1.1 Concetti base

Lunghezza d'onda:

$$\lambda = \frac{c}{f} \qquad \text{con } c = 3 \cdot 10^8 \ m/s$$

Onda:

$$s(t) = A \cdot Cos(2 \cdot \pi \cdot f)$$

Sigle:

[V|U|S|E]HF = [Very|Ultra|Super|Extra] High Frequency

Direttività & Guadagno:

$$D = \frac{\text{densità di potenza alla distanza } d \text{ nella direzione della massima radiazione}}{\text{densità di potenza media alla distanza } d}$$

$$G = \frac{\text{densità di potenza alla distanza } d \text{ nella direzione della massima radiazione}}{\mathrm{P_T} / 4\pi d^2}$$

Orizzonti:

Orizzonte ottico $\Rightarrow d = 3.57\sqrt{h}$

Orizzonte radio $\Rightarrow d = 3.57\sqrt{K \cdot h}$ con $K \approx 4/3$

$d = [Km], \ h = [m]$

1.2 Attenuazione

Decibel:

$$dB = 10 \cdot Log_{10}(P_1 / P_2)$$

dBm & dBW:

$$mW \to dBm \Rightarrow dBm = 10 \cdot Log_{10}\left(P_{[mW]} / 1 \ mW\right)$$

$$dB \to numero \Rightarrow numero = 10^{\frac{dB}{10}}$$

$$numero \to dB \Rightarrow dB = 10 \cdot Log_{10}(numero)$$

$$W \to dBW \Rightarrow dBW = 10 \cdot Log_{10}\left(P_{[W]} / 1 \ W\right)$$

$$1 \ dBW = 30 \ dBm$$

$$1 \ dBm = 1 \ mW$$

Proprietà dei dBm & dBW:

$$dBm = 10 \cdot Log_{10}\left(P_{[W]} / 1 \ mW\right) = P_{[dB]} + 30_{[dBm]}$$

$$P_1 \cdot P_{2[dBm]} = P_{1[dBm]} + P_{2[dB]}$$

$$3 \ dB = \cdot 2$$

$$6 \ dB = \cdot 4$$

$$9 \ dB = \cdot 8$$

$$n0 \ dB = \cdot 10^n \qquad \text{con } n \in \mathbb{N}$$

Perdita del segnale (Path loss):

$$P_{loss[dB]} = P_{t[dBm]} - P_{r[dBm]}$$

Attenuazione della potenza alla distanza *d* (Power attenuation):

$$P_a(d) = \frac{G_t \cdot P_t}{4\pi d^2}$$

Potenza ricevuta da un'antenna isotropica:

$$P_r(d) = P_a(d) \cdot G_r \cdot A_e = \frac{G_t \cdot P_t}{4\pi d^2} \cdot G_r \cdot \frac{\lambda^2}{4\pi}$$

Area effettiva:

$$A_e = \frac{\lambda^2}{4\pi}$$

Modello di Friis (Free-space model):

$$P_r(d) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 \cdot L} = P_t \cdot \frac{G_t \cdot G_r}{L}\left(\frac{c}{4\pi fd}\right)^2 \qquad \text{con } P_t \cdot G_t = \text{EIRP}$$

$$P_r(d)_{[dBm]} = P_{t[dBm]} + 10 \cdot Log_{10}(G_t) + 10 \cdot Log_{10}(G_r) + 20 \cdot Log_{10}(\lambda) - 20 \cdot Log_{10}(4\pi) +$$
$$- 20 \cdot Log_{10}(d) - 10 \cdot Log_{10}(L)$$

Perdita del segnale alla distanza *d*:

$$P_{loss}(d)_{[dB]} = 10 \cdot Log_{10}\left(\frac{P_t}{P_r}\right) = 10 \cdot Log_{10}\left[\frac{L}{G_t \cdot G_r}\left(\frac{4\pi d}{\lambda}\right)^2\right] =$$

$$= 20 \cdot Log_{10}(d) - 10 \cdot Log_{10}\left(\frac{G_t \cdot G_r}{L}\right) - 20 \cdot Log_{10}\left(\frac{\lambda}{4\pi}\right) =$$

$$= 20 \cdot Log_{10}(d) + 20 \cdot Log_{10}(f) - 10 \cdot Log_{10}\left(\frac{G_t \cdot G_r}{L}\right) - 20 \cdot Log\left(\frac{c}{4\pi}\right) =$$

$$= 20 \cdot Log_{10}(d) + 20 \cdot Log_{10}(f) - 10 \cdot Log_{10}\left(\frac{G_t \cdot G_r}{L}\right) - 147.56$$

Free space loss:

$$L_{free}(d) = \left(\frac{\lambda}{4\pi d}\right)^{-2}$$

$$L_{free}(d)_{[dB]} = -20 \cdot Log_{10}\left(\frac{\lambda}{4\pi d}\right) = -20 \cdot Log_{10}\left(\frac{c/f}{4\pi d}\right) = 20 \cdot Log_{10}(d) + 20 \cdot Log_{10}(f) - 147.56$$

Potenza ricevuta a partire da una distanza di riferimento:

$$P_r(d)_{[dB]} = P_r(d_o) \cdot \left(\frac{d_o}{d}\right)^2 \qquad \text{con } d > d_o$$

$$P_r(d)_{[dBm]} = P_r(d_0)_{[dBm]} + 10 \cdot \eta \cdot Log_{10}\left(\frac{d_o}{d}\right) = 10 \cdot Log_{10}\left[P_r(d_o)_{[mW]}\right] + 10 \cdot \eta \cdot Log_{10}\left(\frac{d_o}{d}\right)$$

$$d = 10^{\frac{P_r(d_0)_{[dBm]} - P_r(d)_{[dBm]} + 10 \cdot \eta \cdot Log_{10}(d_o)}{10 \cdot \eta}}$$

Potenza ricevuta a partire da una distanza e da una frequenza di riferimento:

$$P_r(d, f) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d^2 \cdot L} = \frac{P_t \cdot G_t \cdot G_r \cdot (c/f_0)^2}{(4\pi)^2 \cdot d_0^2 \cdot L} \cdot \left(\frac{f_0 \cdot d_0}{f \cdot d}\right)^2 = P_r(d_0, f_0)_{[dBm]} - 20 \cdot Log_{10}(f) - 20 \cdot Log_{10}(d)$$

Geometria del modello a due raggi:

$$d_{dir} = \sqrt{d^2 + (h_t - h_r)^2} = d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2} \approx d\sqrt{1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2}$$

$$d_{ref}=\sqrt{d^2+\left(h_t+h_r\right)^2}=d\sqrt{1+\left(\frac{h_t+h_r}{d}\right)^2}\approx d\sqrt{1+\frac{1}{2}\left(\frac{h_t+h_r}{d}\right)^2}$$

$$d_{ref}-d_{dir}\approx d\sqrt{1+\frac{1}{2}\left(\frac{h_t-h_r}{d}\right)^2}-d\sqrt{1+\frac{1}{2}\left(\frac{h_t+h_r}{d}\right)^2}=2\frac{h_t\cdot h_r}{d}$$

Raggio diretto $\propto A\cdot Cos\left[2\pi f\left(t-\frac{d_{dir}}{c}\right)\right]$ con A = attenuazione lungo il percorso diretto

Raggio riflesso $\propto B\cdot Cos\left[2\pi f\left(t-\frac{d_{ref}}{c}\right)\right]$ con B = attenuazione lungo il percorso riflesso

$$\Delta\varphi=2\pi f\cdot\frac{\Delta d}{c}=\frac{2\pi}{\lambda}\cdot\Delta d=\frac{4\pi\cdot h_t\cdot h_r}{\lambda\cdot d}$$

$$E=E_{dir}\cdot\left(1+\rho e^{-j\cdot\Delta\varphi}\right)$$

$$E=E_{dir}\cdot\left(1+e^{-j\cdot\Delta\varphi}\right)=E_{dir}\cdot\left[1-Cos\left(\Delta\varphi\right)+j\cdot Sin\left(\Delta\varphi\right)\right]\quad\text{con }\rho=-1$$

$$\left|E\right|=\left|E_{dir}\right|\cdot\sqrt{1+Cos^2\left(\Delta\varphi\right)-2\cdot Cos\left(\Delta\varphi\right)+Sin^2\left(\Delta\varphi\right)}=2\cdot\left|E_{dir}\right|\cdot\sqrt{\frac{1-Cos\left(\Delta\varphi\right)}{2}}=2\cdot\left|E_{dir}\right|\cdot Sin\left(\frac{\Delta\varphi}{2}\right)$$

$$P_r\propto 4\cdot\left|E_{dir}\right|\cdot Sin^2\left(\frac{\Delta\varphi}{2}\right)\Rightarrow$$

$$P_r\left(d\right)=\frac{P_t\cdot G_t\cdot G_r}{L}\cdot\left(\frac{\lambda}{4\pi\cdot d}\right)^2\cdot 4\cdot Sin^2\left(\frac{2\pi\cdot h_t\cdot h_r}{\lambda\cdot d}\right)$$

$$\Rightarrow\frac{2\pi\cdot h_t\cdot h_r}{\lambda\cdot d}\approx\text{piccolo}\Rightarrow Sin^2\left(\frac{2\pi\cdot h_t\cdot h_r}{\lambda\cdot d}\right)\approx\left(\frac{2\pi\cdot h_t\cdot h_r}{\lambda\cdot d}\right)^2\Rightarrow$$

$$P_r\left(d\right)\approx\frac{P_t\cdot G_t\cdot G_r}{L}\cdot\left(\frac{\lambda}{4\pi\cdot d}\right)^2\cdot 4\left(\frac{2\pi\cdot h_t\cdot h_r}{\lambda\cdot d}\right)^2=\frac{P_t\cdot G_t\cdot G_r}{L}\cdot\frac{h_t^2\cdot h_r^2}{d^4}$$

Modello di Okumura-Hata:

f = frequenza [MHz]
d = distanza BS-MS [Km]
h_{bs} = altezza effettiva dell'antenna della BS [m]
h_{ms} = altezza effettiva dell'antenna della MS [m]

$$P_r\left(d\right)_{[dBm]}=P_t-P_{loss}\left(d\right)$$

$$\eta=\frac{P_{loss}\left(d\right)-P_{loss}\left(d_0\right)}{10\cdot\left[Log_{10}\left(d\right)-Log_{10}\left(d_0\right)\right]}$$

$$P_{loss}\left(d_0\right)=10\cdot Log_{10}\left[\frac{\left(4\cdot\pi\cdot d_0\right)^2}{\lambda^2}\right]$$

Area urbana:

$$L_{path[dB]}=69.55+26.16\cdot Log_{10}\left(f\right)+\left[44.9-6.55\cdot Log_{10}\left(h_{bs}\right)\right]\cdot Log_{10}\left(d\right)-13.82\cdot Log_{10}\left(h_{bs}\right)-a\left(h_{ms}\right)$$

grandi città $\Rightarrow a\left(h_{ms}\right)=3.2\cdot\left[Log_{10}\left(11.75h_{ms}\right)\right]^2-4.97$

piccole-medie città $\Rightarrow a\left(h_{ms}\right)=\left[1.1\cdot Log_{10}\left(f\right)-0.7\right]\cdot h_{ms}-\left[1.56\cdot Log_{10}\left(f\right)-0.8\right]$

Area sub-urbana:

$$L_{path[dB]}=L_p-2\cdot\left[Log_{10}\left(\frac{f}{28}\right)\right]^2-5.4$$

Area rurale:

$$L_{path[dB]}=L_p-4.78\cdot\left[Log_{10}\left(f\right)\right]^2+18.33\cdot Log_{10}\left(f\right)-40.94$$

1.3 Fading

Fading or multipath/short-term/small-scale/fast fading (distribuzione di Rayleigh):

$$e_r\left(t\right)=\sum_{k=1}^{N}a_k\cdot Cos\left(2\pi\cdot f_0\cdot t+\phi_k\right)\Rightarrow e_r\left(t\right)=Cos\left(2\pi\cdot f_0\cdot t\right)\cdot\underbrace{\sum_{k=1}^{N}a_k\cdot Cos\left(\phi_k\right)}_{X}+$$

$$-Sin\left(2\pi\cdot f_0\cdot t\right)\cdot\underbrace{\sum_{k=1}^{N}a_k\cdot Sin\left(\phi_k\right)}_{Y}=X\cdot Cos\left(2\pi\cdot f_0\cdot t\right)-Y\cdot Sin\left(2\pi\cdot f_0\cdot t\right)$$

$$a=\sqrt{X^2+Y^2}\quad\text{con}\begin{cases}a=\text{amplitude}\\\text{distribuzione di Rayleigh}\\X\text{ e }Y\text{ variabili random Gaussiane (dalla legge dei grandi numeri)}\end{cases}$$

$$f_a\left(x\right)=\Pr\left(x\le a<x+dx\right)=\frac{x}{\sigma^2}\cdot e^{-\frac{x^2}{2\sigma^2}}$$

$$E\left(a\right)=\int_0^\infty x\cdot\frac{x}{\sigma^2}\cdot e^{-\frac{x^2}{2\sigma^2}}\cdot dx=\sigma\cdot\sqrt{\frac{\pi}{2}}=1.253\cdot\sigma$$

$$Var\left(a\right)=\int_0^\infty x^2\cdot\frac{x}{\sigma^2}\cdot e^{-\frac{x^2}{2\sigma^2}}\cdot dx=\sigma^2\frac{\pi}{2}=\sigma^2\cdot\left(2-\frac{\pi}{2}\right)=0.4292\cdot\sigma^2$$

$$p=a^2=X^2+Y^2\quad\text{con}\begin{cases}p=\text{potenza}\\p\text{ distributione esponenziale}\end{cases}$$

Probability density function $\Rightarrow f_p\left(P\right)=\frac{1}{2\cdot\sigma^2}\cdot e^{-\frac{P^2}{2\sigma^2}}$

Probability distribution function $\Rightarrow F_p\left(P\right)=1-e^{-\frac{P^2}{2\sigma^2}}$

Outage probability con fading (potenza in mW):

$$\Pr_{out}=\Pr\left(P_{av}\le P_{th}\right)=F_p\left(P_{th}\right)=1-e^{-\frac{P_{th}}{P_{av}}}\quad\text{con}\begin{cases}P_{th}\text{ e }P_{av}\text{ numeri puri}\\P_{av}=2\cdot\sigma^2\end{cases}$$

$$\Pr_{out}=\Pr\left(P_{av}\le P_{th}\right)=\frac{1-Erf\left(g\right)}{2}=\frac{ErfC\left(g\right)}{2}$$

Shadowing or long-term/large-scale/lognormal/slow fading (distribuzione Lognormale):

$$P_r\left(d\right)_{[dBm]}=10\cdot Log_{10}\left[P_r\left(d_o\right)\right]+10\cdot\eta\cdot Log_{10}\left(\frac{d_o}{d}\right)+Y\quad\text{con}\begin{cases}Y\text{ variabile random Gaussiana}\\E\left(Y\right)=0\end{cases}$$

$$f_Y\left(P_{[dB]}\right)=\frac{1}{\sigma_{[dB]}\cdot\sqrt{2\pi}}\cdot e^{\frac{-\left(P_{[dB]}-P_{av[dB]}\right)^2}{2\sigma_{[dB]}^2}}\quad\text{con }P_{av}=\mu$$

$$Q\left(P_{[dB]}\right)=F_Y\left(P_{[dB]}\right)=\frac{1}{\sqrt{2\pi}}\cdot\int_{-\infty}^{P_{[dB]}}e^{-\frac{t^2}{2}}\cdot dt$$

Outage probability con shadowing & Erf(x):

$$\gamma = \frac{P_{th} - P_{av}}{\sigma_{[dB]} \cdot \sqrt{2}} < 0 \Rightarrow g = -\gamma > 0 \Rightarrow Erf(g) = -Erf(g)$$

$$\Pr_{out} = \Pr(P \le P_{th}) = \frac{1 - Erf(g)}{2} = \frac{ErfC(g)}{2}$$

$$\left.\begin{aligned} Erf(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \cdot dt \\ ErfC(x) &= 1 - Erf(x) \end{aligned}\right\} \Rightarrow \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} \cdot dt = \frac{1}{\sqrt{2\pi} \cdot \left(1/\sqrt{2}\right)} \int_{-x}^{x} e^{-\frac{t^2}{2 \cdot \left(1/\sqrt{2}\right)^2}} \cdot dt$$

1.4 Area di copertura di una cella

Fading margin & outage probability:

$$P_{loss[dB]} = P_{r[dB]} - P_{th[dBm]} - M_{[dB]}$$

$$P_{r[dB]} = P_{th[dBm]} + M_{[dB]}$$

$P_{r[dB]} = P_{th[dBm]} + M_{\text{Rayleigh}[dB]} + M_{\text{Lognormale}[dB]}$ con $M_{\text{Rayleigh}[dB]}$ e $M_{\text{Lognormale}[dB]}$ calcolati indipendentemente

$$M_{[dB]} = P_{r[dB]} - P_{th[dBm]} = 10 \cdot \left[Log_{10}\left(P_{av}(R)\right) - Log_{10}\left(P_{th}\right) \right]$$

$$\Pr_{out}(r) = \int_{-\infty}^{P_{th}} f_Y\left[P_{[dB]}(r)\right] \cdot dp_{[dB]}(r) = \frac{1}{2} \cdot ErfC\left[\frac{P_{av[dB]}(r) - P_{th[dB]}}{\sqrt{2} \cdot \sigma_{[dB]}}\right]$$

Outage probability alla distanza r:

$$P_{av}(r) = P_{av}(R) \cdot \left(\frac{R}{r}\right)^{\eta}$$

$$P_{av[dB]}(r) = P_{av[dB]}(R) - 10 \cdot \eta \cdot Log_{10}\left(\frac{r}{R}\right)$$

$$\Rightarrow P_{av[dB]}(R) = P_0(R) \Rightarrow$$

$$\Pr_{out}(r) = \frac{1}{2} \cdot ErfC\left(\frac{P_0(R) - P_{th} - 10 \cdot \eta \cdot Log_{10}(r/R)}{\sqrt{2} \cdot \sigma_{[dB]}}\right) = \frac{1}{2} \cdot ErfC\left(\frac{M - 10 \cdot \eta \cdot Log_{10}(r/R)}{\sqrt{2} \cdot \sigma_{[dB]}}\right)$$

Outage probability a bordo cella:

$$\Pr_{out}(\text{bordo cella}) = \frac{1}{2} \cdot ErfC\left(\frac{M}{\sqrt{2} \cdot \sigma_{[dB]}}\right)$$

Outage probability area:

$$\Pr_{out}(R) = \frac{1}{\pi \cdot R^2} \int_0^R \Pr_{out}(r) \cdot 2\pi r \cdot dr = \int_0^R \frac{r}{R} \cdot ErfC\left[\frac{M - 10 \cdot \eta \cdot Log_{10}(r/R)}{\sqrt{2} \cdot \sigma_{[dB]}}\right] \cdot \frac{dr}{R} =$$

$$= \int_0^1 x \cdot ErfC\left[\frac{M - 10 \cdot \eta \cdot Log_{10}(x)}{\sqrt{2} \cdot \sigma_{[dB]}}\right] \cdot dx = \int_0^1 x \cdot ErfC\left[\frac{Ln(m) - \eta \cdot Ln(x)}{\sqrt{2} \cdot \sigma}\right] \cdot dx \quad \text{con} \quad \begin{cases} \sigma = \dfrac{\sigma_{[dB]} \cdot Ln(10)}{10} \\ m = 10^{\frac{M}{10}} = \dfrac{P_0(R)_{[mW]}}{P_{th[mW]}} \end{cases}$$

$$\Rightarrow Q_1 = \frac{Ln(m)}{\sqrt{2} \cdot \sigma}; \; Q_2 = \frac{\sqrt{2} \cdot \sigma}{\eta} \Rightarrow$$

$$\Pr_{out}(R) = \int_0^1 x \cdot ErfC\left[Q_1 - \frac{Ln(x)}{Q_2}\right] \cdot dx \Rightarrow \theta = Q_1 - \frac{Ln(x)}{Q_2} \Rightarrow Q_2 \int_{Q_1}^{\infty} e^{2 \cdot Q_2 \cdot (Q_1 - \theta)} \cdot ErfC(\theta) \cdot d\theta =$$

$$= \left[-\frac{1}{2} \cdot e^{Q_2 \cdot (2Q_1 + Q_2)} \cdot Erf(\theta + Q_2) - \frac{1}{2} \cdot e^{2 \cdot Q_2 \cdot (Q_1 - \theta)} \cdot ErfC(\theta) \right]_{Q_1}^{\infty} = \frac{1}{2} \cdot ErfC(Q_1) - \frac{1}{2} \cdot e^{\left(2 \cdot Q_1 \cdot Q_2 + Q_2^2\right)} \cdot ErfC(Q_1 + Q_2)$$

2.2 Cluster & CCI

Area circonferenza ed esagono:

$$A_{circonferenza} = \pi \cdot r^2$$

$$A_{esagono} = \frac{3}{2} \cdot r^2 \cdot \sqrt{3}$$

Possibili cluster:

$$K = N_c = D_R^2 = i^2 + j^2 + ij$$

$$D_R = \sqrt{i^2 + j^2 + ij}$$

Serie romboidale $\Rightarrow K = 1,\ 3,\ 4,\ 7,\ 9,\ 12,\ 13,\ 16,\ 19,\ 21,\ 25,\ 27, \ldots$

Distanza tra due celle:

$$D = \sqrt{\left[(u_2 - u_1) \cdot Cos(30^o)\right]^2 + \left[(v_2 - v_1) + (u_2 - u_1) \cdot Sin(30^o)\right]^2} = \sqrt{(u_2 - u_1) + (v_2 - v_1)^2 + (u_2 - u_1) \cdot (v_2 - v_1)}$$

Distanza dal centro:

$D = \sqrt{i^2 + j^2 + ij} \cdot R \cdot \sqrt{3}$ con R raggio delle celle

$$D_R = \sqrt{i^2 + j^2 + ij}$$

Distanza di riuso:

$$D = R \cdot \sqrt{3 \cdot K}$$

$$K = \frac{1}{3} \cdot \left(\frac{D}{R}\right)^2$$

Fattore di riuso delle frequenze:

$$q = \frac{D}{R} = \sqrt{3 \cdot K}$$

CCI:

$$\left.\begin{aligned} \frac{S}{N} &= \frac{\text{Potenza del segnale } (S)}{\text{Potenza del rumore } (N_S) \ + \ \text{Potenza del segnale interferente } (I)} \\ \frac{S}{I} &= \frac{\text{Potenza del segnale } (S)}{\text{Potenza del segnale interferente } (I)} \end{aligned}\right\} \Rightarrow N_S \simeq 0 \Rightarrow \frac{S}{N} \approx \frac{S}{I}$$

$$CCI = \frac{S}{N} \approx \frac{S}{I} = \frac{\text{cost} \cdot R^{-\eta}}{\sum_{k=1}^{N_I} \text{cost} \cdot D^{-\eta}} = \frac{1}{N_I} \cdot \left(\frac{R}{D}\right)^{-\eta} = \frac{1}{N_I} \cdot \left(\frac{D}{R}\right)^{\eta} = \frac{q^{\eta}}{N_I}$$

$$CCI = \frac{S}{N} \approx \frac{S}{I} = \frac{1}{N_I} \cdot \left(\frac{R}{R \cdot \sqrt{3 \cdot K}}\right)^{-\eta} = \frac{1}{N_I} \cdot (3K)^{\frac{\eta}{2}}$$

$$CCI_{[dBm]} = 10 \cdot \frac{\eta}{2} \cdot Log_{10}(3K) - 10 \cdot Log_{10}(N_I)$$

$$K = \frac{\sqrt[\frac{\eta}{2}]{CCI \cdot N_I}}{3}$$

CCI & settorizzazione:

$$\text{3 settori, } 120° \Rightarrow \begin{cases} \left[\frac{S}{I}\right]_{120^o[dB]} = \left[\frac{S}{I}\right]_{omni[dB]} + 4.77_{[dB]} \\ N_I = 2 \end{cases}$$

$$\text{6 settori,} 60° \Rightarrow \begin{cases} \left[\frac{S}{I}\right]_{60^o[dB]} = \left[\frac{S}{I}\right]_{omni[dB]} + 7.78_{[dB]} \\ N_I = 1 \end{cases}$$

2.3 Pianificazione del traffico

Intensità del traffico (in Erlang):

Traffico generato dall'utente *i*-esimo $\Rightarrow A_i = \frac{n°chiamate}{ora} \cdot \frac{durata}{ora}$

Traffico generato da M utenti $\Rightarrow A = M \cdot A_i$

$$P\left[k \text{ chiamate attive, } M \text{ utenti}\right] = \binom{M}{k} A_i^k \left(1 - A_i\right)^{M-k} = \frac{M!}{(M-k)! \cdot k!} \cdot \left(\frac{A}{M}\right)^k \cdot \frac{\left(1 - \frac{A}{M}\right)^M}{\left(1 - \frac{A}{M}\right)^k}$$

Infiniti utenti:

$$P\left[k \text{ chiamate attive, } \infty \text{ utenti}\right] = \lim_{M \to \infty} \frac{M!}{(M-k)!} \cdot \frac{1}{k!} \cdot \frac{A^k}{M^k} \cdot \left(1 - \frac{A}{M}\right)^M \cdot \left(1 - \frac{A}{M}\right)^{-k} =$$

$$= \frac{A^k}{k!} \cdot \lim_{M \to \infty} \frac{M \cdot (M-1) \cdot \ldots \cdot (M-k+1)}{M^k} \cdot \left[\left(1 - \frac{A}{M}\right)^{-\frac{M}{A}}\right]^{-A} \cdot \left(1 - \frac{A}{M}\right)^{-k} = e^{-A} \frac{A^k}{k!}$$

$P_k(A) = e^{-A} \frac{A^k}{k!}$ se $A<<M$ si approssima la Poissonina alla Binomiale

Probabilità di blocco: Erlang-B:

$$\text{Utenti infiniti} \Rightarrow P(\text{blocco})_{[erl]} = \frac{\frac{A_o^C}{C!}}{\sum_{j=0}^{C} \frac{A_o^j}{j!}} = E_{1,C}(A_o) \Rightarrow E_{1,C}(A_o) = \frac{A_o \cdot E_{1,C-1}(A_o)}{C + A_o \cdot E_{1,C-1}(A_o)}$$

$$\text{Utenti finiti} \Rightarrow P(\text{blocco})_{[erl]} = \frac{A_i^C \binom{M-1}{C}}{\sum_{k=0}^{C} A_i^k \binom{M-1}{i}}$$

Efficienza di utilizzo di un canale:

$$A_{C[erl]} = A_{o[erl]} \cdot \left[1 - P(\text{blocco})_{[erl]}\right] \quad \text{con} \begin{cases} A_C = \text{ traffico smaltito} \\ A_o = \text{traffico offerto} \\ P(\text{blocco})_{[erl]} = \text{ probabilità di blocco} \end{cases}$$

Traffico bloccato = $A_o \cdot P(\text{blocco})_{[erl]}$

$$\text{Efficienza} = \eta = \frac{A_c}{C} = \frac{A_o \cdot \left[1 - P(\text{blocco})_{[erl]}\right]}{C} = \frac{A_o \cdot \left[1 - E_{1,C}(A_o)\right]}{C} \approx \frac{A_o}{C} \quad \text{con } C = n° \text{ di canali}$$