

DOWNLOAD Mathematica Notebook

The best-known two-dimensional <u>cellular automaton</u>, invented by John H. Conway and popularized in Martin Gardner's *Scientific American* column starting in October 1970. Life, sometimes also called "the game of life," was originally played (i.e., successive generations were produced) by hand with counters, but implementation on a computer greatly increased the ease of exploring patterns.

The life <u>cellular automaton</u> is run by placing a number of filled cells on a two-dimensional grid. Each generation then switches cells on or off depending on the state of the cells that surround it. The rules are defined as follows. All eight of the cells surrounding the current one are checked to see if they are on or not. Any cells that are on are counted, and this count is then used to determine what will happen to the current cell.

1. Death: if the count is less than 2 or greater than 3, the current cell is switched off.

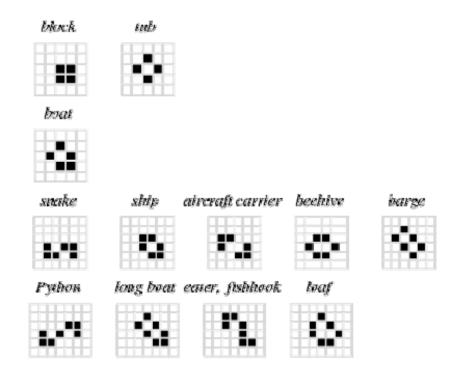
2. Survival: if (a) the count is exactly 2, or (b) the count is exactly 3 and the current cell is on, the current cell is left unchanged.

3. Birth: if the current cell is off and the count is exactly 3, the current cell is switched on.

The game of life is a <u>totalistic cellular automaton</u>, and can be implemented as follows using the built-in command <u>CellularAutomaton</u>, where the <u>initial conditions</u> are specified as a binary matrix m and the results for generations \$1 through \$2 are returned. (Here, \$ = \$ corresponds to the initial pattern.)

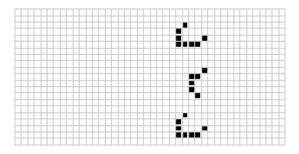
```
Life[m_List?MatrixQ, {g1_Integer?NonNegative,
 g2_Integer?NonNegative}] :=
CellularAutomaton[
    {
        224,
        {2, {{2, 2, 2}, {2, 1, 2}, {2, 2, 2}}},
        {1, 1}
    },
    {m, 0},
    g2,
    {
        {g1, g2},
        Automatic
    }] /; g2>=g1
```

Weisstein gives an extensive alphabetical tabulation of life forms and terms.

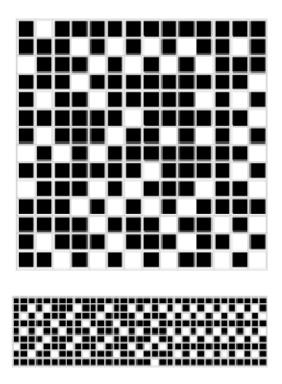


A pattern which does not change from one generation to the next is known as a still life, and is said to have period 1. Several still lifes are illustrated above. The numbers of still lives of n cells for $n \equiv 1, 2, 3, ...$ are 0, 0, 0, 2, 1, 5, 4, 9, 10, 25, 46, 121, 240, 619, 1353, ... (Sloane's A019473).

Patterns that cycle through a set of configurations are called oscillators.



Conway originally believed that no pattern could produce an infinite number of cells, and offered a \$50 prize to anyone who could find a counterexample before the end of 1970 (Gardner 1983, p. 216). Many counterexamples were subsequently found, including guns and puffer trains (illustrated above).



A life pattern which has no father pattern is known as a Garden of Eden (for obvious biblical reasons). The first such pattern was not found until 1971, and at least three are now known. It is not, however, known if a pattern exists which has a father pattern, but no grandfather pattern (Gardner 1983, p. 249).

Amazingly, life is a <u>universal cellular automaton</u>, in the sense that it is effectively capable of emulating any <u>cellular</u> automaton, <u>Turing machine</u>, or any other system that can be translated into a system known to be <u>universal</u>. The outlines of a proof for life's <u>universality</u> were given by Berlekamp *et al.* (1982) and independently by Gosper (Gardner 1983, pp. 250-253). Around 2000, a <u>Turing machine</u> that can be extended to a <u>universal Turing machine</u> was explicitly implemented in life by P. Rendell (Rendell, Adamatzky 2001). While Rendell's machine can be made into a "true" universal computer simply by making his tape infinite, he neither noted this fact nor provided an actual construction of a <u>universal Turing machine</u>. Subsequently, on November 11, 2002, P. Chapman constructed a life pattern based on D. Hickerson's "sliding block memory" approach that implements the actions of a <u>universal register machine</u>. Unlike the finite tape of Rendell's <u>Turing machine</u>, the values in the registers of Chapman's machine are unbounded, making it a true model of universal computation in the game of life. Chapman's construction uses <u>268096</u> live cells in an area of <u>4558×21469</u>, and can calculate approximately 20 generations per second on a 400 MHz computer.

More amazingly still, as shown by Wolfram (2002), even one-dimensional cellular automata (in particular, <u>rule 110</u>), can be <u>universal</u>.

Two-dimensional <u>cellular automaton</u> games similar to life but with different rules have been constructed and given the names HexLife and HighLife. HashLife is a life <u>algorithm</u> that achieves remarkable speed by storing subpatterns in a hash table and using them to skip forward, sometimes thousands of generations at a time.

SEE ALSO: Cellular Automaton, Larger than Life, Rule 110, Totalistic Cellular Automaton, Universal Cellular

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