REAL OPTIONS VALUATION OF AUSTRALIAN GOLD MINES AND MINING COMPANIES

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ABSTRACT

Conventional discounted cash flow valuation techniques are inappropriate for mining companies because operational flexibilities are deemed an essential component of mine values. In this article we review the real options literature on the valuation of mines and their embedded option to close the mine. We use a model based on Brennan and Schwartz (1985) to empirically value Australian gold mines and mining companies. One difficulty with doing empirical research in this area is in obtaining relevant and complete data, given the nature of real assets and the fact that investments are typically private in nature. The mine data for this study is supplied by Brook Hunt mining and metal industry consultants, UK and covers the period from 1992 to 1995. The primary advantage of this data set is its consistency across different mines that cannot be matched by data sets derived from annual reports data. We find that the real options model is a useful tool for the description and valuation of operational flexibilities. However, the values of the embedded options are very sensitive to estimation errors in the input parameters of the model. While average and median closure option values are economically significant, the option values vary over a large range.

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I. INTRODUCTION

The literature in the area of real options is relatively recent. The foundation of the literature comes from financial option pricing theory, which started with pioneering work of Black and Scholes (1973), Merton (1973), Cox and Ross (1976) and Cox, Ross and Rubinstein (1979). As financial option pricing theory became more widely accepted and understood, real options literature began in its own right.

Despite the widespread usage of discounted cash flows (DCF), various studies have found that managers often do not adhere strictly to DCF prescriptions [see Hayes and Abernathy (1980), Hayes and Garvin (1982)]. It is only in the last fifteen or so years that academics and practitioners are beginning to understand why DCF cannot properly capture the total value of an investment. DCF fails because it cannot accurately take into account the managerial flexibility that is embedded in investment opportunities and the interdependencies between them. In particular, according to Dixit and Pindyck (1995), DCF assumes either the investment is reversible or it is a now-or-never proposition. To incorporate the value of their flexibility, managers make apparently ad-hoc adjustments to DCF valuations [see McDonald (1999)]. A more complete solution, however, is to use the real options framework in valuing investment opportunities.

Trigeorgis and Mason (1987) and Trigeorgis (1996) provided further examples of real options. The options to defer, expand, or contract a project are applicable in many investment decisions and especially with natural resource projects whose profitability depends on the highly volatile price of the underlying commodity. Exhibit 1 summarizes the different types of generic real options as well as the representative papers in the academic literature.

Insert Exhibit 1 about here

Exhibit 1, while by no means exhaustive, illustrates that theoretical real options literature has made significant inroads into the valuation of many different types of real options. Admittedly, valuing investment opportunities as options is mathematically more involved than DCF techniques, but Schwartz (1998) describes the clear and immense advantages. Real option valuation:

- obviates the need to make assumptions about the trajectory of spot prices
- does not require the estimation of a risk-adjusted discount rate
- explicitly allows for managerial flexibility in the form of options in the valuation procedure.

In this paper, we focus on the work of Brennan and Schwartz (1985), one of the most often cited papers in the area of real options. It has been credited as the first theoretical paper to pioneer the use of options methodology in valuing physical assets. Within the context of mining natural resources, the mine operator has considerable flexibility in its operation, including the opening, closing and abandoning of the mine. This flexibility embedded in the value of the mine was often neglected or, at best, taken into account unsatisfactorily in valuation with DCF models. Brennan and Schwartz (1985) were among the first to capture the value of this flexibility in a sound mathematical model. Their model uses the standard no-arbitrage approach in pricing derivatives. The ingenuity lies in their replication of a risk-free bond by combining a futures contract for the underlying natural resource with the underlying physical mine asset. Trigeorgis (1996) further noted Brennan and Schwartz were the first to use the convenience yield derived from futures and spot prices of a commodity to value the options to shut down a mine. Similar to the seminal Black and Scholes (1973) paper, a set of Partial Differential Equations (PDE) satisfied by the value of the mine were derived and showed, in general, how assets whose cash flows depend on highly variable output prices could be valued. The model also shed light on how the optimal policy on managing mine investments could be determined.

Although considerable work in conceptual and theoretical real options has been performed, there has been relatively little work with an empirical focus, that is, examining how well real options theory predicts reality. One obvious difficulty of this type of research is obtaining relevant and complete data, given the nature of real assets and the fact that investments are typically private in nature. Paddock, Siegel and Smith (1988) is one of the earliest and most well known theoretical and empirical works on applying real option valuation. The paper focuses on U.S. offshore petroleum leases and compares the model prices with government estimates and with actual market bid prices. Another early but important empirical contribution is Quigg

(1993), which claimed to be the first to examine the empirical predictions of a real option-pricing model using a large sample of market prices, as opposed to Paddock *et. al.* (1986) who used only a limited number of market prices. Their paper focuses on options in a real estate context and incorporates not only the "intrinsic value" of the land, but also the option value to wait, invest and build. Berger, Ofek and Swary (1996) examined the abandonment option of a firm and investigated whether the market prices this option at its exit value in the price of equity. Real options theory postulates that the abandonment option is valuable[1]. If operating income is too low, then the firm can abandon its operation and exit the industry for the salvage value. The paper by Moel and Tufano (2002) is an empirical study of mine closings and openings using a proprietary database that tracks annual opening and closing decisions of 285 North American gold mines over the period from 1988 – 1997. Their objective was to find the factors that affect mine opening and closings and determine how well these fit the theory of real options, as predicted by Brennan and Schwartz (1985). From the empirical evidence, they found *"statistically significant and economically material"* support for real options theory. In particular, they found the presence of the strong hysteresis effect described in Brennan and Schwartz (1985) – that a mine is more likely to remain open if it was open in the prior period; and it is more likely to remain closed if it was closed in the prior period. Tufano (1998) is not a direct empirical test on real options, but its results hold important implications for the presence of real options in the gold mining industry. The paper examined the exposure and elasticity of North American gold mining firms' stock prices to changes in the price of gold from 1990-1994. Its main finding, and one of the motivations of this paper, is that markets take real optionality into account in valuations. In general, the contribution to real options literature from empirical research has been sparse. However, the few formal empirical papers that do exist generally support the significance of real options. It remains for future research to explain empirical phenomena and rigorously test the validity of real options theory in the context of the market.

The remainder of this paper is organized as follows. The next section describes the real options model, the numerical estimation procedure, and the data. The third section applies the model to the valuation of Australian gold mines and the fourth section concludes.

[1] The abandonment option is valued analytically by Myers and Majd (1990) who used the analogy that the option is an American put option on the value of the firm.

II. MODELING THE OPTION TO OPEN, CLOSE AND ABANDON A MINE

Ownership of a mine with the option to open, close and abandon a mine is analogous to an American call option on the gold in the mine. The price of gold, S, is analogous to the price of the underlying asset, and the average extraction cost, denoted by A, is analogous to the option's strike price. Similar to financial option pricing, the other relevant parameters are the volatility of the price of gold, σ, the risk-free interest rate, ρ, and the convenience yield for gold, c. In this model the only stochastic variable is the price of gold.

Note that it is the total mine value relative to the market price of gold, S, that is analogous to the value of an American call option. The closure option component of the mine value that gives the mine owner flexibility is akin to the time value of the option. Without the closure option, the mine owner can only permanently open, close or abandon the mine, but cannot switch between these states. Intuitively, if the price of gold, S, is high enough relative to the average extraction cost, A, then the mine operator will exercise the option to open and operate the mine; otherwise, the mine remains closed. If the mine is closed but not abandoned, an annual maintenance cost, M, must be paid. If the mine is abandoned, then no additional expenses are accrued, but no more gold can ever be mined. If the mine is open, the owner extracts an amount of gold equaling the fixed mine capacity per period, q. In other words, q is the (maximum) rate at which one can mine. We denote the total mine reserves by Q. As in pricing American options, the threshold gold price, denoted S^*, for which the mine will open or close is not necessarily the level of the average extraction cost A. In other words, it is not necessarily true that the mine will be opened if $S > A$, and closed otherwise. In addition, one may choose to abandon the mine, in which case all maintenance costs cease and the mine becomes worthless. The threshold for abandonment is denoted S^0.

1) The Brennan and Schwartz Model

In this article, we implement the model of Brennan and Schwartz (1985). The model will first determine the value of a mine with flexibility (i.e., with the closure option). Valuation is achieved by a no arbitrage argument in which a risk-free portfolio is created by taking a position in the physical mine and in the futures market. With some modifications, the same model is also used to derive the value of the mine without flexibility. The value of the closure option is the difference between the mine value with flexibility and the mine value without flexibility.

The fundamental stochastic variable that drives the mine value is the price of gold, S. A common assumption for commodity prices is that they follow the geometric Brownian motion process with drift μ and instantaneous standard deviation σ. We also consider a futures contract with futures price, F, convenience yield, c, and time to maturity $\tau = T - t$, so that $F = F(S,\tau) = Se^{(\rho - c)\tau}$. The futures price then satisfies the stochastic differential equation

$$dF = (\mu - \rho + c)Fdt + \sigma Fdz \,. \qquad \textbf{(1)}$$

1. *Partial Differential Equation for the Value of the Opened Mine*

Suppose the value of the open mine is denoted by V. As discussed previously, V will be a function of the price of gold, S, the level of reserves, Q, and the mine operating policy ϕ. Intuitively, ϕ represents the set of opening/closing/abandoning decisions. Hence, $V = V(S,Q;\phi)$[2] and in applying Ito's Lemma, we obtain:

$$dV = V_S dS + \frac{1}{2}V_{SS}(dS)^2 + V_Q dQ \,. \qquad \textbf{(2)}$$

If the mine is operated at a capacity of q, then reserves will change according to:

$$dQ = -qdt \,. \qquad \textbf{(3)}$$

Consider a portfolio that is long a unit of the mine V and short $\frac{V_S}{F_S}$ units of the gold futures contract. The instantaneous dollar return, given dt time has elapsed, for this portfolio is:

$$dV + q(S - A)dt - \left(\frac{V_S}{F_S}\right)dF \,, \qquad \textbf{(4)}$$

where the first term is the change in value of the opened mine, the second term is the net cash inflow from extraction of gold and the third term is the changes in value of the short futures

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[2] In the original Brennan-Schwartz model, an additional variable is t. This is only relevant if extraction cost A and convenience yield c are functions of t. In our model, these variables will be modeled as a given constant.

contract. It is not difficult to show that this portfolio is risk-free, and therefore its dollar return is ρVdt. One can then show that V satisfies the partial differential equation

$$\frac{1}{2}S^2\sigma^2 V_{SS} + (\rho - c)SV_S - qV_Q + q(S - A) - \rho V = 0. \tag{5}$$

This is the PDE satisfied by the open mine V. To further specify V, boundary conditions are required. An obvious boundary condition[3] is that when gold reserves are zero, the value of the mine is zero, $V(S,0) = 0$. A similar derivation applies to the closed mine. Suppose the value of the closed mine is denoted by W. Similar to V, W will also be a function of the price of gold, S, the level of reserves, Q, and the mine operating policy ϕ. Hence, $W = W(S,Q;\phi)$ and in applying Ito's Lemma, we obtain:

$$dW = W_S dS + \frac{1}{2}W_{SS}(dS)^2 + W_Q dQ \tag{6}$$

Now, by definition the reserves of a closed mine do not decrease as time passes. Hence, $dQ = 0$. Consider a portfolio that is long a unit of the mine W and short $\frac{W_S}{F_S}$ units of the gold futures contract. The instantaneous dollar return given dt time has elapsed for this portfolio is:

$$dW - Mdt - \left(\frac{W_S}{F_S}\right)dF, \tag{7}$$

where the first term is the change in value of the opened mine, the second term is the maintenance cost incurred and the third term is the change in value of the short futures contract. Again, this is a risk-free portfolio and the dollar return is ρWdt. Thus W satisfies the partial differential equation

$$\frac{1}{2}S^2\sigma^2 W_{SS} + (\rho - c)SW_S - M - \rho W = 0. \tag{8}$$

This is the PDE satisfied by the closed mine, W. To further specify W, boundary conditions are required. An obvious boundary condition is that when reserves are zero, the value of the closed mine will also be zero, $W(S,0) = 0$.

[3] Note $V(0,Q) = 0$ is an incorrect boundary condition. For an open mine, $V(0,Q)<0$ is possible.

2. Value of the Mine

We have shown that the value of an open or closed mine respectively is determined to satisfy the PDEs in (5) and (8) and the two boundary conditions. To complete the model, further boundary conditions are required. The missing information is the threshold between opening and closing the mine. As described before, a mine will be operated if S is higher than some threshold S^*; otherwise it will be closed. With this definition, an additional boundary condition can be obtained assuming zero opening and closing costs:

$$V\left(S^*,Q\right)=W\left(S^*,Q\right) \qquad \textbf{(9)}$$

Note that if $S > S^*$, the mine should be open because $V(S, Q) > W(S, Q)$. On the other hand, if $S < S^*$, the mine should be closed because $W(S, Q) < V(S, Q)$.

To incorporate the abandonment option, consider the closed mine value W. Clearly, W is a monotonic function of S: as S decreases, so does W. It is possible that if S is sufficiently low, $W < 0$. Intuitively, this means that S is low enough that the firm is better off forgoing the possibility of re-opening the mine in the future and no longer incurring the maintenance cost of M per period. If the firm were not able to close down, the mine would have a negative value. The value of the mine, denoted by H, satisfies equation (5) when $V > W$, equation (8) when $W > V$, $W > 0$, and $H = 0$, otherwise. The case $H = 0$, corresponds to the abandonment of the mine. The boundary conditions are, $H = W\left(S,0\right)=V\left(S,0\right)=0$, $H = W(S^*, Q) = V(S^*, Q)$, and $H = W(S^0, Q) = 0$. Brennan and Schwartz (1985) and Dixit and Pindyck (1994) suggest an additional "high contact" or "smooth-pasting" boundary condition:

$$W_S\left(S^*,Q\right)=V_S\left(S^*,Q\right). \qquad \textbf{(10)}$$

Intuitively, this condition ensures the functions W and V join smoothly at S^*. This final boundary condition completes the mathematical description of the model. As Brennan and Schwartz (1985) point out, there is no analytical solution to this problem. The following section develops a numerical estimation technique to evaluate the value of the mine.

2) Procedure for Numerical Estimation

The form of the PDEs derived above suggests the simplifying substitution $z = \ln S$. To estimate the PDE for V and W, an explicit finite difference method will be employed. We find the solution:

$$V_{i+1,j} = \frac{1}{1+\rho\Delta t}\left\{\left(p^u V_{i,j+1} + p^m V_{i,j} + p^d V_{i,1-1}\right) + q\left(e^{z_j} - A\right)\Delta t\right\} \qquad \textbf{(11)}$$

where,

$$p^u = \frac{\sigma^2 \Delta t}{2(\Delta z)^2} + \left(\rho - c - \frac{\sigma^2}{2}\right)\frac{\Delta t}{2\Delta z}$$

$$p^m = 1 - \frac{\sigma^2 \Delta t}{(\Delta z)^2}$$

$$p^d = \frac{\sigma^2 \Delta t}{2(\Delta z)^2} - \left(\rho - c - \frac{\sigma^2}{2}\right)\frac{\Delta t}{2\Delta z}.$$

Notice that $p^u + p^m + p^d = 1$, so that, as long as Δt is small enough that p^u, p^m, and $p^d > 0$, p^u, p^m and p^d can be interpreted as the probabilities of z moving up, staying at the same level and moving down respectively. In fact, it is straightforward to show that these are risk-neutral probabilities, in the sense that

$$p^u z^u + p^m z^m + p^d z^d := E_p\left[\ln \frac{S_{t+\Delta t}}{S_t}\right] = \left(\rho - c - \frac{\sigma^2}{2}\right)\Delta t.$$

Thus, this method of numerical approximation is equivalent to a trinomial tree framework, and $V_{i+1,j}$ = Present Value of (Weighted Probability of $V_{i,j+1}, V_{i,j}, V_{i,j-1}$ + Profit for mining the reserves[4]). Similar discrete estimations can be made for W. With explicit differences, we find that

$$W_{i+1,j} = \frac{1}{1+\rho\Delta t}\left(p^u W_{i,j+1} + p^m W_{i,j} + p^d W_{1,j-1} - M\Delta t\right) \qquad \textbf{(12)}$$

[4] Profit = (Revenue – Cost) × Capacity × Time = $(S - A)\times q\times\Delta t = (e^z - A)\times q\times\Delta t$

where, p^u, p^m, and p^d are as above. The step size Δz, a function of Δt, needs to be specified. Hull (1997 p.376) suggests that setting $\Delta z = \sigma\sqrt{3\Delta t}$ is numerically most efficient within the trinomial tree framework. Finally, at each node of the trinomial tree, we calculate max(V,W,0).

3) Data selection and variable estimation

The mine data for this study is supplied by Brook Hunt mining and metal industry consultants, UK and covers the period from 1992 to 1995. The primary advantage of the data set is its consistency across different mines. This consistency cannot be matched by data sets derived from annual reports data. To correctly attribute individual mine ownership to the listed companies, the Brook Hunt data is supplemented with companies' annual report data, thereby accounting for changes in mine ownership. After the mine attribution process the data set is filtered using the following criteria. To be included in the data set companies must:

(a) own or partly own at least one operating gold mine in the sample period
(b) be classified by the Australian Stock Exchange as a gold mining/exploration company
(c) be listed for at least 2 years during the sample period
(d) have a relatively simple structure with the bulk of its activities in the gold sector
(e) be of sufficient size, that is, have a market capitalization of at least A$100m for at least one of the years in operation

The first four criteria yield 30 companies with a total of 112 company-year data points for which the economic significance of the closure option can be determined. The final criterion, specified for statistical significance tests which require market capitalization, return and accounting information, remove an additional three companies. The final data set includes 27 companies with 103 company-year and 217 mine-year data points.

Brook Hunt report three different extraction cost variables. For this study we use the broadest definition for the average extraction cost variable, A, which includes direct costs, ongoing capital expenditures, depreciation, indirect costs, and interest. To calculate the total gold reserves for each mine, Q, Brook Hunt reports "proven and probable" reserves of the total ore body that contains gold, not actual reserves of gold itself. Hence, Q is estimated by multiplying the quoted ore reserves by the "estimated grade" of the ore, which estimates the amount of gold present in the ore body. To calculate the variable gold mine capacity, q, a similar calculation is performed. The "mill capacity", quoted as the number of kilo-ton of ore body the mine can process per annum, is multiplied by the estimated grade of the ore reserves in the same year. Note that "mill capacity" and the actual level of ore production are not always equal. In general, mines seem to produce at an output rate greater than the reported capacity. This is due to uneven quality of ore bodies, which enabled some ore bodies to be processed faster than others, hence varying the mine output rate from the reported capacity. In addition, firms can choose to slow production and run at a level well below the mine capacity. Since it is impossible to predict *a-priori* whether the gold mill will be over or under-utilized, the quoted capacity q is the best estimate available. Furthermore, our theoretical model assumes that there is a single fixed output rate q for each year.

In practice, the maintenance cost, M, or the cost incurred when a mine is closed, cannot be directly observed. For its estimation we follow the method devised by Moel and Tufano (2002). Conceptually, the known average cash cost, A, includes the fixed cost and variable cost component, where fixed cost is defined as the component of cost that cannot be avoided regardless of the level of production. This fixed cost component can be estimated using an OLS regression model and it acts as an estimate for M. Our model has a relatively high R^2 of 0.87 and the magnitude of the coefficient for the fixed cost component is consistent with the findings of Moel and Tufano (2002). Furthermore, the mine valuation model is very robust to variations in the maintenance cost estimate, M.

The difference between the Treasury bond/bill rate and the convenience yield, $\rho - c$, must also be determined. Because the Australian Reserve Bank does not have an active gold lending market, we use LIBOR and the gold lease rate from the London Central Bank as a proxy $\rho - c$. However, to ensure consistency with the Australian data we use the Australian 10-year Treasury bond rate for ρ where it is used as the discounting term in the denominator of the above equation.

The source for the remaining model variables (S, σ_S^2, and exchange rates) is Datastream. Finally, the Brook Hunt data are all denominated in USD and the mine value with and without flexibility and the closure option will be determined in USD. To perform the significance tests on a consistent basis, these values are converted into AUD – the currency in which most of the annual report data and market capitalization information are quoted. The annual average daily AUD/USD exchange rate from Datastream is used for this conversion.

III. APPLICATION OF THE REAL OPTION MODEL TO AUSTRALIAN GOLD MINES

To illustrate the effects of applying a real option valuation approach to the valuation of Australian gold mines we estimate the characteristics of a hypothetical representative mine in Panel A of Exhibit 2 using the 217 mine data points for the 4 years. In Panel B we estimate the value of this representative mine with various valuation techniques.

Insert Exhibit 2 about here

The simplest valuation method in Panel B uses a discounted cash flow approach under the assumption that extraction margins remain constant until the mine is exhausted. The second valuation technique, modified DCF, considers that in a risk-neutral world, the gold price in future years will not stay constant. Instead, it will grow at a continuously compounded rate of (ρ–c) = 3.3% per annum. Another way to view this is to recall the theoretical relationship between forward and spot price: $F = Se^{(\rho - c)t}$. Thus, the firm can sell its anticipated gold production at the forward price, which grows at a rate of ($\rho - c$) per annum from the current spot price. Using the real option model developed in the previous section leads to higher mine valuations than these first two methods. For the *Mine With (Without) Flexibility* the value of the representative mine is \$49.5m (\$48.5m). An alternate approach suggested by Quigg (1993) calculates the value of the *Mine Without Flexibility* by letting $\sigma \rightarrow 0$ in the real option model with flexibility. This approach also finds a mine value without flexibility to be \$48.5m.

The "naïve DCF approach" significantly undervalues the mine even at the low range of discount rates. The "modified DCF approach" results in considerably higher valuations, but the valuations are still below the mine values of the real options approach. The lower valuations are due to the inability of the DCF technique to adjust for the volatility of gold prices, the stochastic nature of gold price, and the value of the embedded real options. A further disadvantage of the DCF approach is the need to obtain a subjective discount rate, which is not required for the real options model. Note that the alternative ways of calculating the value of the representative *Mine Without Flexibility* are within \$0.1m of each other—further evidence that our numerical estimation procedure is stable and possibly converging. The real option model values the closure option of the representative mine at \$1m. These results confirm the findings in previous literature that DCF valuation techniques can underestimate the theoretically correct mine value.

Furthermore, this is preliminary evidence that the closure option is valuable, though at 2.0 to 2.1% of the theoretical mine value of the representative mine, it does not dominate the total valuation of the mine.

Mine valuation with the real options approach is sensitive to the estimation of the input parameters. Exhibit 3 examines the change in the mine value if the input parameters are varied by ± 20% of the base case scenario.

Insert Exhibit 3 about here

The mine value is most sensitive to changes in the gold price, followed by changes in the average extraction cost. Surprisingly, of all input parameters, the mine value is least sensitive to changes in gold price volatility.[5] Apart from gaining some insight into how real options variables affect mine valuations, there is an important practical issue implied by this analysis. In valuing mine assets, more energy should be focused on estimating the gold price and extraction costs than, for example, interest rates or mine capacity.

1. Sensitivity analysis of the closure option value

Exhibit 4 shows the directional effect of increasing each of the input parameters on the value of the *Mine With Flexibility*, the *Mine Without Flexibility* and the closure option.

Insert Exhibit 4 about here

The interpretation and implications of Exhibit 4 are discussed below for each of the variables.

It is intuitively clear that increases (decreases) in the gold price and decreases (increases) in extraction costs will enhance (reduce) mine valuation. However, changes in these variables have the opposite effect on the value of the closure option (for a mine with positive intrinsic value). That is, increases (decreases) in cost will increase (decrease) the closure option value and increases (decreases) in gold price decrease (increase) the closure option value. As the gold

[5] A similar sensitivity analysis for the value of the closure option also found gold price and average extraction to be the main determinants. While the variations in the mine value are distributed almost symmetrically around the base case, the variations in the value of the closure option are heavily skewed to the right.

price decreases or extraction costs rise, the mine operator is more likely to close or abandon the mine to mitigate potential losses. Hence, closure option value rises even though the overall impact on mine valuation is negative. The percentage increase in the value of a mine with flexibility is much greater if the gold price rises from a low base than if it rises from a high base, even if the absolute change in value is small. Similar results are found if average extraction costs fall from a high level rather than from a low level.

Increasing the volatility of gold leads to an increased closure option value, which in turn increases the total value of a mine with flexibility. However, volatility has no effect on the value of a mine without flexibility. Importantly, this result defies conventional DCF valuation logic, where increases in uncertainty increase the discount rate, which in turn lowers the valuation.

An increase in interest rates, ρ, results in a decrease in the present value of the average cost of extracting the gold, however the risk-neutral expected return for the gold price is $\rho - c$, implying that the present value of the gold that is mined remains unchanged as ρ changes. The over-all effect is that an increase in ρ leads to an increase in the value of the mine. An increase in the convenience yield, c, causes a decrease in the growth rate of the gold price; hence, the value of the mine goes down.

The variables total gold reserves (Q) and mine capacity (q) behave in a similar way to time to expiration in a standard option-pricing framework. Q/q is the number of years that the mine will be productive if it is operated at full capacity. Clearly, the greater the reserve (Q), the greater the mine value (with or without flexibility). The closure option value will also be greater, due to the greater amount of underlying resources on which the closure option is written. The effect of the variable mine capacity q on mine valuation seems counter-intuitive. We would expect that the greater the mine capacity, the greater the value of the option on the gold produced (see McDonald and Siegel (1986)). This unexpected result is an artifact of a part of the structure of the model. Recall that at each stage, the production is either $q\Delta t$ or 0, with no intermediate production choice allowed.[6] This restriction means that additional mine capacity can be a hindrance instead of an increased level of choice. Furthermore, increasing mine capacity in the current set-up is akin to decreasing time to expiry (Q/q). Hence, the total valuation of the mine

[6] This structure makes the valuation of the model more tractable. Otherwise, an additional optimization problem would need to be solved for the optimal production rate in the interval $(0,q)$.

decreases. This result should be interpreted with care. However, Exhibit 3 shows that mine value is relatively robust to changes in mine capacity, so the chosen structure should not unduly affect the valuation results.

2. Economic significance of the closure option value

The value of the closure option can be expressed as an absolute dollar figure (OV) or as a percentage of the intrinsic mine value:

$$\text{Rel. OV} = \frac{\text{Closure Option Value (OV)}}{\text{Mine Value Without Flexibilility (WO)}} \qquad \textbf{(13)}$$

The greater the absolute dollar value or percentage value, the greater the economic significance of the closure option value. We calculate the dollar value of the closure option for each individual mine as well as the relative closure option value as defined in (13). To determine the company-wide value of the various closure options, we attribute to the mines' owners each mine's closure option value (*OV*), mine value with flexibility (*WITH*) and mine value without flexibility (*WO*) and then calculate the absolute and the relative option values. Hence, the absolute measure is the total dollar value of closure options the firm possesses in a particular year; while the relative measure is the company's total closure option value expressed as a percentage of the company's total mine value without flexibility. In addition, the company's total closure option value can be expressed as a percentage of the company's adjusted market capitalization (AMC) which is the market capitalization adjusted for the percentage of mining related assets of the firm.

1) Closure option value at the Individual Mine Level

Panel A of Exhibit 5 shows that out of the 217 mines in the total sample, 54 have negative intrinsic mine value without flexibility (*WO*) due to high average extraction costs relative to the gold price,[7] while two mines have a zero *WO* value due to near exhaustion of reserves. The values of the closure options range widely, varying from 0% to 988% of the intrinsic value of the mine. The distribution of values, however, is highly positively skewed. The mean closure option value as percentage of *WO* (*WITH*) in Panel A is 42% (13%); the median is substantially

[7] Although $WO < 0$, the corresponding *WITH* value will always be ≥ 0 because a physical mine can simply be abandoned. Recall that $WO + OV = WITH$. Thus *OV* will offset the negative *WO* to ensure $WITH \geq 0$.

lower at 1.7% (0.2%), indicating that there are a number of very large closure option values influencing the mean.

Insert Exhibit 5 about here

The distribution of the closure option values is bi-modal in nature; for most of the mines the closure option value is either a very large or a very small percentage of the intrinsic mine value. For example, for over 60% of the mines, the closure option value is less than 2% of the intrinsic mine value, while for 20% of the mines the closure option values are more than 5% of intrinsic value. Clearly, using the mean value to illustrate a typical mine can be misleading and thus the median, a more robust measure regarding distributional assumptions, should be used.

An alternative way of examining the economic significance of the closure option value is to find the average of the dollar values of the mine without closure option (*WO)* and the closure option value (*OV)*, and express the closure option value as percentage of the mine value. Panel B of Exhibit 5 presents the results of this approach. For the 215 pooled mine-year data, the mean closure option value *OV* is $3.0m per mine while the mean intrinsic mine value (*WO)* is $68.8m. Hence the mean closure option value is 4.38% of intrinsic mine value. The descriptive statistics also show high positive skewness, with the medians of *WO* and *OV* substantially lower than the means. Once again, the median is the more robust measure to examine.

Finally, the percentage of the median *WO* mine is shown in the fourth row. As shown, the *OV* value associated with the median *WO* is higher than the median *OV* value, resulting in a higher relative closure option value of 2.93%.

2) *<u>Closure Option Value at Company Level</u>*

The approach used in this section is similar to that of the previous section except that we examine the companies' total closure option values as a percentage of the companies' mine values without flexibility and as a percentage of the companies' adjusted market capitalization (AMC). This measure is potentially more interesting than the individual mine level figures as companies might strategically mix mines of different characteristics, offsetting some of the earlier highly skewed results.

Panel A of Exhibit 6 shows the *OV* using dollar averaging. A similar skewness pattern as at the individual mine level emerges. The mean *OV* is $6.4m, compared to a median dollar *OV* of

$1.2m. Panel B of Exhibit 6 reports the relative closure option values as fraction of the mine value. The mean and median relative closure option values are 4.37% and 1.95%, respectively. The third column of Exhibit 6 uses *AMC* in the denominator. Note that only 103 company-year observations are used, as *AMC* for 9 companies cannot be obtained. The mean *OV* on mean *AMC* is 1.64%, while the median *OV* on median *AMC* is 0.68%. The median *AMC* company has a closure option value of 0.11% of its *AMC*.

Insert Exhibit 6 about here

The median value of the individual mine's closure option is 2.4% of the individual mine's value without flexibility. The average dollar value of the closure option value is $0.4m. At the company level, the median closure option value is almost $1.2m, although expressed as a percentage of the mine value it is about 1.9%. While the percentages are not large, the dollar value of the closure option appears to be an economically meaningful figure.

Our results are generally consistent with past empirical literature, in which embedded real options in physical assets or investments are found to be economically significant. For example, Quigg (1993) found the option to delay developing unimproved land parcel adds about 6% of intrinsic land value, while Davies (1996) found the option value contributes up to 3% to a mineral asset's gross worth. Martzoukos and Teplitz-Sembitzky (1992) found the option to defer transmission line investments to be quantitatively significant and, Laughton and Jacoby (1991) found the option to delay oil development to have significant economic value.

While one may argue that the significance of the closure option as a percentage of total asset value is relatively low, and may even be lost in valuation inaccuracies, one must note that in this paper we present only average results. The averages conceal large variations in the results for the individual mines. In some cases, for example, the closure option value is worth over $50m and reaches almost 100% of the mine value with flexibility. This result highlights the fact that one must be cautious in examining the "average" economic significance in real option studies. The principal message is that mine managers cannot afford to ignore the presence of real options in mine valuations. The economic significance of the real option can only be ascertained on an individual, case-by-case basis.

3. Statistical Significance of Real Option Model and Closure Option Values

To ascertain the statistical significance of the closure option value we specify the following panel regressions. For a gold mining company, its market value will be a function of the gold mines it owns and operates where the total mine value (*WITH*) is the sum of the mine value without flexibility (*WO*) and the value of the closure option (*OV*). Since our focus is on a company's gold related assets, adjustments should be made to reflect the fact that market capitalization may not be entirely attributed to gold mining activities. To do this, the market capitalization is adjusted by the percentage of profits attributable to gold production for each year. Where this figure is not reported (or if the company loses money in a given year), the percentage of assets attributable to gold production is used. Exhibit 7 panel A contains the summary statistics of variables in the market capitalization models.

Insert Exhibit 7 about here

Note that the adjusted market capitalization (*AMC*) ranges from around \$6m to \$1,081m, with a mean of \$421m. In the regressions that follow, White's (1980) heteroscedastic-consistent covariance is used. Exhibit 7 panel B contains the correlation matrix of the regression variables. Notice that, as expected, *WO* and *WITH* are almost perfectly correlated, but the correlation between *WO* (*WITH)* with *OV* is only 0.13 (0.18). At a 5% significance level, the critical value is about 0.25, higher than the observed correlations. This confirms the statement in Exhibit 3 that the directional effect of input variables on *WO* and *WITH* is very similar, but very different for *WO* (or *WITH*) and *OV*. The matrix also shows very high positive correlation between *AMC* and *WO* and between *AMC* and WITH (0.8055 and 0.8278 respectively). Correlation of *AMC* with *OV* is of marginal significance at 0.2275, but *AMC* is significantly correlated with *EXPL* with a correlation of 0.4913. This preliminary evidence supports the real options model of mine intrinsic value and option value as a reasonable description of reality.

Intuitively, mine value should be directly proportional to the total value of the firm. Since the value of the firm accrues to both equity and debt holders, the mine value should therefore be proportional to the adjusted market capitalization.[8] Table 8 presents ordinary least square regression results for a linear function with the following specifications:

Panel A Adj. Market Cap = $\alpha_a + \beta_a$ (Total Mine Value)

Panel C Adj. Market Cap = $\alpha_b + \beta_b$ (Mine Value without Flexibility)

$+ \gamma_b$ (Closure Option Value)

Our hypothesis is that β_a, β_b and γ_b are significant. In regression analysis, an important assumption is that the model includes all relevant explanatory variables to ensure the estimates are unbiased. Hence, the regression model specified should control for other variables that affect market capitalization. *A priori*, apart from the physical ownership of the mines, another source of market value comes from the potential of the firm to find new and profitable mines through its exploration activities. This potential can be considered as the growth options that the firm possesses. Exploration Expenses (*EXPL*) are used to proxy for this growth option in regression panels D, E, and F. Intuitively, the higher the exploration expenses, the greater the value of the growth options. Admittedly, this proxy is far from perfect as not all exploration expenses lead to the discovery of profitable mines and the available data are not always consistently reported. Nevertheless, the characteristics of the growth option should at least be partly captured by this proxy and the highly complex nature of the growth option renders this as the simplest, albeit imprecise, approach for the intended purpose:

Panel F Adj. Market Cap = $\alpha_c + \beta_c$ (Total Mine Value without Flexibility)

$+ \gamma_c$ (Closure Option Value)

$+ \delta_c$ (Exploration Expenses)

The data examined include cross-sectional and time-series observations. Panel data techniques are used to perform multivariate regressions in order to maximize the sample size and statistical power. Suppose there are i = N firms for t = T years, y_{it} denotes the dependent variable and X_{it} denotes the independent variables. The models are:

1) Pooled Regression: $y_{it} = \alpha + \beta' X_{it} + \varepsilon_{it}$.

This is the usual multivariate regression that pools all data across different firms together and assumes a common intercept term. This model has the maximum sample size and the fewest number of variables.

[8] In reality, the valuation effect on shareholders and debtholders values are not the same. A

2) Fixed-Effect Between-Groups Regression: $y_{i.} = \alpha + \beta' X_{i.} + \varepsilon_{i.}$,

$$\text{where } y_{i.} = \frac{1}{T}\sum_{t=1}^{T} y_{it} .$$

This regression specifies a linear relationship between individual means. It averages the observations to reduce noise in the data, and so focuses on cross-sectional differences.

3) Fixed-Effect Within-Groups Regression: $y_{it} = \alpha_i + \beta' X_{it} + \varepsilon_{it}$.

This regression allows a unique intercept for each firm to account for cross-sectional firm differences. Hence, it controls for firm specific effect, while maximizing the sample size relative to a between-group regression.

For all three regression specifications in Panels A and C the coefficients of *WITH* and *WO* are highly significant and positive. Furthermore, the generally high adjusted R^2 obtained when either *WITH* or *WO* are used as explanatory variables indicate that a firm's mine value with or without flexibility explains much of the variation in the adjusted market capitalization of the sample company. For example, in Panel A in the univariate regression using *WITH* as the explanatory variable, the t-statistic of the coefficient estimate is 14.83 for the pooled regression with an adjusted R^2 of 69%. Between-group and within-group regressions display similar results. The univariate regression of AMC against the closure option value variable *OV* is significant at the 5% level for both pooled and between-group regressions, but insignificant for within-group regression. In other words, when each firm is allowed to have a unique intercept through time, the *OV* coefficient is no longer significant. One explanation might be that part of the *OV* may be captured by the firm specific intercept terms.

Insert Exhibit 8 about here

In the regression in panel C that controls for changes in *WO*, *OV* is significant at the 5% level for the pooled regression. *OV* is not significant for the between-group regression. Overall, the significance of the coefficient for *OV* seems to decline in the presence of *WO*. As predicted, the control variable Exploration Expenses (*EXPL*) is significant at the 5% level for the univariate and all multivariate regression specifications (see Panels D, E, and F). Finally, in the full

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priori, however, this is not expected to cause a directional bias in our significance test results.

regression model in Panel F with *WO*, *OV* and *EXPL* as explanatory variables, all coefficients with the exception of *OV* in the "between-group" regression are significant at the 5% level. It should also be noted that this specification results in relatively high adjusted R^2 and F-statistics, suggesting again that the regression model is able to explain much of the variations in market capitalization. The results presented provide encouraging evidence that the real options model developed is significant in explaining the cross-sectional variation in market capitalization. In particular, the company's closure option value is a significant determinant of market capitalization.

IV. CONCLUSIONS

We use a real options model based on Brennan and Schwartz (1985) to value Australian Gold mines and mining companies. We find that the real options model is a useful tool for the description and valuation of operational flexibilities. However, the value of the embedded options is very sensitive to estimation errors in the input parameters of the model. The median value of the closure option is 2.38 percent of the individual mines' value and 1.95 percent of the company's value without operational flexibility. The average and median closure option values are economically significant; however, the option values vary over a large range. Thus the usefulness of the real options model as numerical valuation tool for some individual mines and mining companies may be limited.

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Exhibit 1: Summary of Theoretical Literature

Type of Real Option	*Application in Industry*	*Theoretical and Industry Specific contributions*
Option to Defer	Natural Resources; mining; land development	McDonald & Siegel (1986), Paddock, Siegel and Smith (1988), Quigg (1993), Kulatilaka and Trigeorgis (1994), Mauer and Ott (1995)
Time to Build and Staged Investment Options	R&D in pharmaceutical and other capital intensive projects	Majd and Pindyck (1987), Morck, Schwartz and Stangeland (1989), Martzoukos and Teplitz-Sembitzky (1992), Trigeorgis (1993), Faulkner (1996) Schwartz and Moon (1999)
Option to Alter Operating Scale (expand, contract)	All industries	McDonald and Siegel (1985), Brennan and Schwartz (1985), Pindyck (1988)
Option to Abandon	Airlines, railroads, infrastructure	Myers and Majd (1990), Berger, Ofek and Swary (1996)
Option to Switch (inputs or outputs)	All industries	Kulatilaka and Marks (1988), Kulatilaka and Trigeorgis (1993), Childs, Ott and Triantis (1995)
Multiple Interaction Options	Most projects	Brennan and Schwartz (1985), Trigeorgis (1994), Kulatilaka and Trigeorgis (1994)

Exhibit 2: Characteristics and valuations of a Representative Australian Gold Mine

The representative gold mine consists of the arithmetic averages of the characteristics of the mines included in the data set. Panel B values this representative gold mine with different valuation techniques.

Panel A: Characteristics of the representative mine in the data set.

Total Reserves *(Q)*:	900,000	Capacity *(q)*:	125,000
Gold Price (*S*):	$368	Convenience Yield (*c*):	5.2%
Average Extraction Cost (*A*):	$338	Risk Free Rate (ρ):	8.5%
Gold Price Volatility (σ):	9.0%	Mine Technology:	Open Pit

Panel B: Valuation of the representative mine using different valuation techniques

Naïve DCF [zero growth]	$20.8m to $16.1m*
Modified DCF [ρ - *c* growth]	$45.0m to $31.9m*
Mine with flexibility (real option model)	$49.5m
Mine without flexibility (no closure option)	$48.5m
Mine without flexibility ($\sigma \rightarrow 0$)	$48.5m
Mine without flexibility (Integration)	$48.4m
Option Value	$1.0m
% of Mine Value without flexibility	2.1%
% of Mine Value with flexibility	2.0%

* Discount rates ranging 8.5% to 20.5%.

Exhibit 3: Sensitivity of the Representative Mine Valuation to Variations in the Input Parameters

The graph displays the value of the mine with flexibility, by varying each input from the base case by ±20% while keeping all other inputs constant. All figures are in $US m.

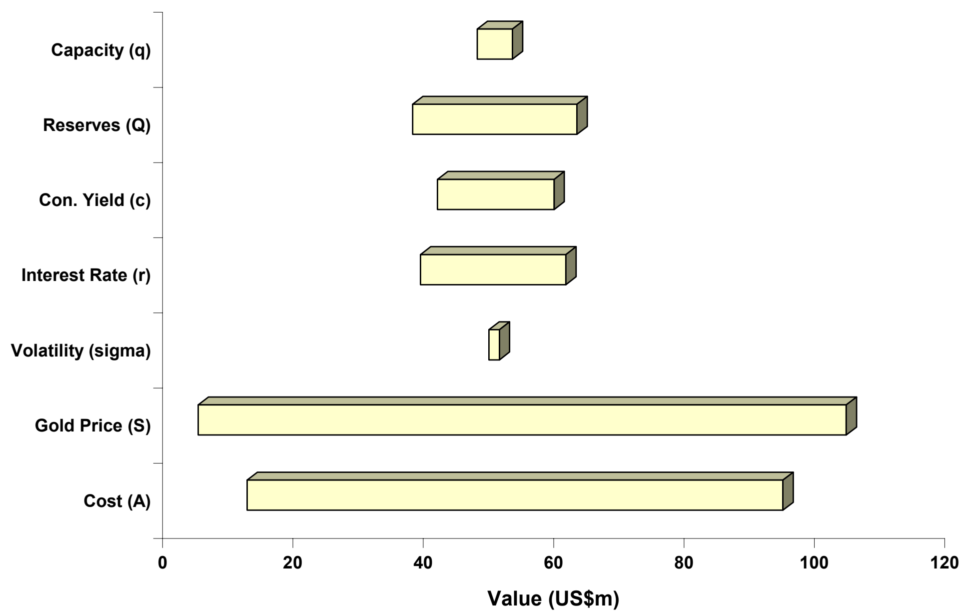

Exhibit 4: Directional Effect of an Increase in Each Input Parameter

Increase in variable	***Mine with flexibility***	***Mine without flexibility***	***Option Value***
Cost (A)	–	–	+
Gold Price (S)	+	+	–
Volatility (σ)	+	0	+
Interest Rate (r)	+	+	–
Convenience Yield (c)	–	–	+
Reserves (Q)	+	+	+
Capacity (q)	+ then –	+ then –	+ then –

Exhibit 5: Descriptive Statistics for the Distribution of the Closure Option Values at the Individual Mine Level

Panel A: Relative closure option value

	OV as percentage of WO	Number of Mines	Percentage of Mines
Positive WO		161	74.2%
	0%	5	2.3%
	0-1%	94	43.7%
	1-2%	11	5.1%
	2-3%	16	7.4%
	3-4%	3	1.40%
	4-5%	1	0.5%
	5-10%	11	5.1%
	> 10%	20	9.3%
Zero WO		2	0.9%
Negative WO		54	24.9%
Total		217	

Panel B: Absolute mine and closure option value

	WO	OV	OV/WO
Maximum	$1,222,057,884	$53,706,724	
Mean	$ 68,848,021	$ 3,017,243	4.38%
Median	$ 17,029,203	$ 405,697	2.38%
Median WO Mine	$ 17,029,203	$ 498,746	2.93%
Minimum	$ - 53,706,724	$ 0	
Standard Deviation	$ 173,740,748	$ 7,339,433	

Exhibit 6: Descriptive Statistics of Closure Option Value at Company Level

Closure option value at company level. The sample size consists of 112 company-years without flexibility (WO), with flexibility (WITH) and the closure option value (OV). For the percentage value figures 3 of the company years are excluded because the mine value was close to zero. 103 company-years make up the sample for the closure option value relative to the adjusted market capitalization (AMC).

Panel A: Closure option value

	WITH (112 comp.-years)	WO (112 comp.-years)	AMC (103 comp.-years)	OV
Maximum	$ 872,285,366	$ 870,975,165	$ 2,081,362,231	$ 51,505,202
Mean	$ 152,854,827	$ 146,456,020	$ 420,670,916	$ 6,398,808
Median	$ 65,295,754	$ 61,112,261	$ 220,495,420	$ 1,191,864
Minimum	$ -	$ -41,560,332	$ 5,714,820	-
Standard Deviation	$ 211,909,094	$ 210,072,613	$ 450,910,289	$ 10,407,036

Panel B: Relative closure option value

Closure option value as percent of	WITH	WO	AMC
Mean	4.19%	4.37%	1.64%
Median	1.83%	1.95%	0.68%
Standard Deviation	99.55%	437.17%	6.82%

Exhibit 7: Summary Statistics of Market Capitalization Regression Variables

The number of observations for the regression is 103. The independent variable is Adjusted Market Capitalization (AMC) and the abbreviations for the dependent variables follow the previous definition, Mine Value ***w****ith****o****ut* Flexibility (*WO*), Closure Option Value (*OV*), Mine Value ***with*** Flexibility (*WITH*), and Exploration Expenses (*EXPL*).

	AMC	*WO*	*OV*	*WITH*	*EXPL*
Panel A: Descriptive statistics for regression variables ($millions)					
Mean	420.7	156.8	6.9	163.7	10.5
Std. Deviation	450.9	216.0	10.7	217.6	15.1
Minimum	5.7	–41.6	0	0	0
Maximum	2,081.4	871.0	51.5	872.3	103.1
Skewness	1.7	1.9	2.0	1.9	3.2
Panel B: Correlation coefficients of regression variables					
AMC	1.0000				
WO	0.8229	1.0000			
OV	0.2275	0.1296	1.0000		
WITH	0.8278	0.9988	0.1778	1.0000	
EXPL	0.4913	0.3541	0.1196	0.3573	1.0000

Exhibit 8: Regression Results – Adjusted Market Capitalization as Dependent Variable

(*t*-statistics in parentheses)

	WITH	WO	OV	EXPL	R^2
		Panel A			
Pooled	1.72				0.69
	(14.83)				
Between	1.90				0.74
	(8.53)				
Within	1.10				0.93
	(8.52)				
		Panel B			
Pooled			9.58		0.05
			(2.35)		
Between			22.69		0.15
			(2.11)		
Within			1.01		0.86
			(0.42)		
		Panel C			
Pooled		1.68	5.18		0.69
		(14.42)	(2.20)		
Between		1.82	7.66		0.75
		(7.66)	(1.23)		
Within		1.10	2.35		0.93
		(8.50)	(1.36)		
		Panel D			
Pooled				14.71	0.24
				(5.67)	
Between				15.95	0.28
				(3.10)	
Within				7.63	0.87
				(3.03)	
		Panel E			
Pooled	1.55			6.71	0.73
	(13.42)			(4.02)	
Between	1.71			6.20	0.78
	(7.39)			(1.95)	
Within	1.03			4.56	0.93
	(8.12)			(2.42)	
		Panel F			
Pooled		1.53	4.48	6.57	0.73
		(13.13)	(2.03)	(3.95)	
Between		1.69	4.44	5.79	0.78
		(6.98)	(0.71)	(1.71)	
Within		1.03	3.29	5.13	0.93
		(8.12)	(1.94)	(2.67)	