermal radiation with temperature $(2\pi)^{-1}\kappa$ coming, apparently, from the event horizon. This result was obtained by considering what an observer with a particle detector would actually measure rather than by trying to define particles in an observer-independent manner. An illustration of the observer dependence of the concept of particle is the result that the thermal radiation in de Sitter space appears isotropic and at the same temperature to every geodesic observer. If particles had an observer-independent existence and if the radiation appeared isotropic to one geodesic observer, it would not appear isotropic to any other geodesic observer. Indeed, as an observer approached the first observer's future event horizon the radiation would diverge. It seems clear that this observer dependence of particle creation holds in the case of black holes as well: An observer at constant distance from a black hole will observe a steady emission of thermal radiation but an observer falling into a black hole will not observe any divergence in the radiation as he approaches the first-observer's event horizon.

A consequence of the observer dependence of particle creation would seem to be that the back reaction must be observer-dependent also, if one assumes, as seems reasonable, that the mass of the detector increases when it absorbs a particle and therefore the gravitational field changes. This will be discussed further in another paper,[16] but we remark here that it involves the abandoning of the concept of an observer-independent metric for spacetime and the adoption of something like the Everett-Wheeler interpretation of quantum mechanics.[36] The latter viewpoint seems to be required anyway when dealing with the quantum mechanics of the whole universe rather than an isolated system.

If a geodesic observer in de Sitter space chooses not to absorb any of the thermal radiation, his energy and entropy do not change and so one would not expect any change in the solution. However, if he does absorb some of the radiation, his energy and hence his gravitational mass will increase. If the solution now settles down again to a new stationary state, it follows from the first law of event horizons that the area of the cosmological event horizon will be less than it appeared to be before. One can interpret this as a reduction in the entropy of the universe beyond the event horizon caused by the propagation of some radiation from this region to the observer. Unlike the black-hole case, the surface gravity of the cosmological horizon decreases as the horizon shrinks. There is thus no danger of the observer's cosmological event horizon shrinking catastrophically around him because of his absorbing too much thermal radiation. He has, however, to be careful that he does not absorb so much radiation that his particle detector undergoes gravitational collapse to produce a black hole. If this were to happen, the black hole would always have a higher temperature than the surrounding universe and so would radiate energy faster than it absorbs it. It would therefore evaporate, leaving the universe as it was before the observer began to absorb radiation.

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