Materiale di discussione n. 12

INTERTHEORY RELATIONS IN CLASSICAL ECONOMICS: SRAFFA AND MARX

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ABSTRACT

It will be shown that if Sraffa's prices are "measured" in terms of quantities of embodied labour, then the total amount of Sraffa's profits is equal to Marx's surplus value.

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INTRODUCTORY REMARKS

The scope of the present short note is to prove a mathematical proposition concerning the relation between Sraffa's and Marx's theories. (1)

As is well known, it is generally impossible to build a one-to-one mapping between Sraffa's prices and Marx's labour values. Such a result implies that the two theories cannot be "transformed" one into the other at disaggregated level. Still, Morishima has shown how Sraffa's rate of profit and Marx's rate of surplus value assume the same type of monotonic trend, (2) thus suggesting that between the two theories some link could be found at a "macroeconomic" level. Yet, in order to achieve such a result, it is necessary to be able to compare the total amount of Sraffa's profits with Marx's surplus value.

Such a problem is often called "the problem of the two normalizations" and it can be expressed as follows. First, in the Sraffa model the author's original price normalization is replaced by setting the price of the gross product equal to the quantity of labour embodied in the gross product, thus obtaining the first normalization. By setting the total amount of Sraffa's profits equal to Marx's surplus value it is possible to define the second normalization.

The problem is the following: are the two normalizations mutually compatible? It is obvious that if the answer is positive we are in possession of a powerful tool for operating a meaningful "translation" between the two theories.

The present contribution has consisted in building a general mathematical procedure showing how, as a matter of fact, the two normalizations are perfectly compatible, thus opening the way to a "macroeconomic" solution of Marx's "transformation" problem.

THE DEMONSTRATION

The Sraffa's model [3] we will employ in the following proof consists of a system of equations based on n technological processes, each producing a single commodity. Joint production and alternative processes will be excluded. For each process, a positive labour coefficient will be defined; the total amount of labour will be set equal to one. The n.n-matrix of input coefficients will be normalized by setting the gross production of each commodity equal to one.

The symbols appearing in the formulas are listed below:

A	n.n-matrix of input coefficients [4];
I	n.n-unit matrix. It will be used to represent the outputs of the production processes.
u	row n-vector with every component equal to one. It will be employed to sum the components of column n-vectors;
f	column n-vector of quantities of labour used in the production processes ($f > 0$, $uf = 1$);
p	column n-vector of Sraffa prices ($p > 0$);
q	column n-vector of quantities of embodied labour ($q > 0$);
w	scalar wage rate ($0 \leq w \leq 1$);

r scalar rate of profit ($0 \le r \le R$, where R is the maximum rate of profit);

$s=1-w$ indicates the scalar rate of surplus value;

H represents the total amount of Sraffa's profits.

Let's consider the Sraffa model including the author's original normalization: (5)

$$Ap\,(1 + r) + fw = p \qquad [1]$$

$$u\,(I - A)\,p = 1 \qquad [2]$$

Following the procedure sketched in the previous paragraph, we now drop Sraffa's original normalization, represented by equation [2], and we replace it by setting the price of the gross product equal to the quantity of labour embodied in the gross product, thus "measuring" Sraffa's prices in terms of quantities of embodied labour. For such a scope, it is necessary to calculate vector q which can be obtained from the following equations:

$$Aq + f = q \qquad [3]$$

from which:

$$q = (I - A)^{-1}\,f \qquad [4]$$

The modified Sraffa model is thus the following: (6)

$$Ap\,(1 + r) + fw = p \qquad [1]$$

$$up = uq \qquad [5]$$

From equations [1], with easy passages, it follows: (7)

$$up = u\,(I - A\,(1 + r)\,)^{-1}fw = \sum_{k=0}^{\infty} uA^k\,(1 + r)^k fw \qquad [6]$$

In the summation after the last sign of equality the addenda can be divided into two groups. The first one includes those which do not contain the factor r or any of its powers, thus representing wage shares. The second group includes the addenda which contain the factor r or its powers, therefore representing profit shares. Remembering the formula of the powers of binomials, expression [6] can be rewritten as:

$$up = (\sum_{k=0}^{\infty} uA^k fw) + H \qquad [7]$$

where H is the summation of all the addenda which include the factor r or its powers. Therefore, in expression [7] at the right of the sign of equality the first summation represents the total amount of wages paid during the production cycle, while H represents the total amount of profits.

In order to complete the demonstration, within Marx's theory it is necessary to find the formula which is linked by analogy to expression [7]. Remembering Marx's definition, equations [3] can be rewritten as follows:

$$Aq + fw + fs = q \qquad [8]$$

and therefore: (8)

$$q = (I - A)^{-1} (fw + fs) = \sum_{k=0}^{\infty} A^k fw + \sum_{k=0}^{\infty} A^k fs \qquad [9]$$

from which we obtain the following expression:

$$uq = \sum_{k=0}^{\infty} uA^k fw + \sum_{k=0}^{\infty} uA^k fs \qquad [10]$$

In expression [10] at the right of the sign of equality the first summation represents the total amount of wages, while the second one indicates the surplus value. Taking into consideration normalization [5], from expressions [7] and [10] we obtain the concluding formula:

$$H = \sum_{k=0}^{\infty} uA^k fs \qquad [11]$$

which sets the total amount of profits equal to the surplus value.

NOTES

(1) The following argumentation does not imply the use of any advanced mathematical tool. On the other hand the reader should be familiar with the two theories and the relative bibliography.

(2) See Morishima (1973), p. 64.

(3) More precisely, we will refer to Sraffa (1960), part I, chapter II, paragraph 11.

(4) In order to obtain economically meaningful solutions, matrix A is required to be non-negative, indecomposable, vital. For a formalization of Sraffa (1960) see, among others, Newman (1962) and Tucci (1976).

(5) Row by column matrix product is used in the following notation. Remember that model [1] – [2] admits economically meaningful solutions if: $0 \leq r \leq R$. See references in note (4).

(6) If: $0 \leq r \leq R$, model [1] – [5] admits economically meaningful solutions. Moreover, with easy passages it can be shown that, as in model [1] – [2], in model [1] – [5] when: $0 \leq r \leq R$, then: $0 \leq w \leq 1$.

(7) Remember that according to a mathematical property:

$$(I - A(1 + r))^{-1} = \sum_{k=0}^{\infty} A^k (1 + r)^k.$$

See, among others, Seneta (1973), p. 186.

(8) It results: $(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$.

See note (7).

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