Limiti notevoli

$$\lim_{x\to 0}\frac{\sin x}{x}=1 \qquad [\text{equiv. } \sin x \backsim x \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2} \qquad \left[\text{equiv. } \cos x \backsim 1-x^2/2 \text{ per } x\to 0\right]$$

$$\lim_{x\to 0}\frac{\tan x}{x}=1 \qquad [\text{equiv. } \tan x \backsim x \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{\arcsin x}{x}=1 \qquad [\text{equiv. } \arcsin x \backsim x \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{\arctan x}{x}=1 \qquad [\text{equiv. } \arctan x \backsim x \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{\log_a(1+x)}{x}=\log_a e \qquad [\text{equiv. } \log_a(1+x) \backsim x\log_a e \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log a \qquad [\text{equiv. } a^x \backsim 1+x\log a \text{ per } x\to 0]$$

$$\lim_{x\to 0}\frac{(1+x)^\alpha-1}{x}=\alpha \quad \forall\alpha>0 \qquad [\text{equiv. } (1+x)^\alpha \backsim 1+\alpha x \text{ per } x\to 0]$$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=e$$

Gerarchia degli infiniti

$$\lim_{x\to+\infty}\frac{a^x}{x^p}=+\infty \qquad \forall p\in\mathbb{R}, a>1$$

$$\lim_{x\to-\infty}|x|^p a^x=0 \qquad \forall p\in\mathbb{R}, a>1$$

$$\lim_{x\to+\infty}\frac{\log_a x}{x^p}=0 \qquad \forall p>0, a>1$$

$$\lim_{x\to 0^+}x^p\log_a x=0 \qquad \forall p>0, a>1$$

Generalizzazione dei limiti notevoli

Se $t(x) \to 0$ per $x \to \overline{x}$ si ottiene

$$\lim_{x\to\overline{x}} \frac{\sin t(x)}{t(x)} = 1 \qquad \text{[equiv. } \sin t(x) \backsim t(x) \text{ per } x \to \overline{x}]$$

$$\lim_{x\to\overline{x}} \frac{1-\cos t(x)}{t(x)^2} = \frac{1}{2} \qquad \left[\text{equiv. } 1-\cos t(x) \backsim \frac{t(x)^2}{2} \text{ per } x \to \overline{x}\right]$$

$$\lim_{x\to\overline{x}} \frac{\tan t(x)}{t(x)} = 1 \qquad \text{[equiv. } \tan t(x) \backsim t(x) \text{ per } x \to \overline{x}]$$

$$\lim_{x\to\overline{x}} \frac{\arcsin t(x)}{t(x)} = 1 \qquad \text{[equiv. } \arcsin t(x) \backsim t(x) \text{ per } x \to \overline{x}]$$

$$\lim_{x\to\overline{x}} \frac{\arctan t(x)}{t(x)} = 1 \qquad \text{[equiv. } \arctan t(x) \backsim t(x) \text{ per } x \to \overline{x}]$$

$$\lim_{x\to\overline{x}} \frac{\log_a(1+t(x))}{t(x)} = \log_a e \qquad \text{[equiv. } \log_a(1+t(x)) \backsim t(x)\log_a e \text{ per } x \to \overline{x}]$$

$$\lim_{x\to\overline{x}} \frac{a^{t(x)}-1}{t(x)} = \log a \qquad \text{[equiv. } a^{t(x)}-1 \backsim t(x)\log a \text{ per } x \to \overline{x}]$$

$$\lim_{x\to 0} \frac{(1+t(x))^\alpha-1}{t(x)} = \alpha \quad \forall \alpha > 0 \qquad \text{[equiv. } (1+t(x))^\alpha-1 \backsim \alpha t(x) \text{ per } x \to \overline{x}]$$

Derivate delle funzioni elementari

$f(x) = x^\alpha \quad (\alpha \in \mathbb{R})$	$f'(x) = \alpha x^{\alpha-1}$
$f(x) = a^x \quad (a > 0, a \neq 1)$	$f'(x) = a^x \log a$
$f(x) = \log_a x \quad (a > 0, a \neq 1)$	$f'(x) = \frac{1}{x} \log_a e$
$f(x) = \sin x$	$f'(x) = \cos x$
$f(x) = \cos x$	$f'(x) = -\sin x$
$f(x) = \tan x$	$f'(x) = 1 + \tan^2 x = \frac{1}{\cos^2 x}$
$f(x) = \cot x$	$f'(x) = -(1 + \cot^2 x) = -\frac{1}{\sin^2 x}$
$f(x) = \arcsin x$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$
$f(x) = \arccos x$	$f'(x) = -\frac{1}{\sqrt{1-x^2}}$
$f(x) = \arctan x$	$f'(x) = \frac{1}{1+x^2}$
$f(x) = \operatorname{arccot} x$	$f'(x) = -\frac{1}{1+x^2}$
$f(x) = \lvert x \rvert$	$f'(x) = \operatorname{sign} x = \frac{x}{\lvert x \rvert} = \frac{\lvert x \rvert}{x}$

Regole di derivazione

$$(f + g)'(x) = f'(x) + g'(x)$$

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$