Magnetic Anomaly in Black Hole Electrons

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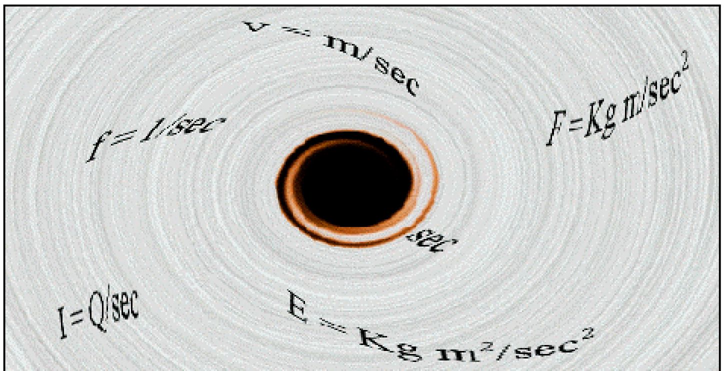
A model for a black hole electron can be developed starting from three basic constants: *h*, *c* and *G*. The quantization of the Planck mass gives a description of the electron with its own associated mass and charge. The precise determination of its rotational speed yields accurate numbers, within one standard deviation, of all quantities including, once a small change in the fine structure constant is considered, its magnetic moment and its magnetic moment anomaly.

Introduction

When we approach a black hole strange things start to happen: time dilation is one of them. What takes place if we go further on, beyond the event horizon, is anybody's guess: our present knowledge does not go that far and we are left only with a number of educated suppositions. One of them is that time does not exist within a black hole, rather it seems to disappear in it. Conversely why not think that a time dimension will appear coming out from what should be seen, more appropriately, as a singularity? Of course, here we mean a force field rather than a material object. *G*, *c* and *h* imply the dimension of mass, length and time only so it should come with no surprise if eventually we get a charge with some unusual dimensions. We can still assign a Coulomb dimension to it so consistency is maintained with the MKSA system which was originally devised without any thought concerning the possible unification of forces. In this way we will be able to link the Planck data with our real world and the resulting numbers will be the ones we are accustomed to see.



In a black hole time loses its meaning and know quantities might show strange dimensions: time itself might become a dimensionless number in a black hole and vice versa. By working our way out from the black hole we discover that the particle is the electron.

This is evident when we calculate a quantity we would call the Planck permittivity ε_p [1]. But first we define the other Planck quantities, i.e., Planck time $t_p=(\pi hG/c^5)^{1/2}$, Planck mass $M=h/t_p c^2$ and Planck length $l_p=t_p c$. The resulting quantities differ by $2^{1/2}$ or $2^{1/2}\pi$ from accepted numbers, however we do not know the intimate structure and geometry of this particle, and it is just a matter of definition. We are now able to define our Planck permittivity:

$$\varepsilon_p = (t_p/4\pi^2)^{1/4} \tag{1}$$

The unusual dimension of the Planck permittivity allows us to build a basic particle, the electron, by means of the fundamental dimensions of mass, length, and time only. The calculation of the Planck charge Q is now relatively easy and it is defined as the charge having the same energy as the Planck mass M:

$$Q = M(4\pi\varepsilon_p G)^{1/2} \tag{2}$$

It is time now to check whether this particle with mass M, charge Q and permittivity ε_p is indeed a black hole.

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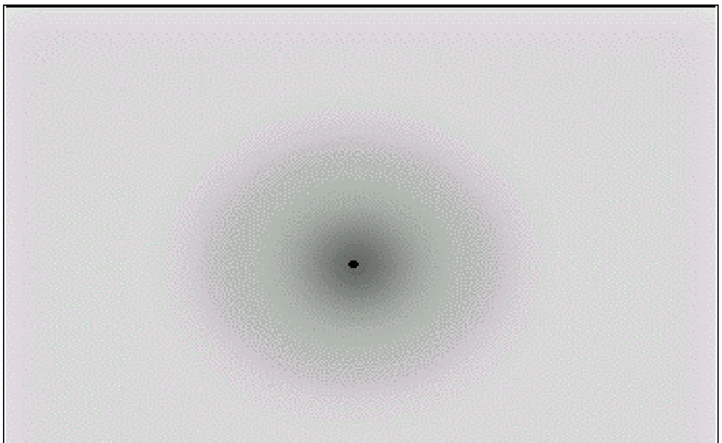
Planck Black hole

The discriminant in the Kerr-Newman equation will tell us whether our particle is a black hole or not. If we do all necessary calculations we find that such a particle satisfies the condition imposed by the discriminant for a static as well as a rotating black hole. Under these circumstances, we cannot, as yet, identify this particle with the electron but let us see first how this black hole is measured in our real world.

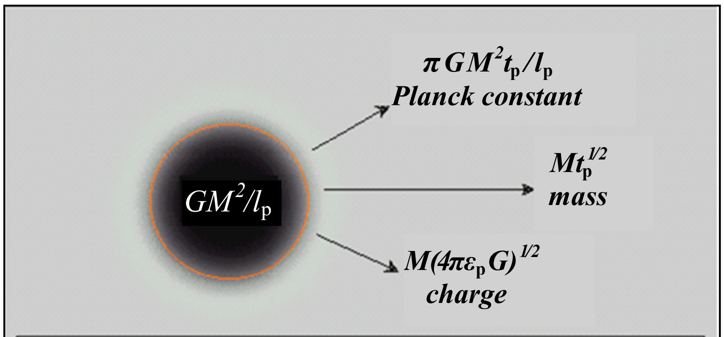


The Planck's black hole could be rendered by a tiny black dot surrounded by a spherical energy field. In this basic model there is a charge but no rotation. Its characteristics makes it a successful candidate for a Kerr-Newman black hole.

The energy of the Planck black hole is given by GM^2/l_p, but the constrain of time t_p outside the black hole would give us something slightly different, the Planck constant in fact:

$$\hbar = (GM^2/2l_p)t_p \tag{3}$$

Although this may seem obvious because the Planck constant was used initially to calculate the other Planck quantities, it is the concept behind it that should be considered: we will never be able to measure the gravitational black hole energy because there is the Planck time which imposes a severe constraint on what we really measure and experience from our reference frame outside the black hole. Here is the first important consequence: we could write eq. 3 in a different way thus getting $\hbar = GM_0^2/2l_p$ where $M_0 = Mt_p^{1/2}$ would be the quantized gravitational mass of the Planck particle; in other words, we would only experience mass M_0 and not mass M because M_0 is the mass that takes in account the effect of time t_p. At this point we would be inclined to use M_0 in the Kerr-Newman discriminant and the result, under these conditions, is not a black hole. Yet, we are still talking about the same particle and it is mass M that should be in the discriminant, this time we surely get a black hole although we would experience it as a much smaller mass M_0. The idea of a black hole electron is not new: other researchers have thought about it in the past [2] while others were quick to point out that it does not satisfy the Kerr-Newman condition. We have seen that it is a black hole after all, but its detectable mass would be the quantized M_0 and not M as expected.



Just as we lose the time dimension in a black hole, we gain it when we go out of it. The Planck time is present in the energy of the black hole thus originating the Planck constant, in the same way we have a mass and a charge. They will be both identified with the mass and charge of the electron once rotation is taken in account.

In any case, the effect of mass M is still present in our real world, not as a gravitational mass but as an electric charge because we would experience it as an electric force, originating from charge Q.

The numbers for charge Q and mass M_0 are an order of magnitude larger compared to the known charge and mass of the electron, while permittivity ε_p is a fraction of a percent off; they will agree once rotation is taken in account.

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Initial electron

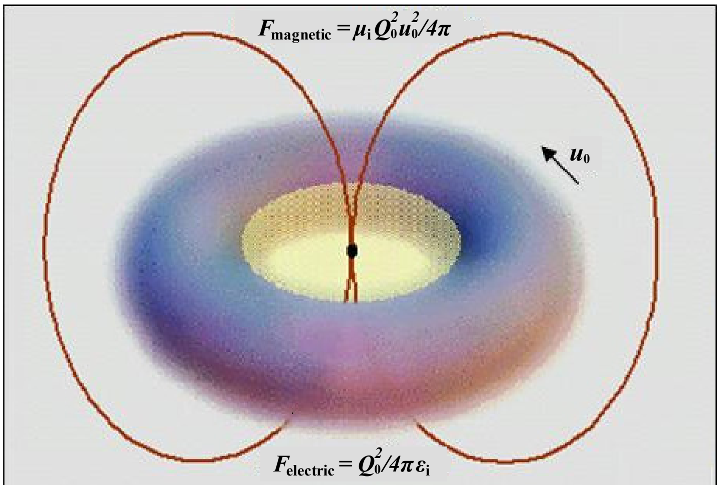
The most suitable physical model that best fits a rotating black hole is the ring model [3,4]. Although there is no direct evidence, there is more than one reason to believe that some sort of toroidal force field is present around the tiny black hole. Fortunately we do not have to know its radius or other data concerning its physical size since the equations in use will eliminate any reference to it and its real size, in this context, is still undetermined.

A rotating charge will set up a magnetic field opposing its own rotation until a stability point is reached. The rotational speed u_0, meant as the speed of the ring, is a fundamental parameter and using a modified equation originally proposed by Sutton and al. [5], we define a relativistic connection with what we would call the initial fine structure constant α_0 applicable to the initial electron:

$$\alpha_0 = 2(1-u_0^2/c^2) \tag{4}$$

In order to calculate u_0 we try to relate α_0 to some electrical properties of the particle and specifically how charge Q compares with a ring of unitary charge Q_u. We define α_0 as the ratio $(w_u/w_p)^{1/2}$ where $w_u = 16\pi^4 Q_u^2/t_u$ would be the unitary charge squared in the unitary time applicable to a toroidal particle and $w_p = Q^2/t_p$ would be charge Q within time t_p. It could be seen also as an energy or force ratio. After elaboration we may write α_0 in terms of fundamental quantities only:

$$\alpha_0 \;=\; 4\pi^2 t_p^{1/2}/Q \;=\; (2\pi^2/c)(\pi/c)^{1/2}(2G/h)^{1/4}(c/\pi hG)^{1/16} \tag{5}$$



$F_{electric} - F_{magnetic} = F_{electron}$

$Q_0^2 - Q_0^2 u_0^2/c^2 = e_0^2$

The measured electron force is the electric force of charge Q_0 minus the magnetic force generated by the same charge rotating at speed u_0. Its relativistic speed increases the energy associated with charge e_0 by a factor $2/\alpha_0$ to become the energy of charge Q_0 thus getting $e_0^2 = Q_0^2 - Q^2$

With the knowledge of fine structure constant α_0 and relevant speed u_0, we are able to go deeper into the details of the ring model. Its rotation at relativistic speed will slightly increase the energy associated with charge Q. At this energy level there is a corresponding charge $Q_0 = Q\, c/u_0$ which will originate the initial electron charge $e_0 = Q_0(\alpha_0/2)^{1/2}$, also written as:

$$e_0 \;=\; Q/(2/\alpha_0 - 1)^{1/2} \tag{6}$$

Q_0 will generate, in turn, an initial permittivity $\varepsilon_i = Q_0^2/4hc$ and permeability $\mu_i = 1/\varepsilon_i c^2$ related to our rotating particle. After elaboration of our equations we can write an important relation for the gravitational constant G given in terms of quantum constants only:

$$G \;=\; \alpha_0^2(2-\alpha_0)^2(e_0/4\pi^2)^4 c^5/\pi h \tag{7}$$

The equation is dimensionally balanced because factor $4\pi^2$ is actually $w_u^{1/2}$. In addition, the quantity $\alpha_0(2-\alpha_0)e_0^2$ is a constant, in other words we do not expect it to change if there is a variation of the rotational speed; this means that eq. 7 is still applicable if we write it in terms of known constants achieved by changing speed u_0 by a small amount, as we will see in the next section. The result is always the same and yields a very accurate constant of gravitation as shown in the numeric table, last page. Conversely, if we extract α_0, we find two values: α_0 and a second value $2-\alpha_0$ which is 273 times larger than α_0.
For the initial electron mass m_b we take in account the relativistic rotation of M_0 in the same way as we had e_0 from Q_0:

$$m_b \;=\; M_0(\alpha_0/2)^{1/2}(1-\alpha_0/2)^{3/8} \tag{8}$$

Factor $(1-\alpha_0/2)^{3/8} = (u_0/c)^{3/4}$ would refer to the contraction of the 3 torus radii affecting mass M_0. The same equation can be written also in terms of the initial electron charge:

$$m_b = (8h^3/\pi e_0^4)(\alpha_0/2)^{1/2}(1-\alpha_0/2)^{3/8}/(2/\alpha_0-1)^2 \quad (9)$$

We could also calculate the initial magneton $\mu_{bi} = e_0\hbar/2m_b$ and initial gyromagnetic ratio $\gamma_i=e_0/m_b$. This approach requires neither infinities nor renormalizations and we are able to calculate exactly the parameters of the initial electron, only problem is that we will never be able to experimentally measure its quantities as the initial electron is in a state prohibited by the uncertainty principle and from the table below we see that there is a substantial difference with known values.

ppm difference of initial electron				
Mass m_b	Charge e_0	Fine structure α_0	Permittivity ε_i	Magneton μ_{bi}
- 2537	*+ 101*	*- 203*	*+ 405*	*+ 2638*

Actually the only adjustment to be considered is a slight decrease of the rotational speed which implies a change in every parameter as they are all mathematically related. This slowdown is due to the spin angular momentum S and we must find a way to quantify this speed decrease.

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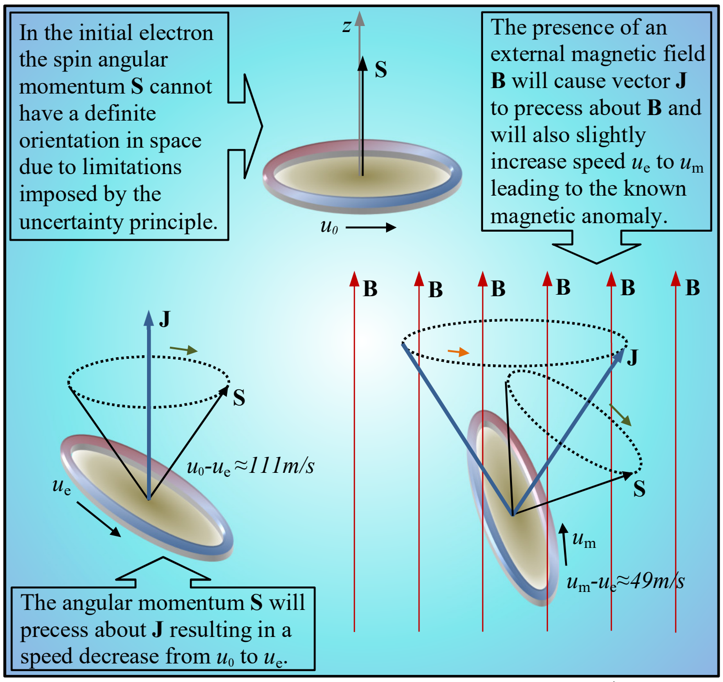
The electron slowdown and magnetic moment

Agreement with known values is achieved by changing the rotational speed u_0 by a very small amount. There is indeed a lower speed u_e yielding all expected data. The spin angular momentum S has the effect of lowering its rotational speed. By elaborating eq. 7 we get a new cubic equation, see last page, giving the known value for α and hence speed u_e in terms of c, h and G only. There is also a more intuitive solution where speed u_0 is decreased until $\varepsilon_0 = 10^7/4\pi c^2$; this is an exact relationship and once such a condition is met we will have the correct numbers for all quantities even if, for the mass, we have to define an additional term.



There is a mass variation proportional to $(\alpha/\alpha_0)^4$. As $M_0=8h^3/\pi Q^4$ we have that any speed variation will change the fine structure constant, hence charge Q, eq. 5, which will affect M_0 with the proportionality factor $(\alpha/\alpha_0)^4$.

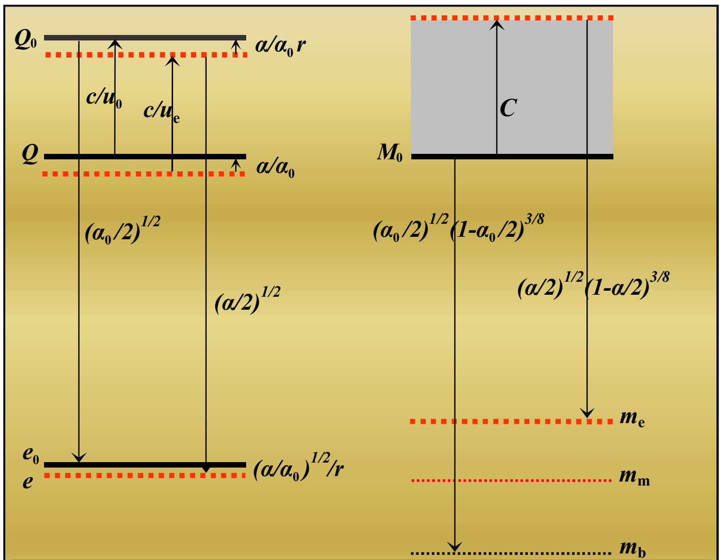
Mass and radius are directly related in a black hole. In our torus model we are dealing with three radii and any change of the rotational speed will affect the three radii which would equally influence the electron mass. We will call $C=(\alpha/\alpha_0)^{12}/r^{3/4}$ the term taking care of the mass and speed variation from u_0 to u_e given by $r=u_0/u_e=((2-\alpha_0)/(2-\alpha))^{1/2}$. The resulting electron mass m_e becomes $CM_0(\alpha/2)^{1/2}(1-\alpha/2)^{3/8}$ and finally:

$$m_e = M_0(\alpha/2)^{1/2}(\alpha/\alpha_0)^{12}((1-\alpha/2)(2-\alpha)/(2-\alpha_0))^{3/8} \qquad (10)$$

C is also present in eq. 8 and 9 giving the initial electron mass m_b but its value is 1.
We get the electron charge e from eq. 6, once the fine structure variation is accounted for:

$$e = Q/(\alpha/\alpha_0)(2/\alpha-1)^{1/2} = 4\pi^2(t_p/\alpha(2-\alpha))^{1/2} \qquad (11)$$

The term on the right, where $4\pi^2=w_u^{1/2}$, is an interesting elaboration linking e and α without the knowledge of permittivity ε_0, or alternatively, we have $e=M(2\pi\varepsilon_0\alpha G)^{1/2}$.
The diagram below shows the mass and charge variation. The black lines refer to the initial values while the red lines refer to the final values after rotation and slowdown.



A detailed picture is necessary to account for the charge and mass variation. Every speed change will originate the ratios α/α_0 and $r = u_0/u_e$ that must be taken in due account for the accurate calculation of charge, mass and magnetic moment. C, the initial electron mass m_b and the magnetic moment mass m_m are also shown.

The initial magneton μ_{bi} would be altered by the total angular momentum J when an external magnetic field B is present.
If we would know the "magnetic" fine structure α_m we could calculate the known magnetic moment.
α_m can be calculated directly from known constants (see addendum) thus getting $\alpha_m = \alpha/1.00008918770992486$. The fine structure is the only parameter that needs to be adjusted and will directly set both mass and charge of the magnetic moment and rotating speed $u_m=c(1\alpha_m/2)^{1/2}$.
By replacing α with α_m in eq. 10 and 11 we get mass m_m and charge e_m respectively, giving the known magnetic moment $\mu_e = e_m\hbar/2m_m$ and gyromagnetic ratio $\gamma_e = e_m/m_m$.
We could rewrite μ_e in many different ways, for instance in terms of charge Q and mass M_0 where there is no direct knowledge of the known electron mass and charge. A similar equation is written for the Bohr magneton. The magnetic moment μ_e is then the following:

$$\mu_e = e_m\hbar/2m_m = (Q\hbar/M_0)(\alpha_0/\alpha_m)^{13}(1-\alpha_0/2)^{1/8}(2-\alpha_0)^{1/4}/(2-\alpha_m)^{5/4} \qquad (12)$$

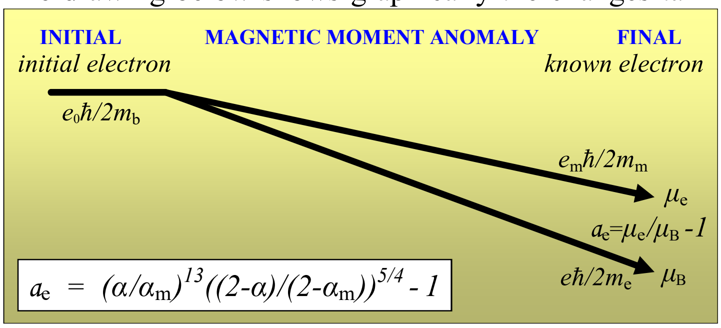
Magnetic anomaly

The magnetic anomaly is measured with experiments and Quantum Electrodynamics offers a solution [6] where a number of terms lead eventually to the theoretical value of the magnetic anomaly. These terms involve fairly complex calculations closely matching the experimental value.

In this theory the starting point is a black hole, leading to an initial electron with its mass and charge. Any slight change of the rotational speed, or fine structure, brings about a variation in each quantity but they are always in a very well defined relationship. We know the initial and final values, meaning the initial and known values, for the fine structure constant, charge, mass and so on. A magnetic anomaly a_e is present when the electron is in a magnetic field thus affecting the fine structure constant and all other parameters. a_e can be calculated directly from eq. 12, divided by the Bohr magneton, but could be written just in terms of the known and magnetic fine structure constants, α and α_m respectively:

$$a_e = (e_m/m_m)/(e/m_e) - 1 = (\alpha/\alpha_m)^{13}((2-\alpha)/(2-\alpha_m))^{5/4} - 1 \qquad (13)$$

The drawing below shows graphically the changes taking place in the magnetic moment from the initial condition, initial electron, to the final magnetic moment and its relevant magnetic anomaly a_e. Eq. 13 gives a_e in a convenient and simple form but all quantities are interconnected so we get the same result in terms of the variation of any other quantity. For example, if we would consider the charge we have $a_e=(e_m^2(2-\alpha_m)/e^2(2-\alpha))^{13}((2-\alpha)/(2-\alpha_m))^{5/4}-1$ or, considering the mass, $a_e=(m_e/m_m)(\alpha(2-\alpha)/\alpha_m(2-\alpha_m))^{1/2}-1$.



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Conclusion

A rotating Planck particle is able to explain all the main electron features. All quantities can be calculated with high accuracy and although the right numbers do not necessarily mean that the theory is correct, as Sir Arthur Eddington knew something about this back in 1919, it is remarkable that there is no need to introduce any other constant but h, c and G in order to have a reasonable description of the electron.

At this point it is convenient to see the steps that led us from the Planck particle to the electron as it is currently known.

1. **Planck** – The Planck particle is derived from fundamental constants. It is a non-rotating black hole. It is a theoretical particle and none if its parameters can be measured directly.
2. **Initial** – The Planck particle is now spinning. Interaction between its electric and magnetic force endows this particle with a defined charge, mass, permittivity and fine

structure constant and some important relations among fundamental constant are established. All these quantities refer to the initial electron where the precession of the spin angular momentum is not taken into account. Also in this case no parameters can be measured directly.

3. **Current** – Precession of the spin angular momentum S will be responsible for the slowdown, around 111m/s, of the rotational speed. This affects all parameters that we will eventually measure, thus fully identifying the electron. One of the solutions of a cubic equation gives the known fine structure constant and therefore the final rotational speed, alternatively, the speed is lowered until we have the known permittivity which is an exact quantity.
4. **Anomaly** – Interaction between the intrinsic magnetic moment and an external magnetic field will yield, as shown in the addendum, a decrease of the fine structure constant by a tiny amount. Such amount, around 89 ppm, will give us a slightly lower fine structure constant α_m generating a new set of parameters thus making the calculation of the magnetic moment, magnetic anomaly and gyromagnetic ratio possible.

The numerical result for the magnetic anomaly, magnetic moment and all other quantities, G being the only exception, show a difference of less than one standard deviation compared with current values (Codata 2014). It must be said that a_e is very sensitive to the fine structure constants. This means that they must be given with the highest precision. The fine structure constants were obtained from h, c and G, the latter being calculated with eq. 7 in order to have a very restricted range of values.
A summary table is shown below together with the main equations and their resulting numbers. Constant w_u is present in many places and its dimension must be accounted for in order to have dimensionally balanced equations.
The constant of gravitation was first calculated with known constants and then a refined value was used as part of the basic data, hence the small difference between the two values.

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Related Documents

1) D. Di Mario, (2003), *Planck permittivity and electron force*, http://digilander.iol.it/bubblegate/planckpermittivity.pdf
2) David Finkelstein, *Black holes*.
3) D. L. Bergman & J. P. Wesley, (1990), *Spinning charged ring model of electron yielding anomalous magnetic moment*, Galilean Electrodynamics, Vol. 2, 63-67.
4) Ph. M. Kanarev, *Planck's constant and the model of the electron*, www.journaloftheoretics.com/links/Papers/Kanarev-Electron.pdf
5) C. Spaniol and J.F. Sutton, (1992), *Classical electron mass and fields*, Part II, Physics Essay, vol. 5(3), 429-430.
6) Peter J. Mohr & Barry N. Taylor, (2000), *Quantum electrodynamics and the fundamental constants,* Electronic journal of differential equations, 4, 238. http://ejde.math.txstate.edu/conf-proc/04/m1/mohr.pdf

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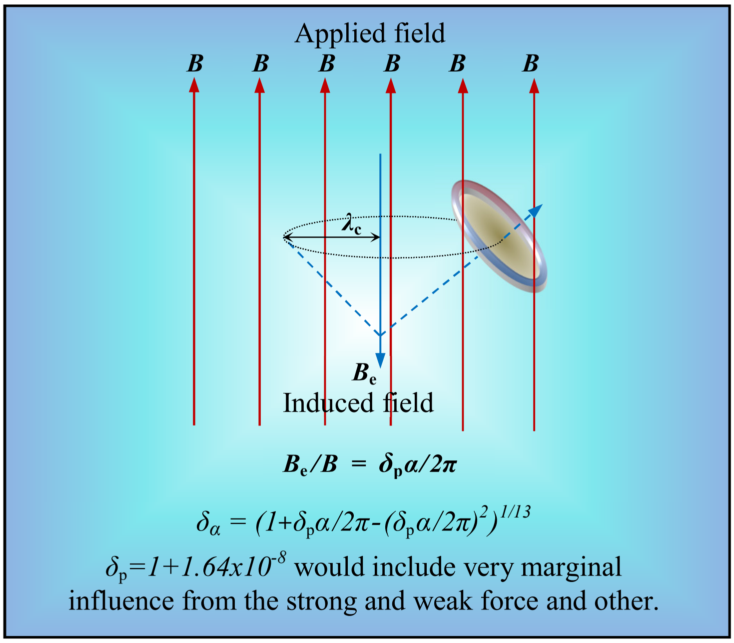
Addendum

Magnetic anomaly with known constants

An electron in a magnetic field B will generate, in turn, an "induced" magnetic field B_e. The ratio B_e/B is always the same regardless of the applied field B and is given by:



$$B_e/B = \mu_0 f_e e/2\lambda_c = \alpha/2\pi \quad (14)$$

We identify $f_e=e/2\pi m_e$ as the Larmor frequency and λ_c = Compton wavelength would be the radius of the rotation circle. A precise calculation for the Larmor frequency would call for an additional factor δ_p.

The ratio B_e/B is slightly lower due to the interaction between the applied and the induced field. The resulting ratio $(\delta_p\alpha/2\pi)(1-\delta_p\alpha/2\pi)$ is then related to the variation $(\alpha/\alpha_m)^{13}-1$ of the fine structure constant (eq. 13). Its variation δ_α is then given by:

$$\delta_\alpha = \alpha/\alpha_m = (1+\delta_p\,\alpha/2\pi-(\delta_p\,\alpha/2\pi)^2)^{1/13} \quad (15)$$

With the knowledge of δ_α we are able to calculate all the electron magnetic parameters using only known data. For example, the ratio magnetic moment/Bohr magneton is calculated as follows:

$$\mu_e/\mu_B = \delta_\alpha^{13}((2-\alpha)/(2-\alpha/\delta_\alpha))^{5/4} = (\alpha/\alpha_m)^{13}((2-\alpha)/(2-\alpha_m))^{5/4} \quad (16)$$

Quantity $\delta_p=1+1.64x10^{-8}$ was introduced in order to take care of secondary effects influencing the Larmor frequency, hence α_m. The value has been set in order to give results in line with known data and to have an idea of its magnitude but it is so small that can be disregarded in many calculations. The table below shows the effect on some parameters.

Quantity	without δ_p	with δ_p	Codata
Magnetic moment μ_e	$9.28476464858x10^{-24}$	$9.28476464876x10^{-24}$	$9.284764620(57)x10^{-24}$
Gyromagnetic ratio γ_e	$1.76085963825x10^{11}$	$1.76085963829x10^{11}$	$1.760859644(11)x10^{11}$
Magnetic anomaly a_e	$1.1596521619165x10^{-3}$	$1.1596521809098x10^{-3}$	$1.15965218091(26)x10^{-3}$

It is only when dealing with the magnetic anomaly that δ_p becomes important due to other marginal effects. They play a very minor role and their exact calculation, which must take in account also the relativistic factor, requires a detailed study leading to quantity δ_p.

Numeric Table

Basic data		
$c = 299792458$ $h = 6.62607008\text{x}10^{-34}$ $G = 6.6729195742\text{x}10^{-11}$		
Planck quantities and initial electron		
Planck time t_p	$(\pi h G/c^5)^{1/2}$	$2.39502\text{x}10^{-43}$
Planck mass M	$h t_p/c^2$	$3.0782613\text{x}10^{-8}$
Quantized Planck mass M_0	$M t_p^{1/2}$	$1.5064685\text{x}10^{-29}$
Planck permittivity ε_p	$(t_p/4\pi^2)^{1/4}$	$8.82545998\text{x}10^{-12}$
Toroid unitary charge squared/unitary time w_u	$(2\pi)^4 Q_u^2/t_u$	1558.5454565
Planck charge Q	$M(4\pi\varepsilon_p G)^{1/2}$	$2.648116\text{x}10^{-18}$
Initial fine structure α_0	$(w_u t_p)^{1/2}/Q$	$7.295873083\text{x}10^{-3}$
Initial charge e_0	$(w_u t_p/\alpha_0(2-\alpha_0))^{1/2} = Q/(2/\alpha_0-1)^{1/2}$	$1.60233847\text{x}10^{-19}$
Initial mass m_b	$M_0(\alpha_0/2)^{1/2}(1-\alpha_0/2)^{3/8}$	$9.08632983\text{x}10^{-31}$
Initial rotational speed u_0	$c(1-\alpha_0/2)^{1/2}$	299245146.4733
Connecting fundamental constants		
Newton's G with known data	$8\hbar c^3(\alpha^2(2-\alpha)/\mu_0 w_u)^2$	$6.6729195334\text{x}10^{-11}$
Fine structure constant α	$\alpha^3-2\alpha^2+w_u t_p/2\varepsilon_0 h c = 0$	$7.2973525665\text{x}10^{-3}$
Electron parameters		
Final rotational speed u_e	$c(1-\alpha/2)^{1/2}$ or u_0 decreased until $\varepsilon_0=10^7/4\pi c^2$	299245035.386
Charge e	$(w_u t_p/\alpha(2-\alpha))^{1/2} = Q/(\alpha/\alpha_0)(2/\alpha-1)^{1/2}$	$1.6021766257\text{x}10^{-19}$
Mass m_e	$M_0(\alpha/2)^{1/2}(\alpha/\alpha_0)^{12}((1-\alpha/2)(2-\alpha)/(2-\alpha_0))^{3/8}$	$9.10938361\text{x}10^{-31}$
Permittivity ε_0	$\varepsilon_p/(\alpha/\alpha_0)^2(1-\alpha/2)$	$8.854187817\text{x}10^{-12}$
Bohr magneton μ_B	$(Q\hbar/M_0)(\alpha_0/\alpha)^{13}((2-\alpha_0)^3/2)^{1/8}/(2-\alpha)^{5/4}$	$9.2740100228\text{x}10^{-24}$
Magnetic field ratio, $f_e = e/2\pi m_e$, λ_c=Compton wl.	$\mu_0 f_e e/2\lambda_c = \alpha/2\pi$	$1.16140973244\text{x}10^{-3}$
Sensitivity to other physics δ_p	*Strong and weak force, other*	$1+1.64\text{x}10^{-8}$
Interaction between main and induced field	$(\delta_p\alpha/2\pi)(1-\delta_p\alpha/2\pi) = (\alpha/\alpha_m)^{13}-1$	$1.160060878881\text{x}10^{-3}$
α variation due to magnetic interaction δ_α	$\alpha/\alpha_m = (1+\delta_p\alpha/2\pi-(\delta_p\alpha/2\pi)^2)^{1/13}$	1.00008918770992486
Fine struct. of intrinsic magnetic moment α_m	α/δ_α	$7.2967017904\text{x}10^{-3}$
Mass in intrinsic magnetic moment m_m	$M_0(\alpha_m/2)^{1/2}(\alpha_m/\alpha_0)^{12}((1-\alpha_m/2)(2-\alpha_m)/(2-\alpha_0))^{3/8}$	$9.09923639\text{x}10^{-31}$
Charge in intrinsic magnetic moment e_m	$(w_u t_p/\alpha_m(2-\alpha_m))^{1/2} = Q/(\alpha_m/\alpha_0)(2/\alpha_m-1)^{1/2}$	$1.6022478097\text{x}10^{-19}$
Magnetic moment μ_e	$(Q\hbar/M_0)(\alpha_0/\alpha_m)^{13}((2-\alpha_0)^3/2)^{1/8}/(2-\alpha_m))^{5/4}$	$9.28476464876\text{x}10^{-24}$
Gyromagnetic ratio γ_e	e_m/m_m	$1.7608596383\text{x}10^{-11}$
Magnetic moment/ Bohr magneton μ_e/μ_B	$\delta_\alpha^{13}((2-\alpha)/(2-\alpha/\delta_\alpha))^{5/4} = (\alpha/\alpha_m)^{13}((2-\alpha)/(2-\alpha_m))^{5/4}$	1.00115965218091
Magnetic anomaly a_e	μ_e/μ_B-1	$1.15965218091\text{x}10^{-3}$