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Quasi Option Value and Irreversible Choices

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Abstract. In this paper we are concerned with optimal investment decisions when dealing with land allocation problems. We aim to emphasize the importance of flexible modelling in order to capture irreversibility. In particular, we stretch a discrete model, firstly developed in Coggins and Ramezani [3], in order to cover a more realistic and complex scenario. Both *environmental* and *economic* uncertainty are included in the model and treated using an integrated approach, in which decision analysis techniques and option pricing theory are jointly applied to evaluate development versus conservation opportunities. Moreover, we take explicit account of how uncertainty interacts with two types of irreversibility: sunk costs associated with investment in developing decisions, including environmental and social costs due to environmental degradation, as well as sunk costs associated to environmental regulation and conservation. Finally, we use the *Quasi Option Value*, *QOV*, to derive decision rules that account for different levels of flexibility of land allocation possibilities.

1 Introduction

In the presence of uncertain future events, the possibility to postpone development investment decisions can lead to strategies diverting from those derived using traditional discounted cash flows approaches. Indeed, the arising of new information may partially resolve uncertainties over time, thus making it profitable to wait and act in the light of it. This flexibility becomes even more valuable when we are in presence of irreversible choices.

In order to take into account the level of irreversibility we introduce the *Quasi Option Value* (*QOV*), that is the extra value that can be captured by performing a fully dynamic analysis of the decision problem.

The issue of irreversibility, uncertainty and environmental policy has been largely discussed in the last three decades. From the first definition of the *QOV* given by Arrow and Fisher [2], the key concept has been developed in several articles, among others, by Conrad [4], [5], Hanemann [11], Krutilla and Fisher [12].

The aim of the present paper is twofold. First, we stretch the discrete model developed in Coggins and Ramezani [3] to focus on a set of decision problems that are particularly relevant in the European context, where decision makers are more concerned with land recovery expenditures and conservation efficiency rather than with wilderness area preservation. Conservation projects imply irreversible investments and operative costs. Hence, we allow for two different sources of irreversibility: sunk costs associated with investment in developing decisions, including environmental and social costs due to the permanent commitment of resources, as well as sunk costs associated to environmental regulation and conservation investment plans. Indeed, information about ecological effects and their economic consequences may reveal land recovery and conservation expenditures as inefficient or redundant. For this reason, in our optimal land allocation model, we compare two investment opportunities, in alternative to the status quo possibility: development, on one side, and land recovery/conservation (Natural Park), on the other side, both entailing initial sunk costs.

Secondly, we deal with the issue of different types of uncertainty, through an integrated methodology which combines the decision analysis techniques to the option pricing theory and that was firstly developed in several papers by Smith and others [18], [17] and [19]. The irreversible choice has to be made in presence of two relevant types of uncertainty: economic and environmental. Indeed, uncertainty can either be related to future social and financial costs/benefits or to the evolution of the environmental scenario. Thus, it is useful to introduce both these two sources of randomness in the model. In the case of the conservation investment alternative, in addition to being uncertain about the demand for wilderness (recreation, ecological services, etc.), *market uncertainty*, the decision maker is uncertain about the efficiency of the Natural Park choice, *environmental uncertainty,* depending on a vector of environmental parameters that are not known until after the investment is undertaken. In fact, until investment in on site research and in land recovery is not undertaken, those parameters composing the ecological value index are partially or completely unknown and one can uniquely deal with subjective probabilities, calculated by experts, regarding possible outcomes of the Natural Park value index.

Market uncertainties are dealt using the option pricing approach [7], [20] and[6], that value real investment projects trying to replicate their return and risk characteristics through a portfolio of existing assets. If we imply no arbitrage opportunities, the deriving probabilities, risk neutral probabilities, can be applied to calculate the expected present value of the investment.

Environmental uncertainties, that are project specific and therefore cannot be entirely diversified, are dealt computing certainty equivalents using subjective probabilities and utilities, as suggested in [19], [17], [18].

We represent the problem as a two periods optimal stopping problem [7], where *stopping* corresponds to invest, while *continuation* corresponds to wait for information to resolve part of the uncertainty.

In section 2 we introduce the analytical framework and we start dealing just with market uncertainty. In section 3 we develop the model to include also environmental uncertainty. Section 4 is devoted to a sensitivity analysis of critical initial values and to a numerical analysis and in section 5 we give some conclusions.

2   Analytical Framework

In this paper we are concerned with the optimal investment decision concerning a land area that can be devoted to three alternative uses: leave the land as it is, *Status Quo* (*SQ*); develop the area investing in a profitable project, *Development* (*DV*); preserve the land converting it into a *Natural Park* (*NP*). The third option is particularly relevant when the decision maker is not dealing with a tract of wilderness land but, as it is often the case in European countries, with a partially degraded area where there is a need to invest in order to re-convert the land to a "natural" state.

Choosing the *Status Quo* option, the only source of profit is a constant amenity value accruing every year from recreational use. In addition, this option is characterized by complete flexibility, thus making it possible to reconvert the area to both other two alternatives in a second period. Moreover, it does not imply any commitment to sunk costs (e.g., research, re-conversion of habitat degradation, active protection). On the contrary, the other two uses involve an initial investment and annual maintaining costs.

The *Development* alternative would result in a stream of random revenues throughout the operating life of the project - we rule out the temporary shut down possibility. However, this choice implies the irreversible sacrifice of any amenity value as well as of any other potential value source related to the conserved site, thus the environmental option is killed.

Preserve the land through a *Natural Park* would entail a stream of random revenues accruing from the amenity value of the site as well as from uncertain revenues due to external effects and ecological services deriving from a completely preserved land. In this state, the developing option is not killed and switching in a second period is conceivable, although, the initial investment outflow may be different now. Indeed, in some cases, it will be sensible to suppose that the new investment amount will be considerably augmented due to compensation for lost social benefits, to increased construction required time, etc. On the contrary, whenever the *NP* investment has beneficial effects on the development project this will entail a decreased initial outflow.

Let r denote the constant risk-free interest rate and R be the compounding factor, $R = (1 + r)$.

In the SQ state, the present value of the land over n years is:

$$\overline{A} = \sum_{i=0}^{n} \frac{A}{(1+r)^i} = \sum_{i=0}^{n} \frac{A}{R^i} \tag{1}$$

Where A is a constant amenity value accruing from the SQ state at each year t. When computing the present value for an indefinite time horizon, i.e. $t \in [0, \infty)$, (1) becomes:

$$\overline{A} = \sum_{i=0}^{\infty} \frac{A}{R^i} = A \frac{R}{(R-1)} \tag{2}$$

In the *Development* state the initial investment for instantaneous construction of the project is $I_d > 0$. The present value of the stream of annual constant per unit operating cost, c, is given by

$$C = \sum_{i=0}^{n-1} \frac{c}{(1+r)^i} = c \frac{R^{n-1}}{R^{n-1}(R-1)} \tag{3}$$

where n coincides with the operating life of the project.

If we assume that the project will yield one unit of good per year, then the yearly revenue will be given by the output price of the produced good that is assumed to follow a stationary multiplicative random walk. Denoting the output price P_t, at time $t + 1$ it will either rise to $P_{t+1}^{+} = uP_t$ (where $u > 1$) or decrease to $P_{t+1}^{-} = dP_t$, (where $0 < d < 1$), with probability q and $(1-q)$, respectively, with $q \in (0, 1)$.

Therefore, the price follows a process that can be described by a binomial tree with parameters (P_0, u, d, q), see Figure 1.
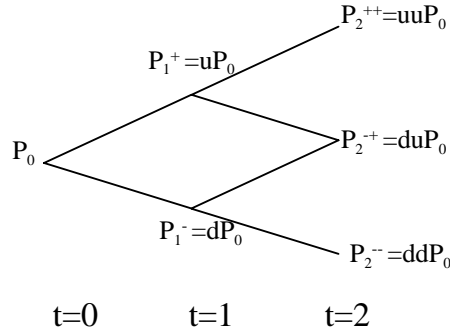

Figure 1: Binomial Tree

Thus, given P_0, the expected value of P_t at time $t = 0$, will be given by:

$$E[P_t] = (qu + (1-q)d)^t P_0 \tag{4}$$

where is E is the expectation operator.

We denote the present value of the expected stream of revenues accruing from the n operative years of the DV project, calculated in $t = 0$, as PV_0^D, and it is given by:

$$PV_0^D = \sum_{i=0}^{n-1} \frac{E[P_i]}{R^i} \tag{5}$$

Denoting

$$\mu = (qu + (1-q)d) \tag{6}$$

and assuming that $\mu < R$, we obtain:

$$PV_0^D(P_0, q) = P_0 \left[\frac{R^n - \mu^n}{R^{n-1}(R - \mu)} \right] \tag{7}$$

The third alternative, the *Natural Park*, is characterized by an initial investment, I_p, where $0 < I_p < I_d$, and a stream of annual constant per unit operating cost, g. The present value, computed for an indefinite time horizon is given by:

$$G = \sum_{i=0}^{\infty} \frac{g}{(1+r)^i} = g \frac{R}{(R-1)} \tag{8}$$

The revenue accruing from the Natural Park investment, $\tilde{A}_t$, with initial value $\tilde{A}_0 > A$, is assumed to follow a stationary multiplicative random walk, in order to include market uncertainty[1] [4]. Then, at time $t + 1$, the value of conservation associated to the Natural Park will either rise to $\tilde{A}_{t+1}^+ = s\tilde{A}_t$ (where $s > 1$) or decrease to $\tilde{A}_{t+1}^- = b\tilde{A}_t$ (where $0 < b < 1$), with probability p and $(1 - p)$, respectively, with $p \in (0, 1)$.

Similarly to previous discussion, the value $\tilde{A}_t$ follows a process that can be described as a binomial tree with parameters $(\tilde{A}_0, s, b, p)$.

Again, we consider the expected value of the stream of revenues accruing from the conservation project, calculated in $t = 0$, PV_0^N, given by:

$$PV_0^N = \sum_{i=0}^{\infty} \frac{E\left[\tilde{A}_i\right]}{R^i} \tag{9}$$

where, as in (4):

$$E\left[\tilde{A}_t\right] = (ps + (1-p)b)^t \tilde{A}_0 \tag{10}$$

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[1] We will treat environmental uncertainty in next section.

where E is the expectation operator.

Denoting

$$\nu = (ps + (1-p)b) \tag{11}$$

and assuming that $\nu < R$, we define the present value of the expected stream of revenues accruing during the life of the Natural Park, calculated in year 0 as:

$$PV_0^N(\tilde{A}_0, p) = \tilde{A}_0 \frac{R}{(R-v)} \tag{12}$$

Let us firstly analyze the problem applying the traditional discounted cash flow rule, without considering different levels of flexibility that each option can offer. In fact if we consider each decision as irrevocable we could represent the problem as in the decision tree of Figure 2.
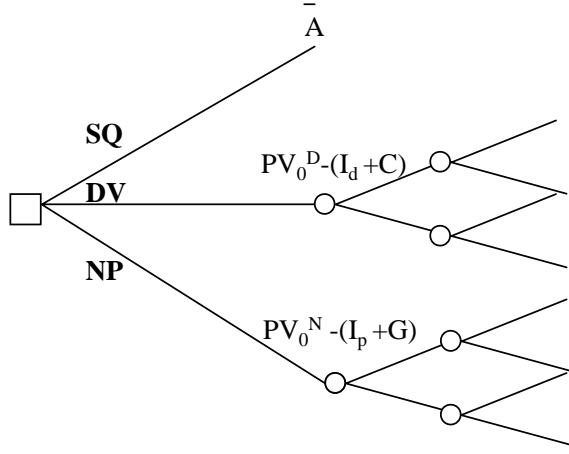

Figure 2: Inflexible Modelling Decision Tree

According to the traditional *Expected Net Present Value (ENPV)* decision rule, the value of the land area is given by the following statement:

$$W_{ENPV} = \max\left[\overline{A}, PV_0^D(P_0, q) - (I_d + C), PV_0^N(\tilde{A}_0, p) - (I_p + G)\right] \tag{13}$$

Now, let us capture potential flexibility sources. We include in the model the possibility to postpone some of the decisions to the second period. The decision rule should now take into account possible future information and embody it in the calculations. Once developed, the land will be irrevocably compromised, whereas the other two alternatives involve the possibility to switch to substitute
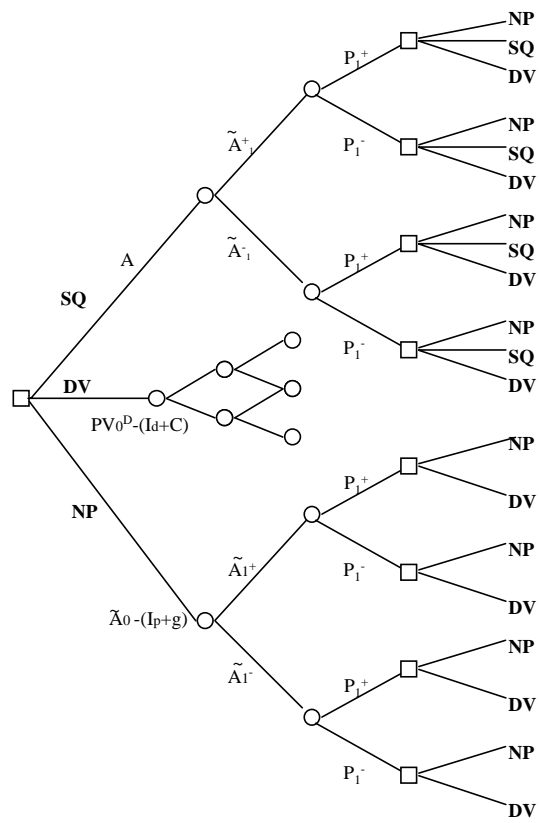
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Figure 3: Flexible Modelling Decision Tree

and more profitable uses[2]. The flexible approach is depicted using the decision tree in Figure (3).

The SQ alternative is a completely flexible choice, i.e., the decision maker is free to postpone the possibility of both investments, to the second period. Hence, during the first period the only revenue will be the SQ amenity value, A. In the second period, the availability of new information concerning the price P_1 and the value of the Natural Park, $\tilde{A}_1$, will leave the decision maker the possibility to choose her strategy according to the maximization criterion, whichever state of nature has occurred. If both investment decisions are delayed, then the value of land is given by:

$$W_0^S = A + \frac{pq\,W_1^{++} + p(1-q)\,W_1^{-\,+} + (1-p)q\,W_1^{+\,-} + (1-p)(1-q)\,W_1^{-\,-}}{1+r} \tag{14}$$

Where the second term of the right hand side of equation (14) is the expectation value, with respect to the probabilities p and q of the quantities defined below, (the state of nature of P_1 and $\tilde{A}_1$ are assumed to be independent) :

$$W_1^{++} = \max\left[\overline{A},\ PV_1^D(P_1^+, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^+, p) - (I_p + G)\right] \tag{15}$$

$$W_1^{-\,+} = \max\left[\overline{A},\ PV_1^D(P_1^-, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^+, p) - (I_p + G)\right]$$

$$W_1^{+\,-} = \max\left[\overline{A},\ PV_1^D(P_1^+, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^-, p) - (I_p + G)\right]$$

$$W_1^{-\,-} = \max\left[\overline{A},\ PV_1^D(P_1^-, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^-, p) - (I_p + G)\right]$$

The NP option leaves the decision maker the opportunity to switch during the second period to the DV alternative, as long as this is more profitable. It can either be that the investment required is now increased, I_d^+, or that the development project can benefit from the Natural Park investment, thus implying a lower initial investment, I_d^- (for example, this might be the case when the developing projects is related to the tourism industry).

The value of land, when the first period choice is NP, is given by:

$$W_0^N = (\tilde{A}_0 - I_p - g) + \frac{pq\,V_1^{++} + p(1-q)\,V_1^{-\,+} + (1-p)q\,V_1^{+\,-} + (1-p)(1-q)\,V_1^{-\,-}}{1+r} \tag{16}$$

where:

$$V_1^{++} = \max\left[\ PV_1^D(P_1^+, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^+, p) - G\right] \tag{17}$$

$$V_1^{-\,+} = \max\left[\ PV_1^D(P_1^-, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^+, p) - G\right]$$

$$V_1^{+\,-} = \max\left[\ PV_1^D(P_1^+, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^-, p) - G\right]$$

$$V_1^{-\,-} = \max\left[PV_1^D(P_1^-, q) - (I_d + C),\ PV_1^N(\tilde{A}_1^-, p) - G\right]$$

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[2] Flexible policies are known as closed loop policies.

Consequently, the decision whether develop in the first period, delay any investment or commit to the semi-flexible option is based on the following comparison:

$$W_0^* = \max \left[W_{ENPV}, \quad W_0^N, \quad W_0^S \right] \tag{18}$$

When $W_{ENPV}(q, p)$ is greater than the other two quantities, development should take place immediately. When $W_0^* = W_0^S$, both decision should be delayed to the second period and no sunk costs should be undertaken. Finally, when $W_0^* = W_0^N$, the suggested policy is to commit to environmental sunk costs (NP), whilst letting the opportunity of future adjustments of the decision.

We can consider the problem as a two periods optimal stopping problem [7] and[15], where *stopping* corresponds to invest either in the irreversible or in the NP project, while *continuation* corresponds to do not commit to any sunk cost. The (18) is the Bellman equation[3] for this problem, where the termination payoff is either given by W_{ENPV} or by W_0^N.

For some initial critical value of P_0 it will be optimal to stop and kill the option, i.e. for $P_0 \geq P_0^*$ it will always be optimal to invest immediately, while for $P_0 < P_0^*$ continuation will be optimal. There exists a similar critical value $\tilde{A}_0^*$ for $\tilde{A}_0$. These two cut-off values divide the optimal strategy space. The SQ choice is optimal provided that neither P_0 or $\tilde{A}_0$ reach the barriers P_0^* and $\tilde{A}_0^*$, respectively.

The QOV, corresponds, in the present work, to the difference, when positive, among the traditionally measured value of the land and the value calculated using the dynamic model:

$$QOV = \max[0, \quad W_0^* - W_{ENPV}] \tag{19}$$

The QOV can capture the value of the right to delay any decision, as well as to delay just the totally irreversible choice, whilst committing to environmental regulation sunk cost only.

3   Valuation Methodology

In the preceding section we have not made any specific assumption either on the probability distributions or on the discount rate to be used in the valuation. It is not sensible, though, to discount risky cash flows using the risk-less discount rate, r, without making any specification.

Whilst, in the DV case we can assume that there exists a complete market for the output good and, therefore, the associated risk can be perfectly hedged by treading securities, this assumption cannot be made in the NP case, where we face both *market uncertainty* (i.e., the consumers' demand for recreation and ecological services) and *environmental uncertainty*, concerning the ecological valuation index of the NP. Indeed, the latter are project specific and cannot

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[3] see, for example, A.K. Dixit and R.S. Pindyck, "Investment Under Uncertainty" Princeton University Press, Princeton, NY, (1994)

be hedged by trading securities. The NP investment entails measurements of a series of attributes of the site, as biodiversity, rareness, size, naturalness, representation, that will have to be combined into an overall evaluation index of the site. Following the criteria-based evaluation method [10], the model will allow for three possible state of nature[4]:

1) if either the size is insufficient or the unnaturalness of the site is classified as permanent, due to previous damage, the NP is not instituted and the only value accruing thereon is coming from the SQ recreational value;

2) if the site attributes are combined to give a middle-low class value, the investment is inefficient and this negatively affects the NP value, $\tilde{A}_t = b\tilde{A}_t$;

3) if the site attributes are combined to give an high class value, the investment is efficient and this positively affects the NP value, $\tilde{A}_t = s\tilde{A}_t$.

In Figure 4, we make explicit the distinction among market uncertainties (represented by white nodes) and environmental uncertainties (represented by black nodes). As it is shown in the picture, if the NP choice is not undertaken at time $t = 0$, the only uncertainty to be considered, before the investment is undertaken at time $t = 1$, is the one related to the market demand. At this point, if the NP decision is taken, we face three possible states of nature related to the site characteristics and to research results. Hereafter, if the NP is effectively instituted, the yearly revenue varies uniquely according to market uncertainty.

As we have already seen in the previous section, investment can take place at $t = 0, 1$, while uncertainties are resolved at times $t = 0, 1, .., n, .., \infty$. We are assuming that investment options are available just in the first two periods, however the model could be easily modified to introduce greater flexibility. The decision maker's state of information at each time t is denoted by $\Im_t$, formally modeled as elements of a filtration on a suitably defined probability space. Moreover, market assumptions can be formalized as a market filtration, $\Im_t^m$, ($\Im_t^m \subseteq \Im_t$) that represents the market risks resolved by time t.

As most of the real options literature assumes, the decision maker can either decide to invest in the project (all-or-nothing decision) or to invest in shares of securities at market prices, as well as lending and borrowing money at the risk-free interest rate r.

Four assumptions on the securities market are critical. Firstly, there are not arbitrage opportunities. Secondly, market is partially complete, i.e., market uncertainties can be hedged by trading securities. Thirdly, market is efficient and does not depend on private information and finally there are not transaction costs.

Under these assumptions, we can always construct a portfolio whose payoffs exactly replicate the payoffs of the project [9]. Thus, we can determine the unique "risk-neutral" distribution such that the current market price of a vector of securities, $s(0)$, that generates a random dividend stream, $c(t)$, is given by:

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[4]This is clearly an oversimplification of the complexity of possible scenarios accruing from the combination of the cited attributes, and we consider this as an important expansion to be made in future development of the model.
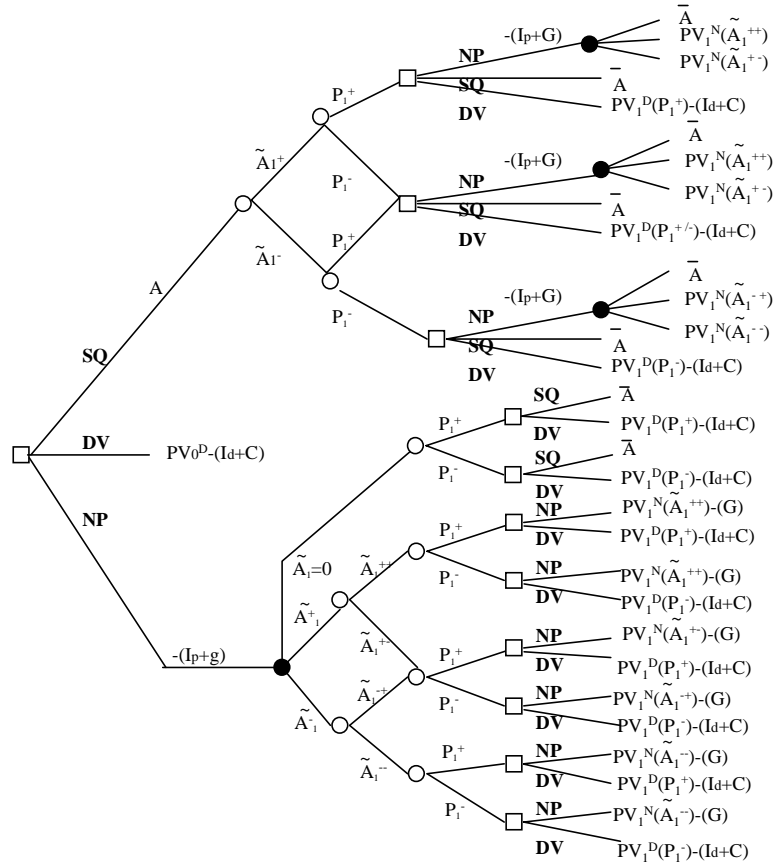
Figure 4: Decision Tree with Specified Uncertainty Types

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$$s(0) = \sum_{t=0}^{T} \frac{E_\pi \left[c(t) \right]}{(1+r)^t} \tag{20}$$

where E_π is the expectation with respect to the "risk neutral" probability, π, (with $\pi \in (0,1)$), and $c(t)$ equals the random stream of revenues accruing from the project. Therefore, whenever the riskiness of a project is diversifiable, it is possible to calculate the value of the project applying the "risk-neutral" probability distribution and using the risk-free interest rate.

If we apply the (20) to calculate "risk neutral" probability distribution for the DV and the NP alternatives, we obtain:

$$\widehat{q} = \frac{(1-d)}{(u-d)}$$

and

$$\widehat{p} = \frac{(1-b)}{(s-b)}$$

For a detailed demonstration we refer the reader to Coggins and Ramezani [3].

It is easy to check that, for such values of the probabilities q and p, the value of (6) and of (11) is equal to one.

Therefore, the expected value of the DV alternative, given in (7), is now:

$$PV_0^D(P_0, \widehat{q}) = P_0 \left[\frac{R^n - 1}{R^{n-1}(R-1)} \right]$$

This result entails that, once invested in the project, the decision maker has the opportunity to contract with a buyer the selling price of the entire stream of production, fixing it at the investment-time price.

However, as we previously noticed, environmental uncertainties cannot be hedged by tradeable securities. In order to include in the model this source of risk, a traditional approach is the capital-asset-pricing model that produces a risk-adjusted discount factor, according to the risk category of the project. However, this is not the first best procedure when the riskiness of a project is not comparable with other's, known, projects and when dealing with long time horizon projects.

An alternative approach has been developed in literature firstly by Smith and others [18], [17] and [19], to deal with project specific uncertainties and it can be applied to deal with environmental uncertainties. We say market is partially complete if security prices depend only on market states, if the market is complete with respect to market uncertainty and if private events do not influence market events. The procedure involves the use of subjective probabilities, suggested by experts, regarding the possible future states combined with the decision maker's preferences for consumption in different periods opportunely defined. This allows us to compute the value of a project as the decision maker's

breakeven buying price. Indeed, the value is calculated as the lump-sum that makes the maximum expected utility accruing from the decision of undertaking the project and choose a trading strategy equal to the maximum expected utility accruing from just investing in a trading strategy.

In this particular case we assume market is not complete with respect to environmental uncertainty that is represented by the three possible scenario following the *NP* investment.

The utility function describing the decision maker intertemporal preferences should be continuous, strictly increasing and strictly concave. Moreover, for consistency and separation theorems to hold, the utility function should satisfy the property of additivity independence[5] and the Δ-property[6]. Without loss of generality we can assume, for the sake of concreteness, that the utility function takes the form:

$$U(x_t) = -exp(-x_t/K_t) \tag{21}$$

where K_t is the decision maker's effective risk tolerance defined as:

$$K_t = \sum_{i=t}^{\infty} \frac{\rho_i}{(1+r)^{i-t}} \tag{22}$$

Hence, K_t is the sum of the discounted future decision maker's period$-i's$ risk tolerance, ρ_i. Assuming that ρ_i grows with a rate r^* ($r^* \neq r$), we obtain, for period $t = 0, 1$, the (22) becomes:

$$K_0 = \sum_{i=0}^{\infty} \rho_0 \frac{(1+r^*)^i}{(1+r)^i} = \rho_0 \frac{(1+r^*)}{(r-r^*)} \tag{23}$$

$$K_1 = \rho_1 \frac{(1+r^*)}{(r-r^*)} \tag{24}$$

Once the tree of prices has been generated, projects are evaluated by starting at the end of the tree, applying a standard dynamic programming "rollback" procedure that can be synthesized in the following steps (see Figure 5 for an example):

- for each node representing market uncertainty, calculate the present value using risk-neutral probabilities ($E_{\widehat{p},\widehat{q}}$) and the risk-free interest rate;

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[5] The decision maker's preferences for risky cash flows depend only on the marginal distribution for each period, not on the joint distribution.

[6] If the decision maker is indifferent between a gamble and its certainty equivalent, then she will be indifferent between the same gamble plus a quantity Δ and the certainty equivalent of the gamble plus the same quantity Δ.
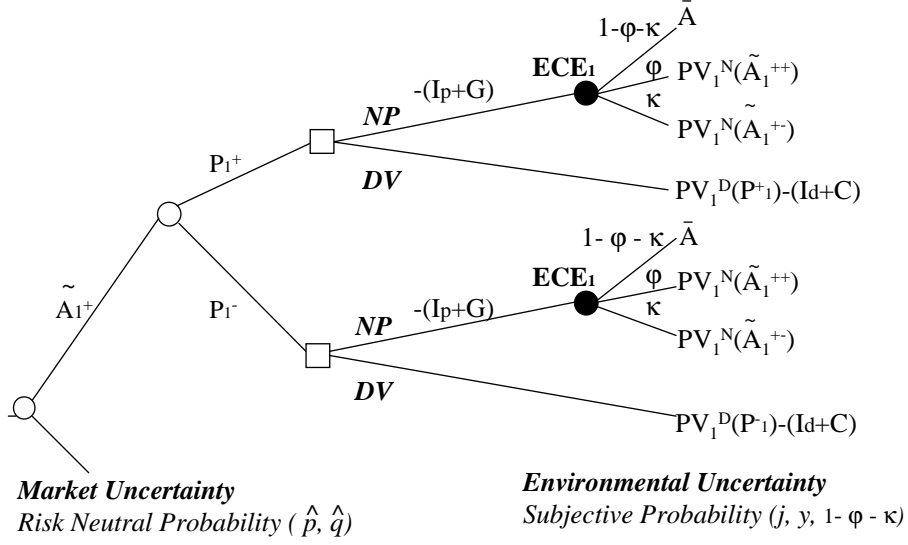
Figure 5: Fragment of the Decision Tree

- for each node representing environmental uncertainty, calculate the effective certainty equivalent at time t, ECE_t, using the subjective probabilities $(\phi, \kappa, 1 - \phi - \kappa$ s.t. $\phi + \kappa \leq 1))$, then subtract the outflows;

- at each of the decision nodes, choose the best strategy according to the maximization criterium.

If assumptions on market partial completeness and on preferences hold, the integrated procedure gives the breakeven buying price of the project using the certainty equivalent replicating portfolio.

In particular, to calculate the ECE_t, we take expectations over period-$t's$ environmental uncertainties, with respect to the "subjective probability", ϕ, κ and $(1 - \phi - \kappa)$, and we apply the exponential utility function described in (21).

$$ECE_{t+1}\left[\widehat{PV}^N_{t+1}(\widetilde{A}^{\pm}_{t+1}, \widehat{p}), \overline{A} \mid \Im_t\right] = -K_{t+1}\ln\left(E_{\phi,\kappa}\left[\exp\left(-\frac{\widehat{PV}^N_{t+1}(\widetilde{A}^{\pm}_{t+1}, \widehat{p}), \overline{A}}{K_{t+1}}\right) \mid \Im_t\right]\right)$$

(25)

Where K_t is the decision maker's effective risk tolerance, and $\widehat{PV}^N_{t+1}(\widetilde{A}^+_{t+1}, \widehat{p})$, $\widehat{PV}^N_{t+1}(\widetilde{A}^-_{t+1}, \widehat{p})$, $\overline{A}$ are the time $t + 1$ possible value of the NP project.

Now, according to the integrated approach, we can calculate the value of different land uses. Firstly, let us consider the value of the land according to

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the traditional ENPV rule, $\widehat{W}_{ENPV}$:

$$\widehat{W}_{ENPV} = \max\left[\overline{A}, \widehat{PV}_0^D(P_0,\widehat{q}) - (I_d+C), ECE_0\left[\widehat{PV}_0^N(\widetilde{A}_0^\pm,\widehat{p}),\overline{A}\mid\Im_0\right] - (I_p+G)\right]$$

(26)

Where $PV_0^D(P_0,\widehat{q})$ is calculated using "risk-neutral" probabilities, while, the effective certainty equivalent, ECE_0, is calculated using subjective probabilities and the decision maker's utility function, see Figure 6.
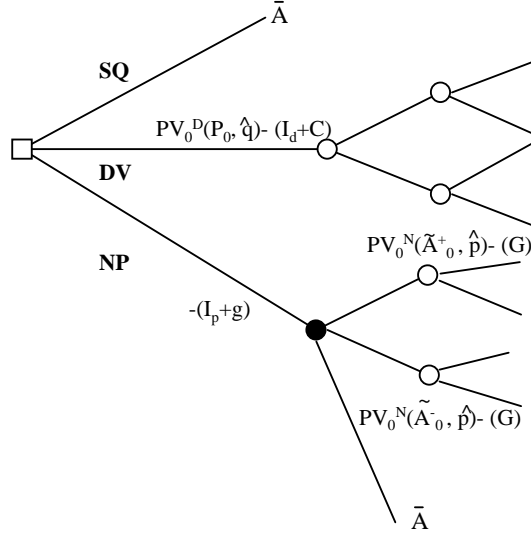

Figure 6: The Unflexible Tree, with Uncertainty Specification

Secondly, we consider the possibility to wait and postpone both the investment options. Thus, the value of land in the SQ state, $\widehat{W}_0^S$, becomes:

$$\widehat{W}_0^S = A + \frac{\widehat{\widetilde{pq}}\widehat{W}_1^{++} + \widehat{p}(1-\widehat{q})\widehat{W}_1^{-\,+} + (1-\widehat{p})\widehat{q}\widehat{W}_1^{+\,-} + (1-\widehat{p})(1-\widehat{q})\widehat{W}_1^{-\,-}}{1+r}$$

(27)

Where:

$$\widehat{W}_1^{++} \;=\; \max\left[\overline{A}, \widehat{PV}_1^D(P_1^+,\widehat{q}) - (I_d^\pm+C), ECE_1\left[\widehat{PV}_1^N(\widetilde{A}_1^{+\pm},\widehat{p}),\overline{A}\mid\Im_0\right] - (I_p+G)\right]$$

$$\widehat{W}_1^{-\,+} \;=\; \max\left[\overline{A}, \widehat{PV}_1^D(P_1^-,\widehat{q}) - (I_d^\pm+C), ECE_1\left[\widehat{PV}_1^N(\widetilde{A}_1^{+\pm},\widehat{p}),\overline{A}\mid\Im_0\right] - (I_p+G)\right]$$

$$\widehat{W}_1^{+\,-} \;=\; \max\left[\overline{A}, \widehat{PV}_1^D(P_1^+,\widehat{q}) - (I_d^\pm+C), ECE_1\left[\widehat{PV}_1^N(\widetilde{A}_1^{-\pm},\widehat{p}),\overline{A}\mid\Im_0\right] - (I_p+G)\right]$$

$$\widehat{W}_1^{-\,-} \;=\; \max\left[\overline{A}, \widehat{PV}_1^D(P_1^-,\widehat{q}) - (I_d^\pm+C), ECE_1\left[\widehat{PV}_1^N(\widetilde{A}_1^{-\pm},\widehat{p}),\overline{A}\mid\Im_0\right] - (I_p+G)\right]$$

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Where we denote $\widetilde{A}_1^{+\pm}$ the second period amenity value when the market state is high, $\widetilde{A}_1^+$, and the investment may reveal both efficient, $\widetilde{A}_1^{++} = s\widetilde{A}_1^+ = ss\widetilde{A}_0$, or inefficient, $\widetilde{A}_1^{+-} = b\widetilde{A}_1^+ = bs\widetilde{A}_0$. Analogously we can define $\widetilde{A}_1^{-\pm}$. The SQ flexible model is depicted in the decision tree of Figure 7.

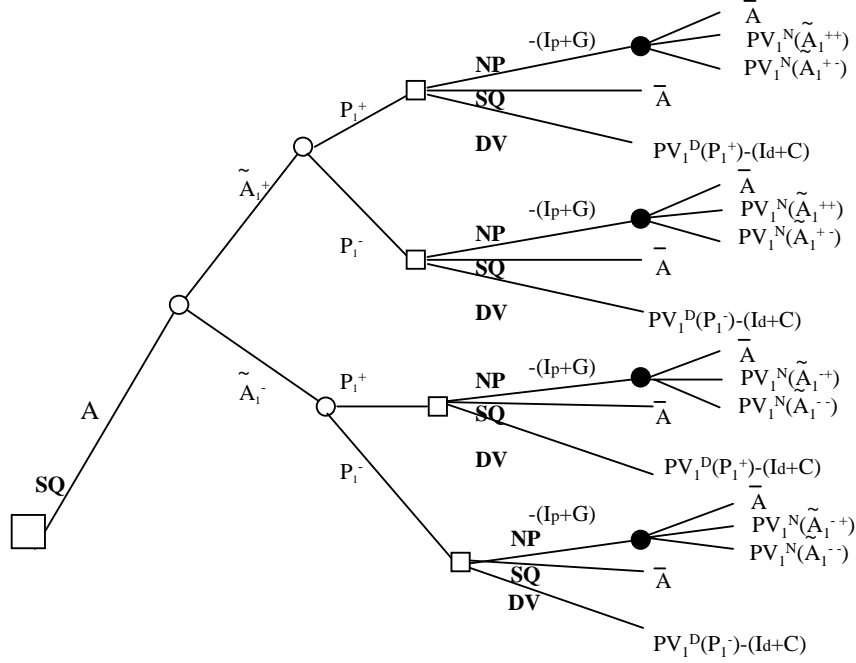
Figure 7: The First Period SQ-Option Value of Land

Finally, we can calculate the value of committing to environmental sunk costs only, $\widehat{W}_0^N$, thus investing I_p and first period operating costs, g, as:

$$
\begin{aligned}
\widehat{W}_0^N &= -(I_p+g) + ECE_0\left[\overline{A}, \widetilde{A}_0^\pm + E_{\widehat{q},\widehat{p}}\max\left[\widehat{PV}_1^D(P_1,\widehat{q}) - (I_d+C), \widehat{PV}_1^N(\widetilde{A}_1,\widehat{p}) - G\right] \mid \Im_0\right] \\
&= -(I_p+g) + ECE_0\left[\overline{A}, \widetilde{A}_0^\pm + \frac{\widehat{pq}V_1^{++} + \widehat{p}(1-\widehat{q})V_1^{-+} + (1-\widehat{p})\widehat{q}V_1^{+-} + (1-\widehat{p})(1-\widehat{q})V_1^{--}}{1+r} \mid \Im_0\right] \quad (28)
\end{aligned}
$$

Where we define:

$$
\widehat{V}_1^{++} = \max\left[\widehat{PV}_1^D(P_1^+,\widehat{q}) - (I_d^\pm+C), \widehat{PV}_1^N(\widetilde{A}_1^{\pm+},\widehat{p}) - G\right]
$$

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$$\widehat{V}_1^{-\,+} = \max\left[\widehat{PV}_1^{D}(P_1^-,\widehat{q}) - (I_d^\pm + C), \widehat{PV}_1^{N}(\widetilde{A}_1^{\pm+},\widehat{p}) - G\right]$$

$$\widehat{V}_1^{+\,-} = \max\left[\widehat{PV}_1^{D}(P_1^+,\widehat{q}) - (I_d^\pm + C), \widehat{PV}_1^{N}(\widetilde{A}_1^{\pm-},\widehat{p}) - G\right]$$

$$\widehat{V}_1^{-\,-} = \max\left[\widehat{PV}_1^{D}(P_1^-,\widehat{q}) - (I_d^\pm + C), \widehat{PV}_1^{N}(\widetilde{A}_1^{\pm-},\widehat{p}) - G\right]$$

With $\widetilde{A}_1^{\pm+}$ denoting a high state of the market conditioned on either a high value of the NP efficiency index, $\widetilde{A}_0^+$, or a low value, $\widetilde{A}_0^-$, while $\overline{A}$ is the SQ payoff accruing if the NP is not instituted.

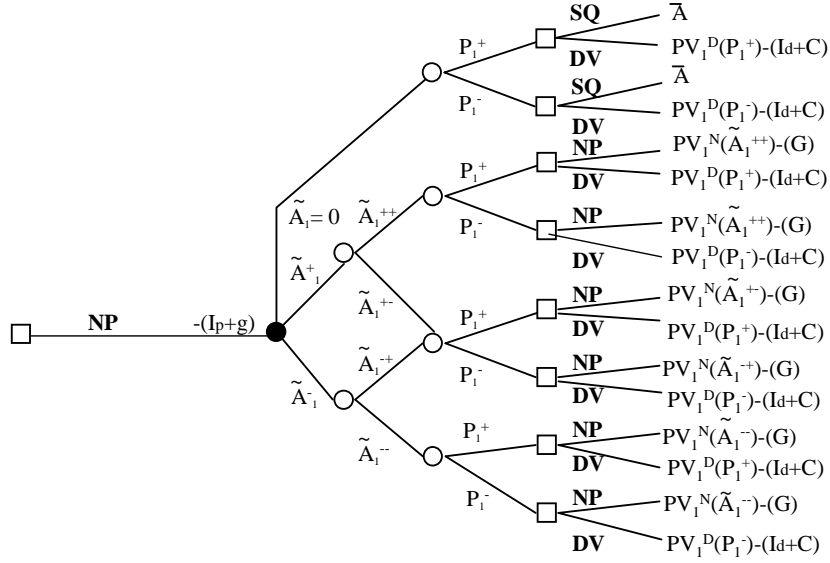The decision tree representation is shown in Figure 8.

Figure 8: The First Period NP-Option Value of Land

Similarly to the previous section, we can calculate the value of the land, at $t = 0$, as:

$$\widehat{W}_0^* = \max\left[\widehat{W}_{ENPV}, \quad \widehat{W}_0^S, \quad \widehat{W}_0^N\right] \tag{29}$$

The decision rule is as the one described in previous section, and, similarly, the QOV, when positive, corresponds to the mistake one would make following the traditional $ENPV$ approach instead of the flexible one. The QOV can capture the opportunity of avoiding either the irreversible development choice or the flexible commitment to the NP sunk costs. Its value is give by:

$$QOV = \max[0, \quad \widehat{W}_0^* - \widehat{W}_{ENPV}] \tag{30}$$

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4   Numeric Example and Sensitivity Analysis

We perform a numerical analysis in order to make clearer what has been discussed in sections 2 and 3. Let us suppose that the parameters in the model are defined as in the table of figure 9.

Parameter	R	n	u	d	q^	c	Id	s	b	p^	g	Ip	A	j	i
Value	1,10	10	1,3	0,8	0,4	50	1000	1,3	0,8	0,4	35	800	75	0,4	0,5

Figure 9: Table of Parameters Values

In example 1 we consider how strategies change, varying the initial value of the DV investment, P_0, while keeping the value of the NP choice, $\widetilde{A}_0$, constant, firstly for a low value of $\widetilde{A}_0$, then for a higher value. Similarly, in example 2, we let $\widetilde{A}_0$ change while we keep P_0 constant again first for a low, then for an higher value.

Example 1 *Letting the other parameters fixed, we consider the trend of $\widehat{W}_{ENPV}$, $\widehat{W}_0^S$, $\widehat{W}_0^N$, their maximum $\widehat{W}_0^*$, and the QOV, as a function of P_0. In figure 10, we firstly perform the analysis setting $\widetilde{A}_0 = 100$. As we would expect, this setting produce results that are very similar to the ones in Coggins and Ramezani [3], the effect of the NP alternative being almost irrelevant. Until the initial DV price reaches the value P_0', the traditional rule and the flexible approach rule coincide, and the suggested strategy is do not commit to any of the two investments. The first kink in the $\widehat{W}_0^S$ curve, in P_0', indicates that we are capturing the second period development opportunity for high market state, P_1^+. In P_0'' the $\widehat{W}_{ENPV}$ curve starts to increase, indicating that a decision maker who follows the traditional rule would now decide to develop immediately, while, according to the flexible approach, waiting and postpone any decision is still the optimal strategy. Above P_0''', that indicates the second kink in the $\widehat{W}_0^S$ curve, the decision to develop in the second period may now be optimal even for a low state of the market, conditioned on the market state for the NP value. The cut-off point, P_0^*, for $\widetilde{A}_0 = 100$, represents the threshold value dividing the continuation region from the stopping region, that is immediate development. The QOV represents the maximum amount a rationale decision maker would accept to pay for the right to delay any decision and it becomes zero above the cut-off value.*

Secondly, we set $\widetilde{A}_0 = 200$, results are shown in Figure 11. As a consequence of the higher $\widetilde{A}_0$ value, the NP opportunity exerts, now, a determinant effect on the threshold level separating decision regions. P_0', P_0'', P_0''' are defined as in the previous case. Up to the cut-off point P_0^*, the $\widehat{W}_0^S$ curve lies above the $\widehat{W}_{ENPV}$ and $\widehat{W}_0^N$ curves. However, the effect of the NP investment, when a favorable state of nature occurs, positively affects the SQ choice, thus shifting P_0^* to higher values. Therefore, the stopping decision, that corresponds to immediate investment, is now optimal for higher DV initial values compared to the previous
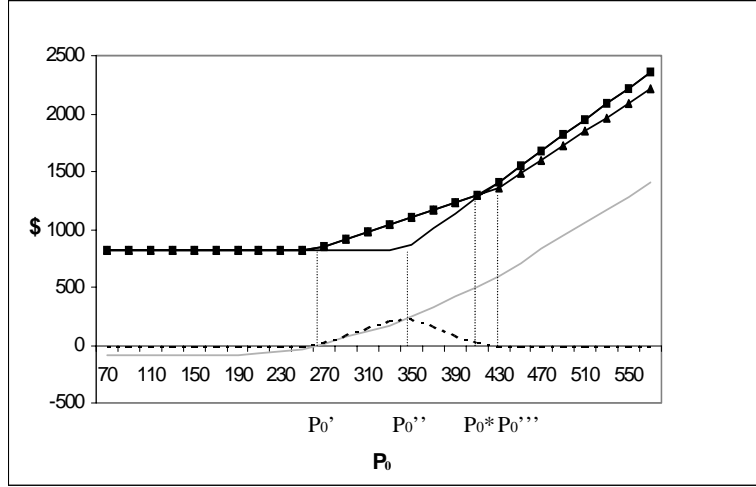
Figure 10: Varying P_0 for $\widetilde{A}_0 = 100$. Where the dashed line represents the QOV, the lighter line is $\widehat{W}_0^N$, the thicker line is $\widehat{W}_{ENPV}$, the line with triangles represents $\widehat{W}_0^S$ and the line with squares is their maximum, $\widehat{W}_0^*$.

scenario. From $P_0^{''}$ to P_0^* the QOV captures the value of the right to postpone DV *decision sunk costs, as in the previous case. While, up to point* $P_0^{''}$ *the QOV captures the flexibility value of the* SQ *choice.*

Example 2 *Similarly, letting all the parameters fixed, we can represent the trend of* $\widehat{W}_{ENPV}$, $\widehat{W}_0^S$, $\widehat{W}_0^N$, *their maximum,* $\widehat{W}_0^*$, *and the QOV, as a function of* $\widetilde{A}_0$. *We firstly assume* $P_0 = 450$. *As it is depicted in Figure 12, for very low values of* $\widetilde{A}_0$ *the best strategy is to develop immediately, hence, the traditional rule is not misleading. In* $\widetilde{A}_0^{'}$, *the* $\widehat{W}_0^S$ *curve starts to lie above the* $\widehat{W}_{ENPV}$ *one, thus, generating a positive QOV, that represents the mistake one would incur into if he developed in the first period. The divergence is due to the second period NP investment possibility, that is optimal for favorable market states of nature. From the value* $\widetilde{A}_0^{''}$ *on, the* ENPV *rule would suggest to commit to the* NP *expenditure, while it is just for* $\widetilde{A}_0^*$, *that the* NP *investment becomes the optimal strategy, according to our approach. Therefore, up to the cut-off point,* $\widetilde{A}_0^*$, *the* QOV *captures the value of not committing to environmental sunk costs and just wait to the second period to take any decision.*

Finally, we investigate the case where $P_0 = 600$, *see figure 13. The increased* DV *price strongly affects the optimal strategy regions. Both* $\widetilde{A}_0^{'}$, $\widetilde{A}_0^{''}$ *and the cut-off value* $\widetilde{A}_0^*$ *are now shifted to the right. Above* $\widetilde{A}_0^*$, *up to* $\widetilde{A}_0^{'''}$, *the* QOV *is still positive. The reason for this is that the flexible approach captures the extra value of the* NP *choice, due to the option to invest in the* DV *alternative, in the second period.*
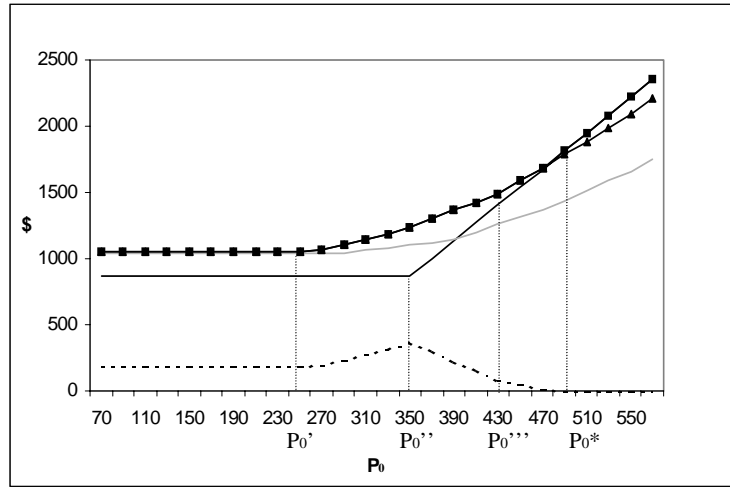
Figure 11: Varying P_0, with $\widetilde{A}_0 = 200$. Where the dashed line represents the QOV, the lighter line is $\widehat{W}_0^N$, the thicker line is $\widehat{W}_{ENPV}$, the line with triangles represents $\widehat{W}_0^S$ and the line with squares is their maximum, $\widehat{W}_0^*$.
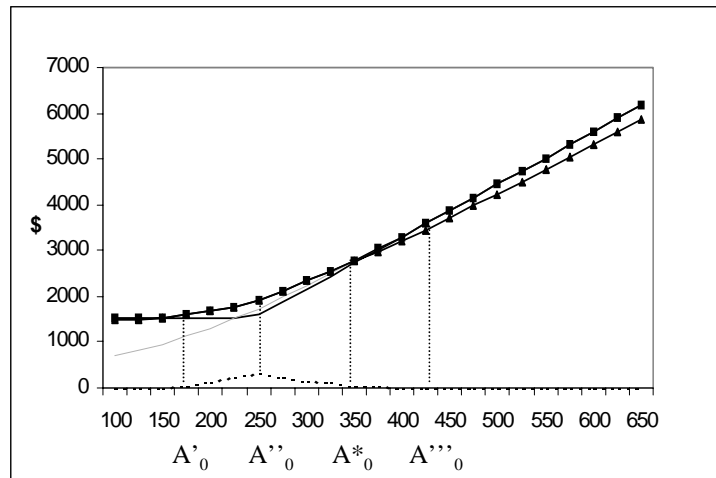

Figure 12: Varying $\widetilde{A}_0$, with $P_0 = 450$. Where the dashed line represents the QOV, the lighter line is $\widehat{W}_0^N$, the thicker line is $\widehat{W}_{ENPV}$, the line with triangles represents $\widehat{W}_0^S$ and the line with squares is their maximum, $\widehat{W}_0^*$.
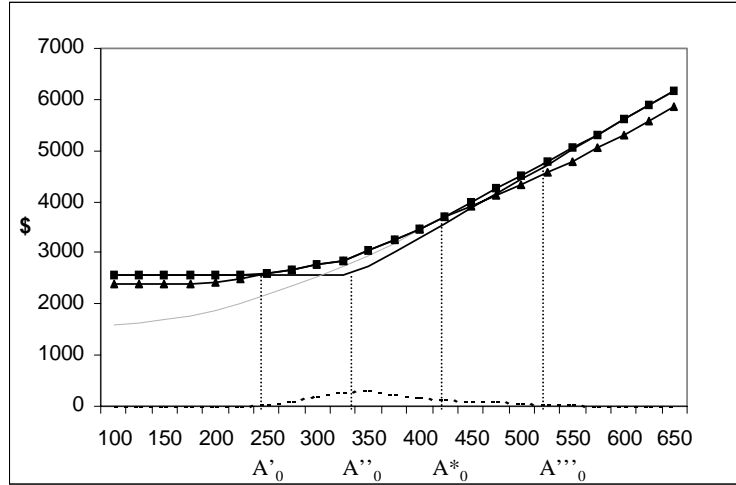
Figure 13: Varying $\widetilde{A}_0$, with $P_0 = 600$. Where the dashed line represents the QOV, the lighter line is $\widehat{W}_0^N$, the thicker line is $\widehat{W}_{ENPV}$, the line with triangles represents $\widehat{W}_0^S$ and the line with squares is their maximum, $\widehat{W}_0^*$.

Now, we analyse how the threshold initial values P_0^* and $\widetilde{A}_0^*$, separating the continuation from the stopping region, may vary with the model relevant parameters supposing all remaining parameters can be considered fixed. In particular, we consider how these trigger values change in respect to the following parameters: A, the first year value of the SQ alternative, I_d, the DV investment, I_p, the NP investment, n, the operative years of the project and R, the discount factor, $R = 1 + r$, where r is the risk free discount rate.

Proposition 3 P_0^* *increases with A and it increases with I_d, while it decreases with n, and R, the discount factor (see Figure 14). The latter relation is true for $R \in (1.01, 1.20)$, for higher values of R, of scarce economic relevance, P_0^* slightly starts to increase.*

Proposition 4 $\widetilde{A}_0^*$ *increases with A, the first year value of the* SQ *alternative, and I_p, the* NP *investment, and it also increases with R, the discount factor (see Figure 15). The latter relation has economic foundation in the idea that very long term projects are usually more sensible to an increased discount factor.*

5   Conclusions

In this paper we have tried to take into account the economic as well as the environmental aspect of irreversibility. While the first feature has been largely in-
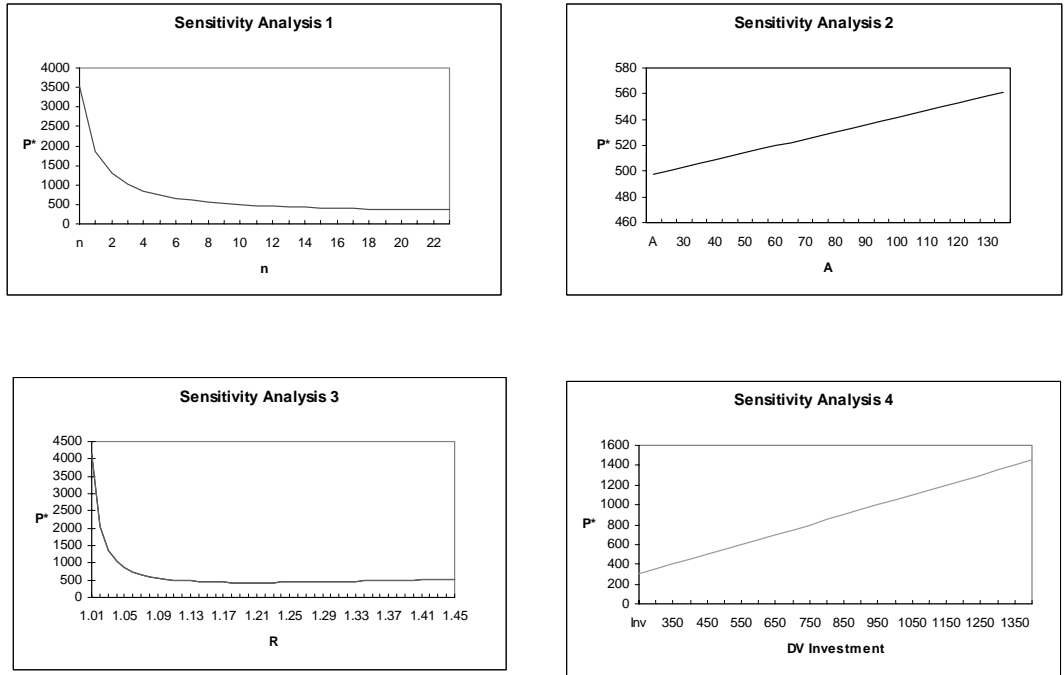
Figure 14: Sensitivity Analysis performed for P_0^*

vestigated in the quasi option value literature the second has been often ignored. However, particularly in European applications of land allocation problems, the importance of initial environmental expenditures should be considered, combining it with the flexibility potential of this option. Moreover, the uncertainty related to the environmental response to recovery and conservation interventions, should be taken into account in addition to uncertainty related to market prices. Therefore, the valuation procedure has been developed to include environmental influence on conservation projects. What we find is that it is possible to calculate, in a similar framework to the Arrow-Fisher Option Value (QOV) of preservation, an option value that may be referred either to the SQ or to the environmental investment project. Therefore, the flexible modelling, enabling us to take into account different sources or irreversibility and uncertainty, prove to be a useful tool in land allocation and environmental expenditure management.

In addition, the output of the model can be easily understood by non technical users and it can be helpful in promoting environmental and economical rationale choices.
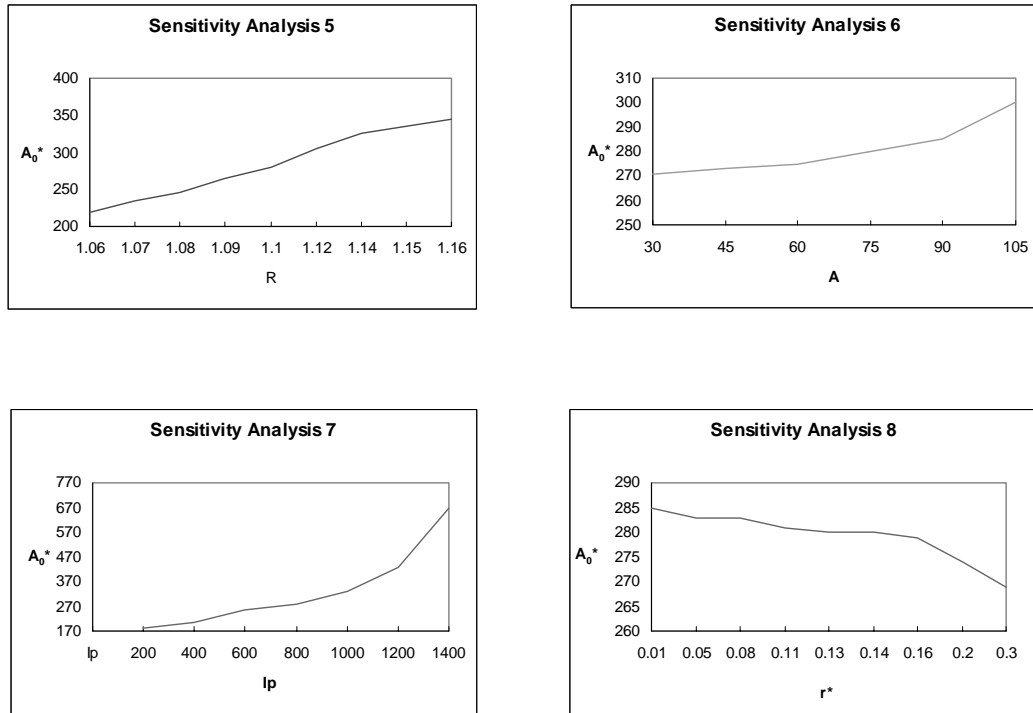
Figure 15: Sensitivity Analysis: $\tilde{A}_0^*$ varying with R

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