SPACECRAFT EQUIPMENT VIBRATION QUALIFICATION TESTING
APPLICABILITY AND ADVANTAGES OF NOTCHING

Andrea Ceresetti
Alenia Spazio S.p.A. - Technical Directorate – Strada Antica di Collegno 253, 10146 TORINO, Italy
Tel: # 39 (0)11- 7180759  -   Fax: # 39 (0)11 – 7180074  -   E-mail: acereset@to.alespazio.it

ABSTRACT
This paper presents for both sine and random vibration cases, the theoretical approaches which qualitatively and quantitatively justify the application of notching profiles during the qualification vibration testing of spacecraft equipment. Usually, these qualification vibration tests are carried-out in “hard mounted” conditions, in which the applied levels are those obtained at the interface between the equipment and its mounting structure as measured during higher level testing (i.e. a spacecraft system, or a spacecraft subsystem structure when subjected to environmental loads such as vibroacoustics, sine, random, shock).

Such a test practice always contains a certain degree of conservatism with respect to the real flight conditions. In fact, for lightweight aerospace structures, during the vibration phases of the flight, the mechanical impedance of the equipment and its mounting structure are typically comparable. Therefore the coupled motion only induces some modest interface forces and limited acceleration amplifications on the equipment when compared to a conventional hard mounted vibration test performed by means of a shaker.

This phenomenon, existing for all types of dynamic excitations, constitutes the application ground for some test techniques such as the ‘NASA force limited vibration testing’ or the more rudimentary ‘response limited vibration testing’, where notching profiles are implemented to adjust the original input requirements.

The implementation principles of notching for the most important vibration test types (sine, random), can be analytically derived by parametrically estimating the difference between the ‘theoretical’ versus ‘effective’ acceleration amplifications of 2-DOF lumped mass models. Thus, with the main goal of equipment structural mass saving, the application of dedicated notching profiles to the vibration input, helps in eliminating unrealistic resonant acceleration loads induced by the ‘nearly infinite’ mechanical impedance of the shaker, thus providing more consistent “in-flight like conditions” which in some cases, could be much less severe than on ground test loads.

Keywords
Transfer functions, hardmounted, tuned/untuned case, effective amplification, dynamic absorber, notching.

Nomenclature
DOF degree of freedom
F(t) excitation force (Newton)
H acceleration transfer function H=H(w)
PSD base excitation power spectral density (g²/Hz)
Q 1-DOF quality factor [1/(2ξ)]
c1, c2 viscous damping of the oscillators 1, 2 (N s/m)
f1 Source uncoupled natural frequency [w1/(2π)]
f2 Load uncoupled natural frequency [w2/(2π)]
k1, k2 stiffness of the oscillators 1, 2 (N/m)
m1, 2 mass of oscillators 1 = Source, 2 = Load (kg)
û acceleration (g) or (m/s²)
û0 base excitation acceleration (g) or (m/s²)
w0 circular frequency of the excitation [f0(2π)]
w1 Source uncoupled circular frequency √[k1/m1]
w2 Load uncoupled circular frequency √[k2/m2]
w circular frequency (rad/sec)
ξ1 Source critical damping ratio [c1/(2√(k1 m1))]
ξ2 Load critical damping ratio [c2/(2√(k2 m2))]
µ Load to Source mass ratio [m2/m1]
ϕ Load to Source frequency ratio [f2/f1]=|w2/w1|
Ψn,m modal analysis eigenvectors (n mode, m DOF)
αn modal analysis eigenvalues (n mode)
Γn modal mass (n mode)
H*, ü* hard mounted test parameters

1. INTRODUCTION
The task of generating the qualification vibration test requirements for spacecraft equipment, is usually achieved using one the following three typical methods

a) Test
b) Analysis
c) Similarity

However, regardless of which of the above methods is chosen, the test requirements for equipment vibration qualification, derive from a higher level assessment.
For example, when a Structural Model (SM) of a Spacecraft (S/C) is available, a vibroacoustic test can be performed, and a zonal map of Power Spectral Density (PSD) can be recovered (method a) to be subsequently used for deriving the Random Vibration (RV) criteria for qualification of the equipment. Alternatively, when a S/C SM is not available, a vibroacoustic analysis can be performed by means of mathematical models (method b), using for example computer codes like Finite Elements (FEA) and Statistical Energy (SEA), thus obtaining in an analytical way the (PSD) zonal profiles as well. Finally, the similarity approach (method c)) is the most suitable for similar S/C cases under comparable loads.

2. DISCUSSION

Although all the above mentioned methods a), b), c) are generally applicable for the derivation of any type of dynamic load, the derived requirements when applied to equipment level testing, may in some cases produce very conservative loads when compared to the flight conditions, as discussed herein.

3. SPACECRAFT DYNAMICS

During the early S/C design phases usually no test models are available, so the S/C dynamics is investigated by means of FEA (even with huge models of hundreds of thousand of DOF). However by analyzing typical results of a S/C modal analysis (ψₙₘ, α²ₙ, Γₙ) a generic S/C can be seen, as far as dynamic behaviour, by simple conceptual models like Fig. 1.

![Fig. 1 Spacecraft Conceptual Dynamic Representation](image1)

Normally a S/C is defined in terms of structural levels, and the most relevant are the following three:

I. (one) Primary Structure (Fig. 1, item 1)
II. (some) Secondary Structures (Fig. 1, item 2)
III. (many) Equipment (Fig. 1, item 3)

However, it is common practice for each of the above structural levels, to establish a requirement of ‘minimum stiffness’ in order to get a limitation of the acceleration responses due to the dynamic couplings.

4. S/C STRUCTURAL LEVELS RESPONSES

When investigating a typical hard mounted modal analysis of / primary structure / secondary structure / equipment /, very often the mass participation factors (Γₙ) are mainly concentrated at the first frequency.

- Primary Structure  Γ₁ = (80-100)% of the mass
- Secondary Structure  Γ₁ = (50-100)% of the mass
- Equipment  Γ₁ = (50-100)% of the mass

The above consideration enables the loads engineer, dealing with the project structural requirements, to think about S/C dynamics by simple models like Fig. 2.

![Fig. 2 3-DOF Spacecraft Simplified Dynamics](image2)

So, if we are interested to analyze the effects of 1(g) base excitation sine sweep for [0-100] Hz (which can be a good way to represent the “low frequency transients” existing at the interface between a S/C and its Launcher adapter), then a frequency response analysis can be performed on the basis of a simplified model as Fig. 2, by considering all the potential cases of structure levels frequency coupling, listed in Tab. I.

<table>
<thead>
<tr>
<th>Case_N.</th>
<th>Typical S/C EXAMPLE</th>
<th>Uncoupled Frequency (Hz)</th>
<th>Tuning DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)Typical</td>
<td>10000 500 10</td>
<td>10 25 35</td>
<td>None</td>
</tr>
<tr>
<td>2)Unusual</td>
<td>10000 500 10</td>
<td>10 10 10</td>
<td>Fully</td>
</tr>
<tr>
<td>3)Unusual</td>
<td>10000 500 10</td>
<td>10 10 35</td>
<td>Partially</td>
</tr>
<tr>
<td>4)Unusual</td>
<td>10000 500 10</td>
<td>10 25 25</td>
<td>Partially</td>
</tr>
<tr>
<td>5)Possible</td>
<td>10000 500 10</td>
<td>10 35 35</td>
<td>Partially</td>
</tr>
</tbody>
</table>

Tab. I 3-DOF Input Base Motion Sine Sweep Analysis
On the basis of the equations of movement eq. [1] and considering the inputs of Tab. I, a frequency response analysis can be performed in order to assess the qualitative and quantitative response upon each DOF.

\[
[m][\ddot{u}]+[c]\frac{d[u]}{dt}+[k][u]=-[m][1]\ddot{u}_0(t) \tag{1}
\]

For this purpose an approximated modal analysis method has been executed, superposing the modulus of each maximum modal response by means of the Square Root of the Sum of the Squares (SRSS). From the results shown in Fig. 3, some conclusions are drawn:

- **Case-1** (typical for a S/C dynamic configuration); limited responses for all 3-DOF structure levels.
- **Case-2** (unusual); strong interaction/amplification between each one of the 2-DOF ‘tuned’ couples.
- **Cases-3, 4 (unusual); 5 (possible);** produced strong amplification only for the 2-DOF ‘tuned’ couples.

Furthermore, from Fig. 3 plots it is pointed out that:

- By the generality of the right side of eq. [1], a similar qualitative behaviour can also be expected for other vibration types (random, acoustic) since:

\[
\{F(t)\} = -[m][1]\ddot{u}_0(t) \tag{2}
\]

- The multi-DOF strong acceleration amplifications are always produced by couples of ‘tuned’ 2-DOF.
- The expected acceleration amplification between two different structural levels is always lower than the theoretical amplification as calculated from the quality factor \((Q=1/(2\xi))\) of the related structure.

Finally, it can be stated that the mass ratio \((\mu)\) and the uncoupled frequency ratio \((\phi)\) of 2-DOF couples, result to be main drivers of S/C structures dynamic response.
4.1 Coupled Response Under Sine Vibration

In order to finalize the ‘nominal’ test requirements of a hard mounted equipment, the spectra derived at higher level are subjected to further processing like:
/ smoothing / peak clipping / envelope of response valleys / test factors /

So, after having established a ‘nominal’ input spectrum \((\ddot{u}_i)^n\) to be used at a certain structural level for a Hard Mounted (H/M) test, the expected structure acceleration response \((\ddot{u}_r)^n\) will be given by the product of input spectrum times its H/M transfer function:

\[
(\ddot{u}_r)^n = (\ddot{u}_i)^n H(w)^n
\]  

[3]

For example, on the basis of the results of the 3-DOF case previously analyzed (see Fig. 3), a typical task of generating ‘nominal’ inputs could be the following:

1) Input for (Secondary Structure + Equipment)2-DOF

Having considered the most realistic case of the 3-DOF S/C dynamic coupling shown in Fig. 3 (Case-1), then:

- Cut primary structure first mode response (~ static)
- Consider \((\ddot{u}_2)^n\) Fig. 3 Case-1 with 3-dB peak clipping
- Envelope \((\ddot{u}_2)\) and antiresonances to a constant value

The outcome of the above process, which stems from a reduction from 3-DOF to 2-DOF, led to the definition of the 2-DOF (secondary structure (S/S) + equipment) nominal input sine sweep \((\ddot{u}_0) = 1(g)\) constant [0-100] Hz, that produces the responses \(\ddot{u}_1, \ddot{u}_2\) shown in Fig. 4.

2) Input for (Hard Mounted Equipment)1-DOF

By repeating the above process on the basis of Fig. 4, from 2-DOF to 1-DOF reduction, the resulting sine-sweep ‘nominal’ equipment H/M spectrum is \((\ddot{u}_i)^n = 10 (g)\) constant between [0-100] Hz. Such a spectrum will produce an H/M equipment response \((\ddot{u}_r)^n = 100 (g)\) at 35 Hz in accordance with eq. [3], as shown in Fig. 4.

In conclusion, the 2-DOF/1-DOF models comparison provides two different acceleration responses for the considered equipment, namely: \((\ddot{u}_1)^n \gg (\ddot{u}_2)^n\). In fact:

- # 2-DOF input \((\ddot{u}_0) \rightarrow 1(g)\) constant [0-100] Hz
- # 2-DOF (S/S) max. response \((\ddot{u}_1) \rightarrow 10(g)\) at ~ 25 Hz
- # 2-DOF equipment max. input \((\ddot{u}_1) \rightarrow 10(g) \sim 25 Hz\]
- # 2-DOF equipment max. response \((\ddot{u}_2) \rightarrow 20(g) \sim 25 Hz\]
- # 1-DOF H/M input \((\ddot{u}_1)^n \rightarrow 10(g)\) [0-100] Hz
- # 1-DOF equipment max. response \((\ddot{u}_1)^n \rightarrow 100(g) \sim 35 Hz\]
- # Max. equipment response ratio \((\ddot{u}_1)^n/(\ddot{u}_2) \gg 5\) times

So, Fig. 4 shows that for this specific investigated 2-DOF case, the main reason of equipment H/M overloading derives from the fact that the applied nominal spectrum \((\ddot{u}_1)^n\) was obtained by enveloping the valleys of the coupled system responses at all frequencies. However, it can be stated that the difference between the equipment H/M versus the integrated test conditions, is not only limited to the case of a sine excitation, but is a more general phenomenon, occurring also for random/acoustic, etc..

4.2 Coupled Response Under Random Vibration

As mentioned above, also the random vibrations show similar phenomena. Let’s consider for example the case of the 2-DOF of Fig. 5, constituted by a secondary structure ‘Source’ and a mounted equipment ‘Load’. Such a model can be thought of as the reduced case of the 3-DOF “more general” model defined in Fig. 2.

\[
\begin{align*}
\text{PSD}_0 & \rightarrow \text{Secondary S.} & \text{Equipment} \\
& \rightarrow \text{Source} & \rightarrow \text{Load} \\
\end{align*}
\]

Fig. 5 2-DOF Under Random Vibration Base Motion

The 2-DOF acceleration transfer functions \(H(w)\) result:

\[
\begin{align*}
H_1 &= \ddot{u}_1(t)/\ddot{u}_0(t) \\
H_2 &= \ddot{u}_2(t)/\ddot{u}_0(t) \\
\end{align*}
\]

[4]

\[
\begin{align*}
H_1 &= -w^2(i2\xi_1w_1) - w^2(w_1^2 + 4\xi_2 w_1 w_2) + i[w(2\xi_1 w_1 w_2 + 2\xi_2 w_2 w_1^2) + (w_1^2 w_2^2)]/\epsilon \\
H_2 &= -w^2(4\xi_1 \xi_2 w_1 w_2) + i[w(2\xi_1 w_1 w_2^2 + 2\xi_2 w_2 w_1^2) + (w_1^2 w_2^2)]/\epsilon \\
\end{align*}
\]

[5]
\[ \varepsilon = A_4w^4 - iA_3w^3 - A_2w^2 + iA_1w + A_0 \quad [6] \]

\[ A_4 = 1 \]

\[ A_3 = [2\xi_2^2w_1 + 2(1+\mu)\xi_2^2 w_2] \]

\[ A_2 = [w_1^2 + (1+\mu)w_2^2 + 4\xi_2^2 w_1 w_2] \]

\[ A_1 = [2\xi_2^2 w_1 w_2^2 + 2\xi_2 w_2 w_1^2] \]

\[ A_0 = [w_1^2 w_2^2] \quad [7] \]

From the random vibrations theory of a 2-DOF model, the (rms) accelerations (\(\ddot{u}\)) and (PSD) are related to the complex frequency responses \(H(w)\) by eq. [8] and [9].

\[
PSD_j = |H(w)_j|^2 PSD_0 \quad [j = 1,...,2] \quad [8]
\]

\[
\ddot{u}_j = \left( \int |H(w)_j|^2 PSD_0 dw \right)^{1/2} \quad [\infty,\ldots,\infty] \quad [9]
\]

Considering Fig. 5 with following 2-DOF parameters:

- A constant random spectrum on the primary structure
  - PSD_0 = 0.01(g^2/Hz), considered as a ‘base motion’
  - Secondary structure (\(f_1 = 25\) Hz; \(\xi_1 = 5\%\); \(m_1 = 500\) Kg)
  - Equipment (\(f_2 = 35\) Hz; \(\xi_2 = 5\%\); \(m_2 = 10\) Kg)

The results of the application of eq. [4] thru eq. [9] for the above 2-DOF case, are shown in Fig. 6 and Fig. 7, where a comparison between the 1-DOF/2-DOF results has been reported respectively for transfer functions and power spectral densities. The PSD results clearly show a large difference between (PSD_j) and (PSD_k). In particular, the following values are found:

- # 2-DOF equip. max. input PSD_j \rightarrow 1(g^2/Hz) at \(-25\)Hz
- # 2-DOF equip. max. response PSD_k \rightarrow 4(g^2/Hz) at \(-25\)Hz
- # 2-DOF equipment (rms) input \((\ddot{u}_1) \rightarrow 1.9\) (g)
- # 2-DOF equipment (rms) response \((\ddot{u}_2) \rightarrow 4.2\) (g)
- # 1-DOF H/M input PSD \rightarrow 1(g^2/Hz) [0-100]Hz
- # 1-DOF equipment max. response (PSD_k) \rightarrow 100 (g^2/Hz)
- # 1-DOF equipment (rms) response \((\ddot{u}_1) \rightarrow 23.5\) (g)
- # Equipment response ratio \((\ddot{u}_1)/(\ddot{u}_2) \rightarrow 5.6\) times.

For the 2-DOF system shown in Fig. 5 constituted by a Source + Load, subjected to either I) a ground sine vibration \(\ddot{u}_0(t)\) or II) a sinusoidal forcing function \(F(t)\) acting on the Source, the Load reaches its maximum response value for ‘dynamic absorber’ case of eq. [10].

\[
W_1 \Rightarrow W_2 \Rightarrow W_0 \quad [10]
\]

The analytical aspects of the dynamic absorber theory simply deal with a particular 2-DOF case in which both the oscillators, Source and Load, have the same uncoupled hard mounted circular frequency \(w_l = w_2\) that is also coincident with the circular frequency \(w_0\) of an external sinusoidal excitation. In such a case, the Load experiences the maximum value of acceleration.

According to Ref. [1], the formulations of the dynamic absorber are herein reported in eq. [11] and [12], where \(Q_2\) is the quality factor of the Load and \(\mu = m_2/m_1\). In particular, thinking in terms of PSD of a random vibration, the left side of eq. [12] represents the “Force Spectral Peak (\(S_{FF}\)) normalized by the squared Load effective mass (\(m_2^2\)) and Acceleration Spectral Peak (\(S_{AA}\))”. Hence, introducing the concept of the ‘effective amplification factor’ also called the Acceleration Feedback Factor (Qeff), which represents the ratio between the acceleration levels \(\ddot{u}_2(t)\) and \(\ddot{u}_1(t)\), then the (Qeff) can be associated with the dynamic absorber formulations by the approximated expression eq. [13].

\[
\beta^2 = (1 + \mu/2) \pm [(\mu + \mu^2/4)]^{1/2} \quad [11]
\]

\[
S_{FF}/(m_2^2S_{AA})=[1+\beta^2/4]/[(1+\beta^2)+(\beta^2Q_2^2)] \quad [12]
\]

\[
Q_{eff} = \ddot{u}_2(t)/\ddot{u}_1(t) \approx S_{FF}/(m_2^2S_{AA})^{1/2} \quad [13]
\]
From previous formulas it is shown that the maximum effective acceleration amplification is always achieved when considering eq. [12] at the lower resonance frequency of the 2-DOF (eq. [11] with negative sign). The behaviour of the \( Q_{\text{eff}} \) as a function of the mass ratio \( \mu \) for four different values of \( Q_2 \) has been calculated and reported in graphical form in Fig. 8.

From plots it can be seen that:

- for very small \( \mu \) values \( (\mu \ll 1) \) the Load tends to behave as 1-DOF, simply like a case of H/M Load.
- in case where the Load and Source have comparable masses \( (\mu \approx 1) \), which is rather common in aerospace structures, there is always a slight amplification \( (\approx 1.6) \) between the Source and the Load, regardless of \( Q_2 \).

Fig. 8 2-DOF Dynamic Absorber Amplification (\( Q_{\text{eff}} \))

4.4 2-DOF Maximum Random Response

For a 2-DOF random vibration case, see Fig. 5 when considering the system being excited by a stationary random process of constant amplitude PSD, 'white noise base motion', the acceleration feedback factor \( Q_{\text{eff}} \) can be derived by eq. [14], based on theory of Ref. [2], where \(<…>\) stands for "mean square value".

\[
(Q_{\text{eff}})^2 = \langle \ddot{u}_2 \rangle / \langle \ddot{u}_1 \rangle \quad [14]
\]


\[
(Q_{\text{eff}}) = \left[ (Q_{\text{eff}})^2 \right]^{1/2} \quad [15]
\]

In particular the \( Q_{\text{eff}} \) solution is given by eq. [16]

\[
(Q_{\text{eff}}) = \{ \phi \ [S1+S2+S3]/[S4+S5+S6]\}^{1/2} \quad [16]
\]

with:

\[
S_1 = \xi_1 (1 + \mu \phi^2) + \xi_2 \phi [\mu + (1+\mu)^2 \phi^2]
\]

\[
S_2 = 4 [\xi_1^3 \phi^2 + \xi_1^2 \xi_2 (\phi + (1+\mu)\phi^3) +
\quad + \xi_1 \xi_2^2 (1 + (1+\mu)^2 \phi^2) + \xi_2^3 (1+\mu) \phi]
\]

\[
S_3 = 16 \xi_1^2 \xi_2 \phi [\xi_1 \phi + \xi_2]
\]

\[
S_4 = \mu \xi_1 \phi^3 + \xi_2 [(1 - (1+\mu)^2 \phi^2) + \mu \phi^2]
\]

\[
S_5 = 4 [\xi_1^3 \mu \phi^3 + \xi_1 \xi_2 (1 - \phi^2)^2 + \mu \phi^4] +
\quad + \xi_1 \xi_2^2 (\phi + (1+\mu)^2 \phi^3 + \xi_2^3 (1+\mu) \phi^2]
\]

\[
S_6 = 16 \xi_1^2 \xi_2^2 \phi [(\xi_1^2 + \xi_2^2)\phi^2 + (1 + \phi^2) \xi_1 \xi_2 \phi]
\]

\[
\phi = \frac{w_2}{w_1}
\]

\[
\mu = m_2/m_1
\]

For example, considering \( Q_1=Q_2=10; \mu = [0.01….1.0] \), the worst case is always achieved when \( \phi=1 \), see Fig. 9. Hence, some tuned conditions have been investigated for \( Q_1=10; Q_2=[5, 10, 25, 50] \), whose results shown in Fig. 10 are very similar to those of Fig. 8 for sine case.

Fig. 9 2-DOF Random Effective Amplification (\( Q_{\text{eff}} \))

Fig. 10 2-DOF Tuned Random Amplification \( Q_{\text{eff}}(\phi=1) \)
4.5 Final Considerations

From above discussion, some final considerations can be drawn as far as the equipment dynamic response:

- The minimum required stiffness of the equipment should guarantee a proper response decoupling with respect to its supporting structure excitation. However, there is always the possibility of having a tuned condition between the equipment and its supporting structure, because the ‘maximum required stiffness’ for structural levels is not usually a design requirement.

- It has been evidenced that for both sine/random vibrations, the equipment in-flight accelerations result always lower than the H/M ones, when applying a ‘nominal’ spectrum derived by higher level assessment.

- The approach of enveloping vibration responses in order to construct a ‘nominal’ input spectrum for equipment H/M test, can be very conservative since it eliminates the beneficial effect of transfer functions valleys typically existing in a dynamically coupled system, especially in the regions of equipment H/M resonances. Such a consideration is strictly related to the effective acceleration amplification factor (Qeff), where the (Qeff) between random/sine are very similar.

- For any expected case of large difference between the equipment H/M accelerations and the true mission accelerations ($u_r^*/u_i^*)$ >> ($u_i^*$), then one or more notching profiles, for both sine/random, can be applied in the relevant zones of the equipment H/M resonances, in order to mitigate the shaker induced overloading.

4.6 Application Example

The inherent efficiency of applying notching profiles to equipment H/M test, can be judged on the basis of the difference between (Qeff) and (Q2). Considering for example the already investigated 2-DOF case:

- **Sine** Fig. 8 [($u_i^*$)/($u_1^*$)] max. >> 5 times.
- **Random** Fig. 7 [($PSD_i^*$)/($PSD_2^*$)] max. >> 25 times; and [($u_i^*$)/($u_2^*$)] max. >> 5.6 times.

More generally, in order to have a good understanding about notching efficiency, the following conditions are deemed as key indicators of high notching gains:

- **Case of comparable mass coupling ($u = m_1/m_2$)**
  - **Sine** Fig. 8 for $\mu = [0.1 \div 1.0]$ $\Rightarrow$ Qeff << Q2
  - **Random** Fig. 10 for $\mu = [0.1 \div 1.0]$ $\Rightarrow$ Qeff << Q2

- **Case of high un-tuned coupling ($\phi<1$, $\phi$>1)**
  - **Sine** Fig. 8, only available a conservative case $\phi=1$
  - **Random** Fig. 9: $\phi<0.5$ or $\phi>2.0$ $\Rightarrow$ Qeff << Q2

The 2-DOF example ($Q_1=Q_2=10; \mu =0.02; \phi=1.4$) gives:

- **Sine Sweep** Fig. 8 $\Rightarrow$ Qeff =6 (dynamic absorber $\phi=1$)
  So, allows a notching to reduce by a factor of 1.66 = = (10/6) the H/M peak ($u_r^*$) ~ 35 Hz, see Fig. 4.
  (In reality the notching could be deeper, being $\phi=1.4$)

- **Random** Fig. 9 $\Rightarrow$ Qeff=2.2
  So, allows a notching to reduce by a factor of 20 = = (10/2.2) the H/M peak ($PSD_i^*$) ~ 35 Hz, see Fig. 7.

5. NOTCHING PROFILES DEFINITION

Notching profiles must mainly be applied in relevant zones of H/M resonances of the equipment, since they strongly influence the acceleration cut in those zones. For example Fig. 11 shows a 1-DOF acceleration transfer function, see eq. [20] along with a notching. The 1-DOF response is given by eq. [21] thru eq. [23]

$$H(w)=[(2i w \xi w_n+w_n^2)/(-w^2+2i w \xi w_n+w_n^2)]$$ [20]

![1-DOF H/M Transfer Function and Notching](image)

**Sine** $\ddot{u}_1 = \ddot{u}_0 \ H(w)$ [21]

**Random** $PSD_1(w) = PSD_0(w) \ |H(w)|^2$ [22]

$$\langle \ddot{u}_2 \rangle = \left( integral \ PSD_1(w) \ dw \right) \ [\pm \infty, \ldots, \pm \infty]$$ [23]

A notching efficiency criterion can be established with:

- $A =$ Depth of Notching (dB)
- $B =$ Width of Notching (Hz)
- $\Delta =$ Half Power Bandwidth of peak resonance (Hz)
- $f_0 =$ Peak resonance frequency (Hz)
The $\Delta$ is related to the 1-DOF quality factor ($Q$) formulation by the following relationships [24], [25].

$$\Delta/(2f_n) = \xi$$  \hspace{1cm} [24]

$$Q = 1/(2\xi) = f_n/\Delta$$  \hspace{1cm} [25]

Thus, in order to find a criteria for the reduction rate in terms of (rms) acceleration response, some 1-DOF parametric cases with $Q=10$ have been calculated for a random white noise excitation $[0-2000]$ Hz, using eq. [22], [23] with an explorative $PSD_0$ of 0.1 ($g^2$/Hz). The results obtained are shown in Tab. II for two cases of significative ‘rectangular notching’ namely: $(B=\Delta)$ and $(B=2\Delta)$. However, it’s underlined that for practical reasons, notching profiles have trapezium shapes.

Example: no-notch: $B=0$; $A=0$; $Q=10$; $f_n=100$ (Hz); $PSD_0=0.1$ ($g^2$/Hz). From Miles eq. $\bar{u}=12.5$ (g) (rms).

Example: with notching: $\Rightarrow$ Quality factor $Q=10$.
Notching depth $A$ (dB) $\Rightarrow [0/3/6/10/20]$ parameter
Notching width $B$ (Hz) $\Rightarrow [\Delta/2\Delta]$ parameter
Resonance frequency $f_n$ $\Rightarrow [100/400]$ parameter

<table>
<thead>
<tr>
<th>$A$ (g)</th>
<th>$\bar{u}$ (g)</th>
<th>$B=\Delta$ Hz</th>
<th>$\bar{u}$ %</th>
<th>$\bar{u}$ (g)</th>
<th>$B=2\Delta$ Hz</th>
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<th>$\bar{u}$ %</th>
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Tab. II Notching Gain 1-DOF for Random White Noise

Fig. 12 shows the NASA approach, Ref.[1] to obtain the notching Reduction Fraction ($R$) in terms of mean square response, see eq. [26], [27] for different values of $Q=10$, 20, 50; ($Q$ appears not to influence the gain).

$$R = 1-(2/\pi )\tan^{-1}((A^2-1)^{1/2}-(A^2-1)^{1/2}/A^2)$$  \hspace{1cm} [26]

$$R = [\langle \bar{u}^2 \rangle_{\text{notched}}/\langle \bar{u}^2 \rangle_{\text{un-notched}}]$$  \hspace{1cm} [27]

Finally, Fig. 13 shows a comparison of ($\sqrt{R}$) between 1-DOF parametric analysis shown in Tab. II and eq. [26].

6. CONCLUSIONS

Notching profiles for hard mounted equipment under sine/random qualification tests can be estimated on the basis of the amplifications ratio ($Q_{\text{eff}}/Q$). Such a ratio results to be very similar for both sine/random cases. Notching profiles derived at $\phi=1$ can produce low gain.

7. REFERENCES