The Strategic Value of Flexible Quality Choice: a Real Options Analysis*

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Abstract

In this paper the advantages of flexibility of quality choice are studied in a real option framework. Before firms can decide about quality they first have to incur a sunk cost investment in order to enter the market. Flexibility of quality induces (ceteris paribus) earlier investment, and the value of being able to adjust quality over time increases with demand uncertainty. It is also found that competition raises the excess value due to flexibility and this excess value, in turn, increases with uncertainty. Furthermore, we extend the general theory of strategic real options. From this theory it is known that the follower's investment timing is irrelevant for the decision of the leader. However, due to the addition of a second control in the form of quality choice, the investment timing of the first investor is influenced by the decision of the other firm. Moreover, introducing the second control variable in combination with strategic interaction can result in the option value of the leader decreasing in uncertainty. Finally, we show that the follower can be driven out of the market due to "aggressive" quality choice of the leader in high states of demand.

Keywords: real options, dynamic programming, market entry, second-mover advantage.

JEL classification: C61, D81, G31

1 Introduction

In a period of rapid technological change and high market uncertainty, many results from option pricing theory become a useful tool in evaluating corporate investment opportunities. The option approach allows for determining the value of managerial flexibility concerning, for example, timing of investment and scale of operations. The implementation of option-based techniques requires taking into consideration the differences between real and financial options. The former type can be attributed two distinguishable features. First, in most cases real options are not exclusive, i.e. exercising a given option by one party results in the termination of corresponding options held by other parties.[1] For example, an option to open an outlet in an attractive location is alive only until a competitive firm opens its own store there. Second, the firm can influence both the value of the underlying asset as well as the exercise price of the corresponding option. In many situations there exists a positive relationship between the amount of the sunk cost and the profitability of the project (i.e. via the level of automatization of the production process or via the product quality). Consequently, the firm is often faced with a menu of mutually exclusive real options with different exercise prices and payoff structures.

In the analysis we incorporate both these aspects of real options and apply them to investigate the investment decision in an uncertain product market with positive network externalities and competitive entry threat. We develop a strategic model in which a firm chooses the timing of irreversible investment and the quality of the product. Competitive entry is modeled as a timing game with a second firm. We compare the cases of a fixed and flexible quality choice, which can be interpreted as corresponding to a licensed and, respectively, internally developed technology. This paper thus studies the additional value of flexibility in quality choice. This flexibility, resulting in being able to adjust quality over time, requires sufficient know-how within the firm and the use of a more advanced technology. In practice, the case of flexible quality will be associated with higher (sunk) costs. According to the results of this paper these higher costs are especially justified in competitive environments with large demand uncertainty. Furthermore, we derive the optimal investment thresholds, optimal quality choices and projects' valuations in terms of market parameters and firms' costs characteristics.

Consequently, we aim at unifying two streams of literature: strategic real options and industrial organization-based endogenous quality choice. As far as the real option framework is concerned, our model builds up upon such contributions as Smets [24], Grenadier [11], Lambrecht and Perraudin [14], Perotti and Rossetto [22], Mason and Weeds [17], Huisman [13], and Nielsen [21], which all have in common that they analyze the effects of both competition and uncertainty on investment timing. Reinganum [23], and Fudenberg and Tirole [10] provide the game-theoretical foundations within a deterministic framework.

Introducing quality choice as a strategic variable results in the extension

[1] Cf. Zingales [29].

of the existing strategic real option framework to a class of models in which firms are equipped with two control variables. Besides choosing the timing of investment, the firms now have to decide also about the optimal quality of the product they are going to offer in the product market. The result of this is that some of the classic real-option results cease to hold. For example, the optimal investment timing of the second firm is no longer irrelevant for the investment decision of the leader (cf., e.g., Huisman [13]). This is due to the fact that the entry decision of the follower interacts with the second control variable of the leader (quality), which, in turn, influences the leader's optimal investment timing. With flexible quality the follower's investment decision becomes again irrelevant since the leader can change quality instantaneously. As a consequence, it can act as a monopolist until the follower's entry.

In the paper we show that due to strategic interaction with the follower, the value of investment option of the leader can decrease in uncertainty in a situation where fixed-quality technology is used. Moreover, the value of the leader is lower than the one of the follower. This latter result is due to strategic disadvantage of the first mover in a Stackelberg game in which the firms compete in strategic complements. The situation reverses under the flexible quality technology of the leader. Now, the value of the follower can decrease in uncertainty since its project's value becomes concave in the realizations of random demand.

Furthermore, we show that under the flexible-quality technology, the leader can drive its competitor out of the market in high states of the demand. This is caused by the fact that the leader has an incentive to invest in quality when demand is high. This reduces the demand for the product offered by the follower to zero for states of demand exceeding a certain trigger.

We also discuss the impact of network externalities on the optimal investment timing, quality choice and firms' valuations. Since, from the point of view of a consumer, an increase of the degree of network externalities can compensate the decrease in quality, the optimal quality choice of firms is inversely related to network externalities. Moreover, firms wait with investing shorter and their valuations are higher when the product market exhibits strong network externalities.

As far as the literature on strategic quality choice is concerned, our model is related to the contributions, to mention only few, by Motta [18], Aoki and Prusa [1], Foros and Hansen [9], Dubey and Wu [6], Hoppe and Lehmann-Grube [12] and Banker et al. [2].

Motta [18] considers a two-stage duopoly model with either fixed or variable costs of quality improvements. Fixed costs can be associated with R&D or advertising activities. Variable costs, that correspond to our framework, reflect more skilled labor and more expensive raw materials and inputs. The result of the paper is that firms differentiate qualities which is possible due to setting different prices. In our model, goods are differentiated horizontally so the firms set different qualities even if the cost of the good to consumer is equal and beyond their control. In a similar framework Aoki and Prusa [1] analyze optimal sequential and simultaneous quality choice. Again, due to the fact that the authors assume only vertical product differentiation and price competition, there exists a first-mover advantage in the quality choice game. In our case, products are differentiated also horizontally and firms do not influence prices. As a consequence, qualities become strategic complements, reaction curves are continuous, and the profit of the second-mover is higher.

Foros and Hansen [9] apply a two-stage model extended to allow for horizontal differentiation and network externalities to the market of Internet Service Providers. They find that the optimal choice of quality is positively related to network externalities. Their result differs from ours due to the fact that the substitution effect between quality and network externalities is dominated in Foros and Hansen [9] by the impact of lower competitive pressure resulting from higher network externalities.

Dubey and Wu [6] investigate firms' incentives to invest in product innovation, which ultimately leads to a quality increase. They show that the relationship between the number of firms and the propensity to innovate is bell-shaped. In other words, if the number of firms is "too large" or "too small" the innovation process does not occur. The results of Dubey and Wu [6] are consistent with our model that predicts that the possibility of entry increases the quality provided by the otherwise monopolistic firm. Using a different analytical framework Banker et al. [2] conclude that in the absence in the synergies among the firms in the quality cost, an increasing number of firms leads to decreasing quality. This finding coincides with the argument of Dubey and Wu [6] for a "too large" number of firms and is caused by the fact that improving quality is assumed to be sufficiently costly.

An alternative dynamic model of strategic quality choice is developed by Hoppe and Lehmann-Grube [12]. In their framework, the firms chose the optimal timing of entry, given that the available quality is a deterministic function of time. Prior to the investment, firms are assumed to pay R&D costs which are proportional to time until investing. The authors show that, depending on the cost of R&D, there can be either rent equalization (cf. e.g. Fudenberg and Tirole [10]) or a second-mover advantage in the quality choice game. The assumption made by Hoppe and Lehmann-Grube [12] that the costs of higher quality are incurred prior to investment differs from ours in which the costs of quality occur after the investment is made (similar to the notion of variable quality costs in Motta [18]). As a consequence, contrary to Hoppe and Lehmann-Grube [12], we do not observe the first-mover advantage (corresponding to payoff equalization without exogenous firms' roles) in the fixed-quality case in our model.

The paper is organized as follows. In Section 2 we present the model of a monopolistic firm with a fixed-quality technology. Section 3 extends the model to a duopolistic environment. The discussion of the monopolistic model with a flexible quality choice is presented in Section 4 and the analysis of its duopolistic extension is included in Section 5. In Section 6 we compare the impact of flexible quality on the value of the firm. Section 7 concludes.

2 Non-Strategic Model with Fixed Quality

Consider a situation in which a firm has an investment opportunity to launch a product/service in an uncertain market. It chooses the optimal investment timing and quality of the product. In this section we assume that once chosen quality cannot be changed. The idea of the fixed quality choice is therefore similar to Ueng [28], who considers an infinitely repeated oligopoly game in which the qualities are chosen before the first period. It is realistic to assume that the revenue per customer is not constant but evolves stochastically over time.[2] The instantaneous revenue per customer at time t is equal to x_t, where x_t follows the geometric Brownian motion

$$dx_t = \alpha x_t dt + \sigma x_t dw_t. \tag{1}$$

Here α denotes the deterministic drift rate and σ is the instantaneous volatility of the process. In the analysis we assume that the initial realization of (1), x_0, is sufficiently low, so that in all possible cases the market is too small for immediate investment to be optimal.

There is a continuum of heterogenous consumers with valuations ω_i distributed uniformly over the interval $[0, 1]$. As mentioned in the introduction, a consumer derives utility not only from the stand alone good but also from the number of other consumers using it. A consumer's utility function satisfying these characteristics is[3]

$$U_i = \omega_i q + an - k, \tag{2}$$

where $q \in \mathbb{R}_+$ is the quality of the good, $k \in \mathbb{R}_+$ is the cost the consumer has to bear to acquire the good, and $a \in \mathbb{R}_+$ is a parameter that measures the intensity of the network externalities. Consequently, ω_i can be interpreted as the marginal rate of substitution between income and quality, so that a higher ω_i reflects a lower marginal utility of income and, as a consequence, a higher income (see also Tirole [27], p. 98). Large a implies that the consumer's utility grows fast with the number of other users. In the opposite case, when a tends to zero, the number of users of the same good does not affect the utility of the consumer.[4] The size of the network, $n \in [0, 1]$, is interpreted as the fraction of the total market for which a given product is offered. Without loss of generality, we normalize the absolute size of the total market to 1.

Network externalities are present if the number of other consumers using the same product influences the utility of a given consumer. Positive (negative)

[2] For instance, the revenue per customer of a mobile telephone network depends on the intensity of voice traffic, competitive pressure, and arrival of new services that can be offered to the customer against an additional fee. It is natural to assume that the evolution of these economic variables over time contains an unpredictable component.

[3] Heterogeneity of consumers with respect to the value associated with the stand-alone good and their homogeneity with respect to the degree of network externalities is a common assumption in the economics of network literature (cf. Mason [16] and references therein).

[4] Of course, there are examples of negative a as well. For instance, the utility from having a Rolls-Royce is decreasing in the number of other owners of this brand in the neighborhood.

network externalities imply that the utility of the consumer increases (decreases) with the number of other users. An example of such a good is an access to the web via a given Internet Service Provider, a computer operating system, an audio recorder using a particular standard (DCC, MD, CD-R), or a mobile phone (GSM, CDMA). We analyze a good for which the consumer's utility depends on the network size and the quality (MacOS vs. Windows). The purchase decision is determined mainly by these two parameters, so that we do not incorporate a pricing strategy.

Such a choice of modeling approach follows recent empirical evidence. In the analysis of the on-line book retail market Latcovich and Smith [15] claim that "consumers do not respond much to significant price differences between sellers [...]. But they [...] care about vertical characteristics such as reliability, security, and ease of use". This support the idea of a quality-oriented market analyzed in our paper. Also Varian and Shapiro [26] point out that the price is an insignificant determinant of a purchase decision for many information goods, such as software. Referring to the market for spreadsheets they claim that "the purchase price of the software is minor in comparison with the cost of deployment, training and support. Corporate purchasers, and even individual customers, were much more worried about picking the winner of the spreadsheet wars than they were about whether their spreadsheet costs \$49.95 or \$99.95".

On the basis of the consumers' utility function, we can determine the size of the network as a function of the quality chosen by the firm. Define the consumer of type $\overline{\omega}$ to be indifferent between acquiring the good or not. Consequently, it holds that

$$\overline{\omega}q + an - k = 0. \tag{3}$$

By setting $a < k < q$, which is to ensure an interior solution for the size of the network (we waive these restrictions later), and observing that the size of the network, n, equals $1 - \overline{\omega}$, we obtain

$$n(q) = \frac{q-k}{q-a}. \tag{4}$$

Now, we are ready to provide a valuation framework for the project of a single firm. In solving the valuation problem we do not apply the contingent claim approach since it is, in general, not possible to replicate the underlying asset, i.e. the stochastic factor in the firms' profits. Therefore we use the dynamic programming methodology. For simplicity we assume risk neutrality of the firm.[5] As a consequence, the assets are priced such that the expected rate of return equals the risk-free rate.[6]

We further assume a constant value per customer, constant economies of scale on the supply side, and that the marginal cost of operation, $c(q)$, satisfies $c'(q) > 0$ and $c''(q) \geq 0$. The firm chooses quality q so as to maximize

[5] As known from, e.g., Dixit and Pindyck [5], dynamic programming under risk neutrality yields the same results as contingent claims analysis in the complete markets framework.

[6] The assumption of risk-neutrality may be waived by adjusting the underlying assets' drift rate to as proposed by Cox and Ross [3] in order to account for the risk premium.

the value of the investment opportunity. In order to determine the value of the investment opportunity, we begin with calculating the value of the project after the investment decision is made. The value of the project is found by integrating over time the discounted difference between the instantaneous value of the installed base of consumers, $xn(q)$, and the operating costs $c(q)\,n(q)$.[7] Therefore, if we denote the project value at time t by V, it holds that

$$
\begin{aligned}
V &= E\left[\int_t^{\infty} (x_t - c(q))\, n(q)\, e^{-r(s-t)} ds\right] && (5) \\
&= \frac{n(q)\, x_t}{r-\alpha} - \frac{c(q)\, n(q)}{r} \\
&\equiv R(q)\, x_t - C(q) && (6)
\end{aligned}
$$

where r is the risk-free rate.

The firm has to incur a sunk investment cost, $I \in \mathbb{R}_{++}$. Although I does not depend on the choice of quality, the cost associated with pursuing the project increases with quality due to a higher present value of operating costs.[8] The decision of the firm is to choose the optimal quality, q, and timing of entry, x^*, in order to maximize the expected value of the investment opportunity.

To find the optimal investment threshold and product quality we proceed in two steps. First, we solve the optimal stopping problem using the methodology of McDonald and Siegel [19] for an arbitrary level of q. As an intermediate result we obtain the optimal investment threshold and the value of the investment opportunity as a function of q. Second, we maximize the value of the investment opportunity with respect to q.

The threshold $x^*(q)$, being the lowest value of x_t at which the firm enters the market, is

$$
x^*(q) = \frac{\beta_2}{\beta_2 - 1} \frac{I + C(q)}{R(q)}, \tag{7}
$$

where

$$
\beta_2 = -\frac{\alpha}{\sigma^2} + \frac{1}{2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{8}
$$

By differentiating (7) we immediately obtain that $\partial x^*(q)/\partial q > 0$. Consequently, the level of demand sufficient for triggering optimal investment increases with the quality of the product.

[7] The instantaneous value of the installed base of consumers can be obtained integrating the utilities of participating consumers over their types, $\int_{\bar{\omega}}^{1} (\omega q + an - k)\, d\omega$. This equals $0.5n^2 + 0.5(q-k)\,n$ which is convex in n. However, here we assume that the firm does not price discriminate so that it does not extract the whole consumer surplus. Instead, we impose linearity in n of the firm's profit.

[8] An alternative interpretation of the cost structure is that the initial investment outlay equals $I + c(q)\,n(q)/r$, and the marginal production cost is zero for all levels of q.

The value of the investment opportunity, $F(q, x_t)$, equals

$$F(q, x_t) = \frac{(\beta_2 - 1)^{\beta_2 - 1}}{\beta_2^{\beta_2}} \frac{R(q)^{\beta_2} x_t^{\beta_2}}{(I + C(q))^{\beta_2 - 1}}. \tag{9}$$

The proofs of (7), (8) and (9) follow directly from Dixit and Pindyck ([5], p. 142). Subsequently, we maximize the value of the investment opportunity with respect to q, given the optimal investment rule, $x^*(q)$. In order to ensure that our solution is a maximum, we introduce the following assumption.

Assumption 1 *Let q^* be the solution to $\partial F(q, x^*)/\partial q = 0$. Then it holds that*

$$\left.\left(\beta_2 (C + I) R_{qq} + C_q R_q - (\beta_2 - 1) C_{qq} R\right)\right|_{q=q^*} < 0. \tag{10}$$

The solution to the problem of quality choice is given in the following proposition.

Proposition 1 *Under Assumption 1 the optimal quality of the product, q^*, is implicitly given by the following equation*

$$C_q = x^* R_q. \tag{11}$$

Proof. See Appendix. ■

From Proposition 1 it is obtained that the value of the investment opportunity is maximized if at the optimal investment threshold the marginal cost of increasing the quality is equal to the marginal benefit. (11) implies that in the optimum the ratio of elasticities of functions $C(q) + I$ and $R(q)$ equals the wedge occurring in the threshold value $x^*(q)$ (cf. (7)), i.e.

$$\left.\frac{\varepsilon_{C+I,q}}{\varepsilon_{R,q}}\right|_{q=q^*} = \frac{\beta_2}{\beta_2 - 1}, \tag{12}$$

where $\varepsilon_{f,x} \equiv \frac{x f_x}{f}$.

In order to provide more insight into the obtained result, we analyze the relationship between market uncertainty, intensity of the network externalities, size of the network and the optimal quality. Proposition 2 provides part of the results.

Proposition 2 *The quality of the product increases with revenue uncertainty and its growth rate, i.e.*

$$\frac{dq^*}{d\sigma} > 0, \text{ and} \tag{13}$$

$$\frac{dq^*}{d\alpha} > 0. \tag{14}$$

Proof. See the Appendix. ■

The fact that higher uncertainty concerning the demand side of the market influences the quality choice of the firm positively results from the option-like structure of the project value and upside potential from higher quality investment. Furthermore, a higher growth rate of the market also implies a higher quality choice since the firm prefers to incur additional cost to increase quality when revenue is expected to grow faster.[9]

Furthermore, numerical simulations indicate that the impact of network externalities on the optimal quality choice is negative. The latter relationship results from the fact that the level of quality and the degree of network externalities act as substitutes in the marginal consumer's utility function. Since a higher quality is equivalent to a larger consumer base (cf. (4)), the size of the network in optimum, n^*, also rises with σ and α.

Market uncertainty and intensity of network externalities also have an impact on the optimal investment threshold. Since both factors affect the optimal investment threshold directly and indirectly (via the change of the optimal quality), the total impact is determined by calculating the following total derivative:

$$\frac{dx^*(q)}{d\theta} = \frac{\partial x^*(q)}{\partial \theta} + \frac{\partial x^*(q)}{\partial q}\frac{dq}{d\theta}, \;\; \theta \in \{a, \sigma\}. \tag{15}$$

In the Appendix we prove the following proposition:

Proposition 3 *It holds that*

$$\frac{dx^*(q)}{d\sigma} > 0. \tag{16}$$

Hence, the relationship between uncertainty and the optimal investment threshold is positive. Therefore we conclude that the flexibility in the quality choice does not change the classical result of real option theory (cf. Dixit and Pindyck [5]).

Extensive numerical simulations show that the optimal investment threshold decreases in the degree of network externalities. This is associated with the fact that a higher degree of network externalities makes the product market more valuable for the firm. This results in a higher value of the investment project (other things equal) and, thus, a lower value of x suffices to achieve the required profitability ratio, $\beta_2/(\beta_2 - 1)$, of the project at the time of investing.

3 Strategic Model with Fixed Quality

Here we introduce the possibility of competitive entry by a second firm (Firm 2). After entering the market, Firm 2 starts offering the good having a

[9] The positive sign of the derivative with respect to α is equivalent to the negative derivative with respect to the cost-of-carry, defined as $\delta \equiv r - \alpha$.

quality q_2. In general, q_2 will differ from q_1, i.e. from the quality choice made by Firm 1. The fact that the firms do not compete in prices implies that for the consumers the cost of accessing each network is equal across the networks. Consequently, if the products were perfect substitutes, consumers would always choose the product with a higher quality and the resulting market outcome would always be a monopoly. In case of imperfect substitution this does not hold any longer. Denote the degree of substitution by $\rho \in (0,1)$. For ρ close to unity, the goods are close substitutes, whereas a very small ρ implies that the firms operate in virtually separated markets.

In order to analyze the impact of entry on the valuation of the first firm in the market (Firm 1), we adopt a simple structure for the market with differentiated goods (as, e.g., in Spence [25]) and allow for the presence of network externalities as in Section 2. The system of inverse demand functions is given by

$$\left\{ \begin{array}{l} k = (1-n_1)\, q_1 - \rho n_2 q_2 + a\,(n_1 + \rho n_2) \text{ for Firm 1's network, while} \\ k = (1-n_2)\, q_2 - \rho n_1 q_1 + a\,(n_2 + \rho n_1) \text{ for Firm 2's network,} \end{array} \right. \tag{17}$$

and n_i, $n \in \{1,2\}$, is the size of Firm i's network. Each of the inverse demand functions can be interpreted as follows. The LHS represents the instantaneous cost (utility loss) of accessing the network. The RHS corresponds to the linear demand schedule that decreases in the offered quantities, n_i and n_j, while its negative slope is reduced by the presence of a component $a\,(n_1 + \rho n_2)$ which reflects network externalities. The impact of the quantity offered by Firm j on Firm i's demand, and the network externalities among its consumers is scaled down by factor ρ reflecting imperfect substitution among the goods. It can be easily noticed that for n_j equal to zero, (17) reduces to the monopolistic demand function of Section 2.

The size of the network of Firm i obtained by solving (17), subject to $n_i \in [0,1]$, equals

$$n_i\,(q_i) = \left\{ \begin{array}{ll} 0 & q_i < \underline{q}_i, \\ \frac{1}{1-\rho^2} \frac{q_i - \underline{q}_i}{q_i - a} & q_i \in \left[\underline{q}_i, \overline{q}_i\right], \\ \frac{q_i - k}{q_i - a} & q_i > \overline{q}_i, \end{array} \right. \tag{18}$$

where

$$\underline{q}_i = k\,(1-\rho) + \rho \max\,[k, q_j]\,, \tag{19}$$

$$\overline{q}_i = \frac{\max\,[k, q_j] - k\,(1-\rho)}{\rho}, \tag{20}$$

and $i, j \in \{1,2\}$, $i \neq j$. Depending on the quality offered, Firm i competes with Firm j for moderate values of q_i, it is a monopolist for high q_i, or has no customer base if q_i is low. Both qualities $\underline{q}_i$ and $\overline{q}_i$ depend positively on quality q_j offered by the competitor. Moreover, higher substitutability of the goods, captured by ρ, results in shrinking the range of qualities in which firms compete. This is intuitive since the closer substitutes the goods are, the less they can differ in qualities for both firms to be present in the product market. Since the once chosen qualities remain fixed and neither $\underline{q}_i$ nor $\overline{q}_i$ depends on x, both firms being active implies that $q_i \in \left[\underline{q}_i, \overline{q}_i\right]$ for $i \in \{1, 2\}$. Otherwise, one of the firms would be better off by not entering.

For analytical convenience, we impose the following linear specification of the cost function:

$$c(q_i) = c_0 (q_i - a), \; c_0 \in \mathbb{R}_{++}, \; q_i \in [a, \infty), \tag{21}$$

where c_0 can be interpreted as an efficiency parameter. Consequently, higher values of c_0 correspond to industries that are less efficient in R&D. Setting a quality equal to $a \, (< k)$ is equivalent to the firm producing no output and incurring no cost (since $n_i(a) = c(a) = 0$ in this case). The instantaneous profit function corresponding to (21) is

$$\pi_i = (x - c_0 (q_i - a)) \, n_i. \tag{22}$$

We solve the problem backwards in time. First, the optimal investment threshold and quality choice of Firm 2 is determined. The value of Firm 2's investment opportunity at $t \leq T_2$ equals

$$V_2 = E\left[\int_{T_2}^{\infty} (x_s - c(q_2^*)) \, n_2(q_2^*) e^{-r(s-t)} ds - I e^{-r(T_2 - t)}\right], \tag{23}$$

where T_2 denotes the random stopping time associated with x_t reaching Firm 2's optimal investment threshold. A well-known procedure (cf. Dixit and Pindyck [4], p. 145) allows for deriving Firm 2's optimal threshold, x_2^*, and the value of its investment opportunity, $F_2^* x^{\beta_2}$:

$$x_2^* = \frac{\beta_2}{\beta_2 - 1}\left(\frac{I(1-\rho^2)}{q_2 - \underline{q}_2} + \frac{c_0}{r}\right)(q_2 - a)(r - \alpha), \tag{24}$$

$$F_2^* x^{\beta_2} = \max_{q_2} \frac{\left(q_2 - \underline{q}_2\right) x_2^*}{\beta_2 (1-\rho^2)(r-\alpha)(q_2 - a)} \left(\frac{x}{x_2^*}\right)^{\beta_2}. \tag{25}$$

From (25) it follows that the quality maximizing the value of Firm 2's investment opportunity, q_2^*, is

$$q_2^* = \frac{1}{2(\beta_2 - 1)} \times \tag{26}$$
$$\left[(2\beta_2 - 1)\underline{q}_2 - a + \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + 4\beta_2 I r (\beta - 1)(1-\rho^2) c_0^{-1}}\right].$$

Upon analyzing (26) it can be concluded that the qualities chosen by the firms are strategic complements. Since $\underline{q}_2$ is an increasing function of q_1 (see (19)) and q_2^* rises with $\underline{q}_2$, the quality chosen by Firm 2 is positively related to the quality choice made by Firm 1.

This relationship, in combination with a closer inspection of (24), leads to the following proposition.

Proposition 4 *Firm 2 responds optimally to an increased quality of Firm 1 not only by rising its own quality but also by delaying its timing of entry, i.e. the following inequalities hold*

$$\begin{aligned} \frac{dq_2^*}{dq_1} &> 0, \text{ and} \\ \frac{dx_2^*}{dq_1} &> 0. \end{aligned}$$

Proof. See the Appendix. ■

Consequently, it can be concluded from Proposition 4 that the choice of higher q_1 is equivalent to entry-deterrent behavior of Firm 1.

Having calculated the optimal investment threshold of Firm 2, we are in position to analyze the investment decision of Firm 1. First, we note that the value of Firm 1's investment project at the time of investing, t, is given by

$$\begin{aligned} V_1 &= E\left[\int_t^{T_2} (x_s - c_0(q_1^* - a))\, n(q_1^*) e^{-r(s-t)} ds - I\right] \\ &\quad + E\left[\int_{T_2}^{\infty} (x_s - c_0(q_1^* - a))\, n_1(q_1^*, q_2^*) e^{-r(s-t)} ds\right]. \end{aligned} \tag{27}$$

Working out the expectations yields

$$V_1 = \underbrace{\frac{q_1^* - k}{q_1^* - a}\left(\frac{x}{r-\alpha} - \frac{c_0(q_1^* - a)}{r}\right)}_{\text{Monopolistic value}} - I + \tag{28}$$

$$\underbrace{\left(\frac{1}{1-\rho^2}\frac{q_1^* - \underline{q}_1}{q_1^* - a} - \frac{q_1^* - k}{q_1^* - a}\right)\left(\frac{x_2^*}{r-\alpha} - \frac{c_0(q_1^* - a)}{r}\right)\left(\frac{x}{x_2^*}\right)^{\beta_2}}_{\text{Value lost due to the competitive entry}}.$$

Again, applying the well-known procedure (cf. Dixit and Pindyck [4], p. 145) yields the optimal threshold, x_1^*, and the value of investment opportunity, $F_1^* x^{\beta_2}$, of Firm 1

$$x_1^* = \frac{\beta_2}{\beta_2 - 1}\frac{I + C(q_1)}{R(q_1)}, \tag{29}$$

$$F_1^* x^{\beta_2} = \tag{30}$$

$$\max_{q_1} \frac{(q_1 - k)\frac{x_1^*}{r-\alpha} + \beta_2\left(\frac{q_1 - \underline{q}_1}{1-\rho^2} - q_1 + k\right)\left(\frac{x_2^*}{r-\alpha} - \frac{c_0(q_1 - a)}{r}\right)\left(\frac{x_1^*}{x_2^*}\right)^{\beta_2}}{\beta_2(q_1 - a)}\left(\frac{x}{x_1^*}\right)^{\beta_2}.$$

It is worthwhile noticing that the optimal investment timing of Firm 1 does not *explicitly* depend on the action taken by Firm 2. This outcome results from the fact that the roles of the firms (leader vs. follower) are exogenously determined. However, this result still differs from the classical result from the real option theory (see, e.g., Huisman [13], p. 170) concerning the irrelevance of the follower's investment timing for the decision of the leader. The reason is that Firm 1's timing decision is affected by the choice of quality, q_1, and, according to (30), q_1 depends on Firm 2's threshold x_2^* and on the threshold quality $\underline{q}_1$, which is a function of q_2 (cf. (19)).

The dependence of Firm 1's investment threshold results from the fact that in our model firms have two control variables (timing and quality) as opposite to one variable in classic real option models. It still holds that introducing the competitor does not change the optimal *ceteris paribus* choice of the timing variable. However, competitive entry changes the optimal choice of quality (the second control variable). This makes the monopolistic choice of timing no longer optimal and, as a consequence, it holds that $x_1^* \neq x^*$.

As far as the value of the investment opportunity is concerned, it can be determined by maximizing the argument of the RHS of (30). The derivative of F_1^* with respect to q_1 can be computed since x_1^*, x_2^* and q_2 are known functions of q_1. The unique (in the relevant interval) root of the derivative can be easily found numerically.

3.1 Comparative Statics: Valuation of Firms

We are interested in the sensitivity of the value of the firms with respect to changes of market parameters. Figure 3.1 depicts the relationship between the market volatility (left window) and network externalities (right window), and the value of investment opportunities of both firms.

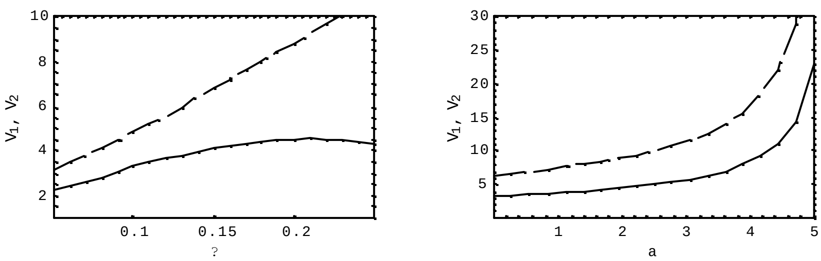

Figure 3.1

Figure 3.1. The value of the investment opportunity of Firm 1 (solid line) and Firm 2 (dashed line) for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (left window), $\sigma = 0.2$ (right window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

On the basis of Figure 3.1 two interesting observations can be made. First, the value of Firm 1's investment opportunity is lower than the one of Firm 2. Second, the value of Firm 1's project is non-monotonic in uncertainty. The first phenomenon results from the strategic disadvantage of the first mover in a game in which the firms compete in strategic complements. As it can be shown in a simple Stackelberg setting, the follower's payoff is higher than the payoff of the leader if the control variables are strategic complements (cf. Tirole [27], p. 331, footnote 53). Despite the fact that Firm 1 enjoys profit from investment for a longer period (it invests as first), its value is still lower than of Firm 2.

Non-monotonicity of Firm 1's value results from the more "aggressive" choice of quality of Firm 2 in a more uncertain market (cf. (13)). Consequently, despite the fact that the option value increases in market volatility in a non-strategic setting, the interaction among firms drive down the value of Firm 1 when uncertainty is high.

Moreover, it can be seen that the presence of the network externalities significantly enhances the value of the investment opportunities of both firms. The rate of increase is most dramatic where the degree of network externalities approaches the cost of joining the network (i.e. when the marginal consumer's valuation of the stand alone-good is equal to zero).

3.2 Comparative Statics: Firm 1's Strategic Choice of Variables

Finally, we compare the non-strategic and strategic case with respect to Firm 1's optimal investment threshold and its optimal quality choice (see Figures 3.2 and 3.3)

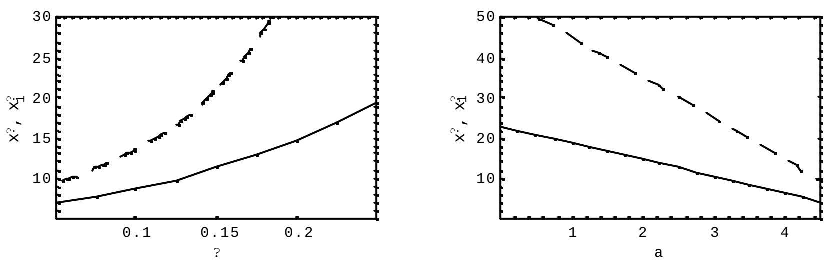

Figure 3.2

Figure 3.2. The optimal investment threshold of Firm 1 in the non-strategic (solid line) and strategic (dashed line) case for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (right window), $\sigma = 0.2$ (left window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

From Figure 3.2 it can be concluded that the optimal investment threshold is higher if a subsequent competitive entry threat exists. This contradicts the result known from the strategic real option literature that the optimal investment threshold of the market leader is not influenced by the entry threat if the roles of the firms are exogenous. As we already concluded from (29), Firm 1's investment threshold depends on the investment timing and quality decision of its competitor.

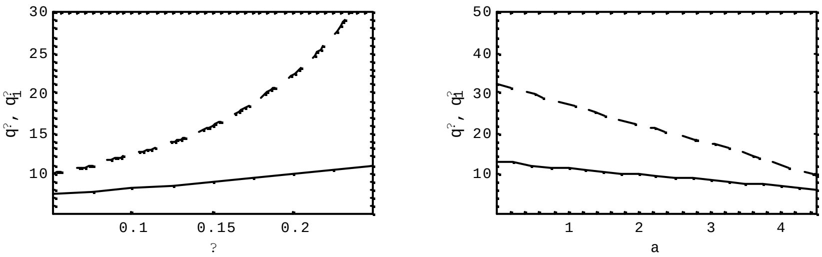

Figure 3.3

Figure 3.3. The optimal quality choice of Firm 1 in the non-strategic (solid line) and strategic (dashed line) case for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (right window), $\sigma = 0.2$ (left window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

On the basis of Figure 3.3 we conclude that the presence of a (potential) competitor increases the quality chosen optimally by Firm 1. Higher quality (as shown in Section 2), as well as the fact that, from the timing of the second firm onwards, the market must be shared with the competitor, results in the optimality of a higher - than in the non-strategic case - investment threshold which, in turn, leads to the outcome depicted in Figure 3.2.

This result and the one concerning the project's value contradict the findings of Foros and Hansen [9], who analyze a duopoly model of Internet Service Providers. In a modified Hotelling framework they show that the profits decrease and the offered quality increases with the degree of network externalities. A point has to be made why this differs from our results. Here, in a non-strategic framework, network externalities can act a substitute of quality in a consumer's utility function. Consequently, a firm can have less of an incentive to invest in (costly) quality when network externalities. This effect also takes place in a strategic framework if the increase of quality occurs for a single product. In case of Foros and Hansen [9], the increase of interconnection quality affects at the same both products so that the substitution effect is dominated by the impact of increased competitiveness.

4 Non-strategic Model with Flexible Quality

Here, it is assumed that within the firm sufficient know-how is present for adjusting quality, which can be valuable in case of changing demand characteristics. The fact that the firm can change quality could be caused for instance by the fact that its technology is the result of its own R&D process. Such an interpretation implies that in the previous section quality was fixed because the production technology was provided by an external vendor.

Once the entry threshold, x^{**}, is reached, production commences. The marginal cost, $c\left(q\left(x\right)\right)$, is a function of the instantaneously chosen product/service quality. This quality is chosen in such a way that the value of the firm is maximized. In this section we assume that no competitive entry threat exists.

Consequently, at each point in time the firm chooses quality q_t such that[10]

$$q^{**}\left(x\right) = \arg\max_{q}\left[\left(x - c\left(q\right)\right)n(q)\right]. \tag{31}$$

From this the present value of the firm's expected cash flow at time t can be determined

$$V = E\left[\int_t^{\infty}\left(x - c\left(q^{**}\left(x\right)\right)\right)n(q_s)e^{-r(s-t)}ds\right]. \tag{32}$$

Since we in general allow for $q < k$, let us redefine $n\left(q\right)$ (cf. (4)) as

$$n\left(q\right) = \max\left[0, \frac{q-k}{q-a}\right]. \tag{33}$$

Maximizing (31) with cost specification (21) leads to the optimal quality choice

$$q^{**}\left(x\right) = a + \sqrt{\frac{\left(k-a\right)x}{c_0}}\mathbf{1}_{\{x>\eta\}}, \tag{34}$$

where

$$\eta = c_0\left(k-a\right)$$

and $\mathbf{1}_B$ is an indicator function.[11] (34) implies that for low states of demand (i.e. for $x < \eta$) the optimal choice of quality is a $(< k)$, which corresponds to the situation in which the market is not served and the firm incurs no cost (see (21)). As soon as x hits η from below, quality jumps to k and, subsequently, adjusts continuously to changes in x. When x hits η from above, the quality drops to a and the firm again becomes idle without incurring variable costs.

Define the instantaneous profit function, π, to be equal to the expression under the arg max operator in (31). Substituting q^{**} into the instantaneous profit function yields

$$\pi = \left(\sqrt{x} - \sqrt{\eta}\right)^2\mathbf{1}_{\{x>\eta\}}. \tag{35}$$

[10] Our formulation differs from the optimal control models of quality as, e.g., presented by El Ouardighi and Tapiero [8] (cf. also references therein) since these authors consider a deterministic setting in which they include elements absent here such as pricing strategy and learning effects.

[11] $\mathbf{1}_B$ denotes an indicator function of B such that $\mathbf{1}_B\left(x\right) = \begin{cases} 1 & x \in B \\ 0 & x \notin B \end{cases}$.

Solving the Bellman equation[12]

$$0.5\sigma^2 x_t^2 V'' + \alpha x V' + \pi = rV \tag{36}$$

for appropriate value matching and smooth pasting conditions yields:

$$V = \begin{cases} B_{M2}x^{\beta_2} & \text{for} \quad x < \eta, \\ B_{M1}x^{\beta_1} + C_0 + C_1 x^{0.5} + C_2 x & \text{for} \quad x > \eta, \end{cases} \tag{37}$$

where

$$B_{M1} \equiv C_0 \frac{\eta^{-\beta_1}\beta_2}{\beta_1 - \beta_2} + C_1 \frac{\eta^{0.5-\beta_1}(\beta_2 - 0.5)}{\beta_1 - \beta_2} + C_2 \frac{\eta^{1-\beta_1}(\beta_2 - 1)}{\beta_1 - \beta_2}, \tag{38}$$

$$B_{M2} \equiv C_0 \frac{\eta^{-\beta_2}\beta_1}{\beta_1 - \beta_2} + C_1 \frac{\eta^{0.5-\beta_2}(\beta_1 - 0.5)}{\beta_1 - \beta_2} + C_2 \frac{\eta^{1-\beta_2}(\beta_1 - 1)}{\beta_1 - \beta_2}, \tag{39}$$

$$C_0 \equiv \frac{\eta}{r}, \tag{40}$$

$$C_1 \equiv \frac{-2\sqrt{\eta}}{r - 0.5\alpha + 0.125\sigma^2}, \tag{41}$$

$$C_2 \equiv \frac{1}{r - \alpha}, \tag{42}$$

$$\beta_1 = -\frac{\alpha}{\sigma^2} + \frac{1}{2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \tag{43}$$

and β_2 is given by (8). The value functions in the two regimes of the stopping region are the solutions of the standard ODE (36) with the non-homogeneity term defined by (35). Under the regime $x < \eta$ demand is too low and no service/product is offered. Consequently, the value of the firm consists entirely of the option value to relaunch the activities should the market turn out to be favorable. For $x > \eta$ the firm offers the service and makes positive profit. Now, the value of the firm consists of two parts: the perpetuity value of the current instantaneous profit and the option-like component reflecting the possibility of ceasing the operations if x falls below η. The perpetuity value of the instantaneous profit has the structure of a portfolio of continuously paid dividends proportional to various powers of the GBM (1). By either solving the Bellman equation of type (36) with a non-homogeneity term being proportional to the n-th power of x, or by calculating the drift coefficient in the GBM for $y \equiv x^n$ using Itô's lemma, it can easily be shown that the effective discount rate corresponding to the n-th power has a form $r - n\alpha - 0.5n(n-1)\sigma^2$ (cf. Dixit [4], p. 13).

The optimal investment threshold and the value of the investment opportunity are found by applying the standard procedure for the optimal exercise

[12] The value of the firm, V, (cf. (32)) still satisfies the differential equation (36) since q is an $\mathcal{F}$-previsible process. Consequently,

$$dV = V_x dx + 0.5V_{xx}(dx)^2 + V_q dq = V_x dx + 0.5V_{xx}(dx)^2,$$

which after substituting (1) yields the LHS of (36).

of an American option when the value of the investment project in the stopping region is described by (37). It should just be noticed that it is never optimal to exercise the investment option for $x < \eta$ since by waiting an increment dt the present value of investment cost diminishes by $Irdt$, whereas the expected present value of the cash flow remains unchanged. The value-matching and smooth-pasting conditions regarding the expression for V when $x > \eta$ in (37) are

$$A_M x^{\beta_2} = B_{M1} x^{\beta_1} + C_0 + C_1 x^{0.5} + C_2 x - I, \tag{44}$$

$$\beta_2 A_M x^{\beta_2 - 1} = \beta_1 B_{M1} x^{\beta_1 - 1} + 0.5 C_1 x^{-0.5} + C_2. \tag{45}$$

From (44) and (45) the following implicit equation for the optimal investment threshold, x^{**}, can be obtained

$$\begin{aligned} &(\beta_2 - \beta_1) B_{M1} (x^{**})^{\beta_1} + \beta_2 (C_0 - I) + (\beta_2 - 0.5) C_1 (x^{**})^{0.5} \\ &+ (\beta_2 - 1) C_2 x^{**} = 0. \end{aligned} \tag{46}$$

The value of the investment opportunity equals

$$F = (V(x^{**}) - I) \left(\frac{x}{x^{**}} \right)^{\beta_2} \equiv A_M x^{\beta_2}. \tag{47}$$

Here, we would like to make an additional remark concerning the implications of the flexible quality choice on the cost structure. Compared with the fixed-quality case, the effective sunk cost in the current case equals I, as opposed to $I + C$ in the former. Consequently, the choice of flexible quality not only allows for optimizing the product parameter when demand changes but also for avoiding commitment to fixed production costs in the future.

5 Strategic Model with Flexible Quality

In this section we introduce the possibility of entry of a second firm (Firm 2). As in the fixed quality case, such an entry threat is going to influence both the optimal investment timing and the value of the investment opportunity of Firm 1. We proceed as follows. First, we discuss possible market outcomes dependent on the realization of the stochastic variable, x. Subsequently, we determine the value of Firm 1 in the situation where both firms have already invested. Then, we move backwards and calculate the value of Firm 1 after it entered the market but before Firm 2 invested. Finally, we determine the value of Firm 1's investment opportunity and its optimal investment threshold, and provide some comparative statics.

As in Section 3, Firm 2 is assumed to have the fixed-quality technology. Profit maximization of Firm 1 yields the following optimal quality schedule

$$q_1^{**} = \begin{cases} a, & \text{when Firm 1 is idle,} \\ a + \sqrt{\frac{(\underline{q}_1 - a)x}{c_0}}, & \text{when Firm 1 is a duopolist,} \\ a + \sqrt{\frac{(k-a)x}{c_0}}, & \text{when Firm 1 is a monopolist.} \end{cases} \tag{48}$$

The first (idle) and the third (monopoly) case have already been derived in Section 4. The result for the duopoly case can be obtained by maximizing the profit function (22) with respect to q_i, $i = 1$, and using the observation that n_1 is in this case defined by the second equation in (18). Before we derive Firm 1's profit as a function of x, we formulate the following lemma.

Lemma 5 *There are three regimes of the product market structure when Firm 1 has the flexible quality. For low realizations of x, market is served only by the entrant (Firm 1 stays idle), intermediate realizations of x correspond to the duopoly outcome, whereas high realizations of x are associated with Firm 1's monopoly. The three regimes correspond to the following intervals*

$$\begin{aligned} x &\in (0, \varphi)\,, \\ x &\in (\varphi, \varkappa)\,, \text{ and} \\ x &\in (\varkappa, \infty)\,, \end{aligned}$$

where

$$\varphi \equiv c_0 \left(\underline{q}_1 - a \right), \tag{49}$$

$$\varkappa \equiv \frac{c_0^2 \psi^2}{\rho^2 \varphi}, \tag{50}$$

where $\psi \equiv \rho \left(\overline{q}_1 - a \right)$, and $\underline{q}_1$ and $\overline{q}_1$ are given by (19) and (20).

Proof. See the Appendix. ■

The existence of three regimes of quality choice result from the fact that now Firm 1 is able to adjust its quality, q_1, as x evolves. Since from (19) and (20) we learn that $\underline{q}_2$ and $\overline{q}_2$ explicitly depend on q_1, it follows that $\underline{q}_2$ and $\overline{q}_2$ become functions of x. Consequently, for low realizations of x (lower than φ) Firm 1 remains idle (in order to avoid operating loss), whereas for intermediate values of x it competes against Firm 2. If x becomes large (larger than $\varkappa$), Firm 1 can afford to choose quality that is high enough to prevent Firm 2 (with a fixed quality q_2) from serving the market. Consequently, the quality choice (48) reflects the optimal response in the state of inaction, duopoly and monopoly, respectively. This relationship is illustrated in Figure 5.1.

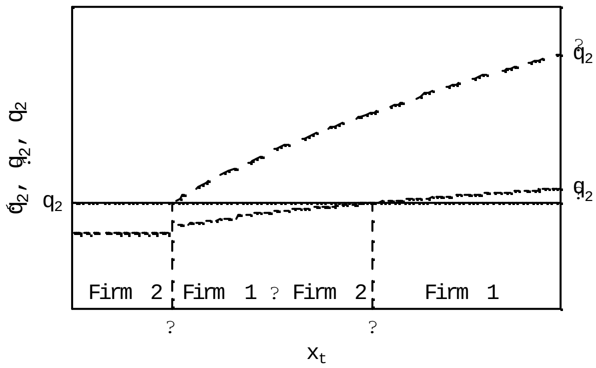

Figure 5.1

Figure 5.1. Trigger qualities $\underline{q}_2$ (short-dotted line), $\overline{q}_2$ (long-dotted line) as a function of x, for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$, $c_0 = 1$, and $q_2 = 7.5$ (solid line). For low realizations of x (below φ) only Firm 2 is active in the market whereas for high realizations (above $\varkappa$) Firm 1 becomes a monopolist - the quality of Firm 2 is too low. For intermediate values of x both firms serve the market since q_2 remains within the bounds determined by $\underline{q}_2$ and $\overline{q}_2$.

Denote the value of the Firm 1, provided that Firm 2 has already entered the market, by V_1^d. V_1^d satisfies the following Bellman equation

$$0.5\sigma^2 x_t^2 \frac{\partial^2 V_1^d}{\partial x^2} + \alpha x \frac{\partial V_1^d}{\partial x} + \pi_1 = r V_1^d, \tag{51}$$

where

$$\pi_1 = \begin{cases} 0 & \text{for} \quad x < \varphi, \\ \frac{1}{1-\rho^2}\left(\sqrt{x} - \sqrt{\varphi}\right)^2 & \text{for} \quad \varphi < x < \varkappa, \\ \left(\sqrt{x} - \sqrt{\eta}\right)^2 & \text{for} \quad x > \varkappa. \end{cases} \tag{52}$$

For $x < \varphi$ Firm 1 is idle, for $x > \varkappa$ it earns monopoly profit, whereas for $x \in (\varphi, \varkappa)$ it has a duopoly profit. The latter can be calculated by substituting the intermediate cases of (18) and (48) into (22). Solving (51) with the value matching and smooth pasting conditions satisfied for realizations φ and $\varkappa$ yields

$$V_1^d = \begin{cases} (D_2 + D_4)\, x^{\beta_2} & \text{for} \quad x < \varphi, \\ D_1 x^{\beta_1} + D_2 x^{\beta_2} + E_0 + E_1 x^{0.5} + E_2 x & \text{for} \quad \varphi < x < \varkappa, \\ (D_1 + D_3)\, x^{\beta_1} + C_0 + C_1 x^{0.5} + C_2 x & \text{for} \quad x > \varkappa, \end{cases} \tag{53}$$

where

$$
\begin{aligned}
D_1 &\equiv E_2\frac{\varphi^{1-\beta_1}\left(\beta_2-1\right)}{\beta_1-\beta_2}+E_1\frac{\varphi^{0.5-\beta_1}\left(\beta_2-0.5\right)}{\beta_1-\beta_2}+E_0\frac{\varphi^{-\beta_1}\beta_2}{\beta_1-\beta_2}, && (54)\\
D_2 &\equiv -E_2\frac{\varkappa^{1-\beta_2}\left(\beta_1-1\right)\rho^2}{\beta_1-\beta_2}-E_1\frac{\varkappa^{0.5-\beta_2}\left(\beta_1-0.5\right)}{\beta_1-\beta_2}\frac{\left(1-\rho^2\right)\varphi-\eta}{\varphi} && \\
&\quad -E_0^d\frac{\varkappa^{-\beta_2}\beta_1\rho}{\beta_1-\beta_2}, && (55)\\
D_3 &\equiv -E_2\frac{\varkappa^{1-\beta_1}\left(\beta_2-1\right)\rho^2}{\beta_1-\beta_2}-E_1\frac{\varkappa^{0.5-\beta_1}\left(\beta_2-0.5\right)}{\beta_1-\beta_2}\frac{\left(1-\rho^2\right)\varphi-\eta}{\varphi} && \\
&\quad -E_0^d\frac{\varkappa^{-\beta_1}\beta_2\rho}{\beta_1-\beta_2}, && (56)\\
D_4 &\equiv E_2\frac{\varphi^{1-\beta_2}\left(\beta_1-1\right)}{\beta_1-\beta_2}+E_1\frac{\varphi^{0.5-\beta_2}\left(\beta_1-0.5\right)}{\beta_1-\beta_2}+E_0\frac{\varphi^{-\beta_2}\beta_1}{\beta_1-\beta_2}, && \\
E_0 &\equiv \frac{1}{1-\rho^2}\frac{\varphi}{r}, && (57)\\
E_1 &\equiv \frac{1}{1-\rho^2}\frac{-2\sqrt{\varphi}}{r-0.5\alpha+0.125\sigma^2}, && (58)\\
E_2 &\equiv \frac{1}{1-\rho^2}\frac{1}{r-\alpha}, && (59)
\end{aligned}
$$

and C_0, C_1, and C_2 are defined by (40), (41) and (42). Again, it can be seen that the value of Firm 1 consists of the present value of the expected cash flow and the option-like components reflecting possible switches across regimes. Parameters E_k and C_k, $k \in \{1,2,3\}$, correspond to the duopolistic and monopolistic profit function, respectively. Components of the form $D_l x^{\beta_1}$, $l \in \{1,2,3,4\}$, reflect the possibility of switching to the regime corresponding to lower than current realizations of x, whereas the opposite is true for components $D_l x^{\beta_2}$.

Equipped with the valuation formula for Firm 1 when both firms are already present in the market, we are ready to derive the value of Firm 1, V_1^m, prior to Firm 2's entry

$$
V_1^m = V + \left(V_1^d\left(x_2^{**}\right) - V\left(x_2^{**}\right)\right)\left(\frac{x}{x_2^{**}}\right)^{\beta_2}, \tag{60}
$$

where x_2^{**} denotes Firm 2's entry threshold (derived in the Appendix). V_1^m equals the monopolistic value of Firm 1 (as defined by (37)) adjusted for the component reflecting competitive entry. The latter component equals the value loss from switching from monopoly to duopoly multiplied by the probability-weighted discount factor corresponding to the random time of Firm 2's entry.

In the last step, we determine the value of Firm 1's investment opportunity. We already know that the valuation formulae for V differ across the two regimes (cf. (37)) and that it is never optimal for Firm 1 to invest in the first regime. Consequently, when applying the value-matching and smooth-pasting conditions, we have to use the expression corresponding to the second regime. A simple algebraic manipulation yields the following implicit formula for the optimal investment threshold of Firm 1, x_1^{**}

$$(\beta_2 - \beta_1) B_{M1} (x_1^{**})^{\beta_1} + \beta_2 (C_0 - I) + (\beta_2 - 0.5) C_1 (x_1^{**})^{0.5} + (\beta_2 - 1) C_2 x_1^{**} = 0. \tag{61}$$

A comparison of (61) with (46) leads to the observation that $x_1^{**} = x^{**}$. This is in line with the classic strategic real option models in which the roles of the firms (leader vs. follower) are determined exogenously and where the firms have a single control variable (investment timing). This finding can be explained by the fact that in our case the decision problem of the Firm 1 with one discrete control variable (timing) and with one continuous control variable (quality) can be transformed into the problem of a single discrete variable whereas the relevant payoff functions are at each moment optimized with respect to the continuous variable. Consequently, the value of Firm 1 is no longer a function of quality since this is chosen optimally given the realization of x_t and the choice of exogenous parameters.

The value of the investment opportunity of Firm 1, F_1, equals

$$\begin{aligned} F_1 &= \left(V(x_1^{**}) + \left(V_1^d (x_2^{**}) - V(x_2^{**}) \right) \left(\frac{x_1^{**}}{x_2^{**}} \right)^{\beta_2} - I \right) \left(\frac{x}{x_1^{**}} \right)^{\beta_2} \\ &\equiv A_1 x^{\beta_2}. \end{aligned} \tag{62}$$

It can immediately be noticed that $F_1 < F$ (cf. (47)) because of the present value of future revenues lost due to competitive entry, which is equal to

$$\left(V_1^d (x_2^{**}) - V(x_2^{**}) \right) \left(\frac{x_1^{**}}{x_2^{**}} \right)^{\beta_2}.$$

As soon as competitive entry becomes very remote, i.e. when $x_2^{**} \to \infty$, it holds that the problem reduces to the valuation of a monopolistic firm and $F_1 = F$.

5.1 Comparative Statics: Valuation of Firms

Analogous to Section 3, we are interested in the sensitivity of the firms' value with respect to changes of market parameters. Figure 5.2 depicts the relationship between the market volatility (left window) and network externalities (right window), and the value of the investment opportunities of both firms.

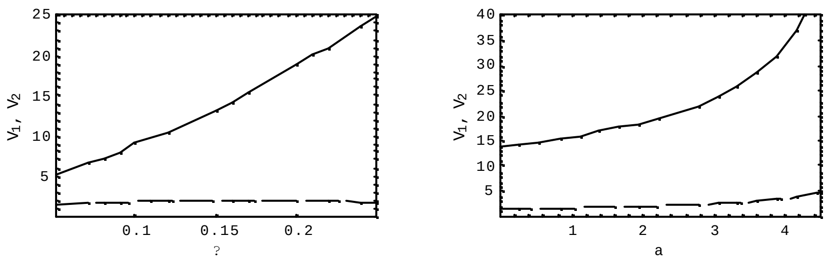

Figure 5.2

Figure 5.2. The value of the investment opportunity of Firm 1 (solid line) and Firm 2 (dashed line) for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (left window), $\sigma = 0.2$ (right window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, $x_0 = 4$, and $I = 10$.

Inspection of Figure 5.2 leads to two main conclusions First, contrary to the fixed quality case, the value of Firm 1's investment opportunity is higher than of Firm 2. Second, the value of Firm 2's project is non-monotonic in uncertainty (like the value of Firm 1 in the previous case). The first result is implied by the fact that Firm 1 is a leader in the investment game but, thanks to its flexibility with regard to quality choice, acts as a follower in the Stackelberg quality game. Consequently, Firm 1 not only receives cash flow from the project over a longer period but also is able to adjust its quality optimally given the quality choice of Firm 2, q_2, and the realization of the stochastic demand, x_t.

The non-monotonicity of Firm 2's value in uncertainty results from two factors. First, Firm 1 can exploit (relatively) more the changes in stochastic demand by changing its quality when uncertainty is high. Moreover, higher uncertainty affects the effective discount rates of the components of Firm 2's value that are concave in x_t. Consequently, the presence of such concavities leads to a lower valuation in a more uncertain environment. A positive relationship between Firm 2's value and uncertainty at the low levels of uncertainty can be explained by the traditional option argument that, in this case, dominates the strategic effects.

As far as the relationship between the degree of network externalities and the value of the firms is concerned, it resembles the picture of the fixed quality case. Again, the presence of the network externalities leads to an increase in the value of the investment opportunities of both firms and the rate of this increase is highest when the degree of network externalities approaches the cost of joining the network, k.

5.2 Comparative Statics: Firm 1's Strategic Choice of Variables

In the case in which quality is flexible, the following observations can be made. First, the optimal investment threshold in the presence of entry threat is identical to the level of x triggering the investment of the monopolist. This is due to the well-known fact that if the roles of the firms are predetermined and the only choice variable of the leader is the investment timing, future entry of the follower does not impact the investment timing of the leader. Second, upon examining (48), we can conclude that the quality chosen by Firm 1 does not change in a continuous way. In the following subsection, we present a short discussion of the properties of $q_1^{**}(x_t)$.

5.2.1 Properties of $q_1^{**}(x_t)$

The optimal quality choice, q_1^{**}, piecewise (weakly) increases in the state of the market, x_t. At φ and $\varkappa$ the quality exhibits discontinuities. Calculating the relevant limits yields

$$\lim_{x\downarrow\varphi} q_1^{**}(x) - \lim_{x\uparrow\varphi} q_1^{**}(x) = \rho q_2 + (1-\rho)k - a > 0, \tag{63}$$

$$\lim_{x\downarrow\{} q_1^{**}(x) - \lim_{x\uparrow\{} q_1^{**}(x) = (\overline{q}_1 - a)\left(\sqrt{\frac{k-a}{\underline{q}_1 - a}} - 1\right) < 0. \tag{64}$$

Realizations φ and $\varkappa$ are reversible switch points in which the functional form of the optimal quality changes. As pointed out by Mella-Barral and Perraudin [20], the function describing the optimal choice of a control variable is in general discontinuous in the switch points (see also Dumas [5]). Continuity is implied if the switch points are chosen optimally so as to maximize the value of the firm. Here, the switch points are not chosen optimally by Firm 1 but result from the change of the product market structure. From (48) it can be seen that for low x Firm 1 ceases operations as the revenues do not cover the operating costs. When x reaches φ from below, Firm 1 resumes operations and the resulting outcome is duopolistic. Finally, when x reaches $\varkappa$ Firm 1 covers the entire market and the monopoly prevails. Consequently, the discontinuity of $q_1^{**}(x)$ occurs at both φ and $\varkappa$.

A positive sign of (63) results from the fact that the quality of the idle firm equals a (cf. (48)), whereas resuming the operations requires the quality exceeding $k\,(> a)$. A negative sign of (64) can be explained as follows. At the moment x equals $\varkappa$ (cf. Figure 5.1), quality chosen by Firm 1 is that high that Firm 1 captures all customers. Hence, Firm 2 leaves the market after which Firm 1 reduces quality. It can do so since Firm 2 will not re-enter (unless x falls below $\varkappa$). Firm 2 knows that if it re-entered, Firm 1 would immediately raise quality to the optimal duopoly level..

6 Valuation Effects of Flexible vs. Fixed Quality

In this section we analyze the effects on the valuation of the flexible vs. fixed technology choice made by Firm 1. We address the following two related questions: *i*) what is the relationship between the loss in value due to the expected competitive entry (in comparison with monopoly) and the fixed or flexible quality choice, and *ii*) what is the impact of flexibility on the valuation with and without competitive entry threat. Figure 6.1 contains a comparison of the ratio of Firm 1's value in the monopoly vs. duopoly case for flexible and fixed quality choice.

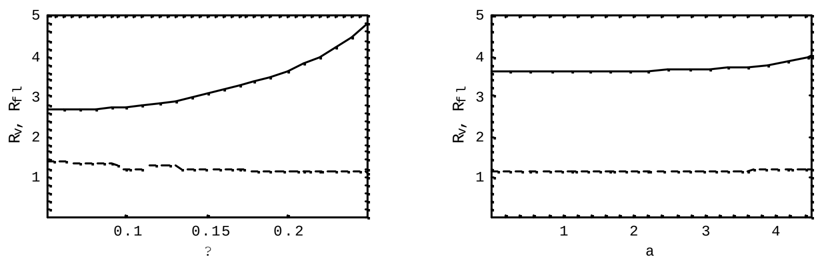

Figure 6.1

Figure 6.1. The relationship between the ratio of Firm 1's duopolistic to monopolistic value under fixed (solid line), R_v, and flexible (dashed line), R_{fl}, quality choice and market uncertainty (left window) and network externalities (right window) for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (left window), $\sigma = 0.2$ (right window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

On the basis of Figure 6.1 it can be concluded that the value lost due to competitive entry is much lower when quality is flexible (as opposed to fixed quality). This results from the fact that the flexible quality choice is associated with Firm 1's follower's role in the quality game played by the firms at each instant. The follower's advantage is stronger when the demand uncertainty is higher (see left window). Finally, we can observe that the degree of network externalities have little effect on the firms' relative valuation until they become very high in the fixed quality case. Then the fraction of Firm 1's value lost due to the competitive entry as compared to monopoly is even higher (cf. right window).

Now, let us analyze the impact of flexible quality choice on the firms' valuation from a slightly different angle. Instead of looking at the value lost due to competitive entry, we investigate the value impact of a switch from the fixed- to flexible-quality technology. Figure 6.2 depicts this effect as a function of demand uncertainty and network externalities.

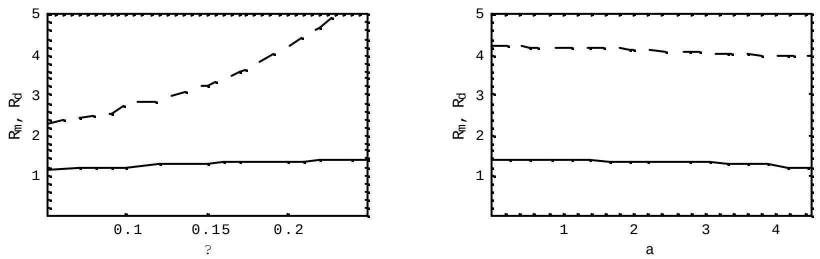

Figure 6.2

Figure 6.2. The relationship between the ratio of Firm 1's flexible to fixed technology value without (solid line), R_m, and with (dashed line), R_d, competitive entry threat and market uncertainty (left window) and network externalities (right window) for the parameter values $\rho = 0.5$, $k = 5$, $a = 2$ (left window), $\sigma = 0.2$ (right window), $c_0 = 1$, $r = 0.05$, $\alpha = 0.015$, and $I = 10$.

From Figure 6.2 we draw the following conclusions. First, the excess value of the flexible technology is higher in a strategic than in a monopolistic framework. Moreover, the strategic impact of flexibility is increasing in demand uncertainty (cf. left window). Whereas in the monopolistic framework the value gain occurring due to the flexible technology is moderate and does not increase sharply in σ, both the value gain and its sensitivity towards growing uncertainty are much more dramatic. Like previously, the value impact of network externalities is relatively small and affects the advantage of the flexible technology adversely.

7 Conclusions

In the paper we determine advantages of flexibility in quality choice of a firm considering an uncertain product market sector exhibiting network externalities. The firm is able to adjust quality over time when it possesses sufficient know-how, invented the technology itself, or adopted a more advanced technology. In general, this requires larger sunk costs and the aim of this paper is to determine in which cases it is particularly justified to incur these larger costs.

First, we derive the optimal investment threshold and the quality choice of the firm using the fixed-quality in both the monopolistic and duopolistic framework. Second, we repeat the analysis for the flexible technology choice. Finally, we perform a comparison of outcomes resulting from applying the two alternative technologies.

We show that the qualities chosen by the firms in the fixed-quality framework are strategic complements. This implies that a higher quality chosen by the market leader is associated with a higher quality provided by the second firm to enter. Moreover, the market leader uses the quality as a means to deter entry since its level of quality chosen under competitive entry threat is higher than in an isolated monopolistic market. Finally, since the firms play a version of a Stackelberg game in strategic complements, the value of the second firm to enter exceeds the one of the leader.

We also extend general results of strategic real option theory. From this theory it is known that if roles of the firms are exogenous or they sufficiently differ in characteristics, the follower's investment timing is irrelevant for the decision of the leader. However, due to the addition of a second control in the form of quality choice, the investment timing of the first investor is influenced by the decision of the other firm.

If the market leader is able to adjust quality over time, its optimal investment strategy is identical to the monopolistic case. This observation results from the fact that the loss due to the competitive entry equally affects the value of its investment opportunity before investing and the value of the project once the sunk cost is incurred. Moreover, the flexible quality choice of the leader implies three different market structures as functions of the underlying demand. When demand is low, only the second firm is active, moderate demand is associated with both firms serving the market, whereas high demand implies that the entire market is served by the leader.

A comparison of firms' values under two alternative technologies leads to further conclusions. It appears that the strategic value of the flexible (as opposed to fixed) technology is much higher than its value in an isolated monopoly. A related observation is that value loss from the competitive entry is much lower when the quality is flexible. Second, the value of flexible quality choice increases with uncertainty since an immediate quality adjustment to the changes in stochastic demand is possible. Moreover, the case of flexibility also allows for achieving the second-mover advantage in the Stackelberg game after the competitive entry. The latter result is amplified if the market uncertainty is high.

8 Appendix

Proof of Proposition 1. The optimal quality level is calculated by maximizing (9) with respect to q. The corresponding first-order condition is (dependence on q is dropped for the sake of transparency)

$$0 = \frac{(\beta_2 - 1)^{\beta_2 - 1}}{\beta_2^{\beta_2}} \frac{x^{\beta_2}}{(C+I)^{2\beta_2 - 2}} \times \left(\beta_2 R^{\beta_2 - 1} (C+I)^{\beta_2 - 1} R_q - (\beta_2 - 1) R^{\beta_2} (C+I)^{\beta_2 - 2} C_q\right), \tag{65}$$

for which it follows that

$$\beta_2 (C+I) R_q - (\beta_2 - 1) R C_q = 0. \tag{66}$$

Dividing by $(\beta_2 - 1) x^* R$ and observing that $\frac{\beta_2}{\beta_2 - 1} \frac{C+I}{x^* R} = 1$ yields the desired result. The corresponding second-order condition is

$$\left(\beta_2 (C+I) R_{qq} + C_q R_q - (\beta_2 - 1) C_{qq} R\right)\big|_{q=q^*} < 0. \tag{67}$$

This is a necessary and sufficient condition for the relevant functions which ensures that q^* corresponds to a local maximum and is formulated as Assumption 1. If (66) has multiple solutions satisfying (67), then the one corresponding to the highest value of (9) is chosen. ■

Proof of Proposition 2. We begin by defining (cf. (66))

$$H\left(q\right)=\beta_2\left(C\left(q\right)+I\right)R_q\left(q\right)-\left(\beta_2-1\right)C_q\left(q\right)R\left(q\right). \tag{68}$$

For q^* it holds that $H\left(q^*;\cdot\right)=0$. Therefore, the impact of a change in $\theta\in\{a,\sigma\}$ can be determined by applying the envelope theorem:

$$\frac{dq^*}{d\theta}=-\frac{H_\theta}{H_q}. \tag{69}$$

By Assumption 1 we know that

$$\left.\frac{\partial H\left(q\right)}{\partial q}\right|_{q=q^*}<0. \tag{70}$$

Consequently, from (69) and (70) it follows that (we drop the dependence of variables on q)

$$\operatorname{sgn}\left.\frac{\partial H}{\partial\theta}\right|_{q=q^*}=\operatorname{sgn}\left.\frac{dq}{d\theta}\right|_{q=q^*}\quad\text{for }\theta\in\{a,\sigma\}. \tag{71}$$

We have

$$\left.\frac{\partial H}{\partial\sigma}\right|_{q=q^*}=\frac{\partial\beta_2}{\partial\sigma}\left(\left(C+I\right)R_q-C_qR\right)>0, \tag{72}$$

$$\begin{aligned}\left.\frac{\partial H}{\partial\alpha}\right|_{q=q^*}&=\frac{\partial\beta_2}{\partial\alpha}\left(\left(C+I\right)R_q-C_qR\right)\\&\quad+\beta_2\left(C+I\right)R_{q\alpha}-\left(\beta_2-1\right)C_qR_\alpha\\&=\frac{\partial\beta_2}{\partial\alpha}\left(\left(C+I\right)R_q-C_qR\right)>0.\end{aligned} \tag{73}$$

■

Proof of Proposition 3. Result (16) follows immediately from (15). Repeating (15), we have

$$\frac{dx^*\left(q\right)}{d\theta}=\frac{\partial x^*\left(q\right)}{\partial\theta}+\frac{\partial x^*\left(q\right)}{\partial q}\frac{dq}{d\theta},\ \theta\in\{a,\sigma\}, \tag{74}$$

and we are interested in the signs of the components of (15). We know that

$$\frac{\partial x^*\left(q\right)}{\partial q}=\frac{\beta_2}{\beta_2-1}\frac{C_qR-\left(C+I\right)R_q}{R^2}. \tag{75}$$

Consequently, in the optimum

$$
\begin{aligned}
\left.\frac{\partial x^*(q)}{\partial q}\right|_{q=q^*} &= \frac{\beta_2}{\beta_2-1}\frac{C_q R-(C+I)R_q}{R^2} \\
&> \frac{\beta_2}{(\beta_2-1)^2}\frac{(\beta_2-1)C_q R-\beta_2(C+I)R_q}{R^2} \\
&= 0. \qquad (76)
\end{aligned}
$$

The last equality directly results from (66). Moreover, by differentiating (7), we immediately obtain that

$$\frac{\partial x^*(q)}{\partial \sigma} > 0. \qquad (77)$$

What we still have to establish is the sign of $\frac{dq}{d\sigma}$. Using the results of Proposition 2 we obtain

$$\frac{dq^*}{d\sigma} > 0. \qquad (78)$$

This completes the proof. ■

Proof of Proposition 4. The sign of derivative dq_2^*/dq_1^* immediately follows from (26) and the argument thereafter. In order to determine the sign of dx_2^*/dq_1^*, we first express x_2^* as

$$x_2^* = \frac{\beta_2}{\beta_2-1}\frac{I(1-\rho^2)r+c_0\left(q_2-\underline{q}_2\right)}{r(r-\alpha)^{-1}}\frac{q_2-a}{q_2-\underline{q}_2}. \qquad (79)$$

Subsequently, we show that two last factors of (79) increase with $\underline{q}_2$. We already know that $\underline{q}_2$ is an increasing function of q_1. Consequently, we derive expressions for $q_2-\underline{q}_2$ and q_2-a on the basis of (26):

$$q_2-\underline{q}_2 = \frac{1}{2(\beta_2-1)}\times \qquad (80)$$

$$\left[\underline{q}_2-a+\sqrt{\underline{q}_2-a}\sqrt{\underline{q}_2-a+4\beta_2 Ir(\beta-1)(1-\rho^2)c_0^{-1}}\right],$$

$$q_2-a = \frac{1}{2(\beta_2-1)}\times \qquad (81)$$

$$\left[(2\beta-1)\left(\underline{q}_2-a\right)+\sqrt{\underline{q}_2-a}\sqrt{\underline{q}_2-a+4\beta_2 Ir(\beta-1)(1-\rho^2)c_0^{-1}}\right].$$

By inspecting (80) we immediately conclude that the second factor of (79) is increasing in $\underline{q}_2$. Now, we concentrate on the derivative of the ratio $\frac{q_2-a}{q_2-\underline{q}_2}$. It can be written (using (80) and (81)) as

$$\frac{d}{d\underline{q}_2}\left(\frac{q_2-a}{q_2-\underline{q}_2}\right) = \frac{d}{d\underline{q}_2}\left(\frac{(2\beta_2-1)\left(\underline{q}_2-a\right)+f\left(\underline{q}_2\right)}{\underline{q}_2-a+f\left(\underline{q}_2\right)}\right), \qquad (82)$$

where

$$f\left(\underline{q}_2\right) = \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + K}.$$

and $K = 4\beta_2 Ir\left(\beta_2 - 1\right)\left(1-\rho^2\right)c_0^{-1} > 0$. Now, (82) can be expressed as

$$\frac{d}{d\underline{q}_2}\left(\frac{q_2 - a}{q_2 - \underline{q}_2}\right) = \frac{2\left(\beta_2 - 1\right)\left(f\left(\underline{q}_2\right) - f'\left(\underline{q}_2\right)\left(\underline{q}_2 - a\right)\right)}{\left(\underline{q}_2 - a + f\left(\underline{q}_2\right)\right)^2}.$$

In the final step, we determine the sign of the second factor in the numerator

$$\begin{aligned} & f\left(\underline{q}_2\right) - f'\left(\underline{q}_2\right)\left(\underline{q}_2 - a\right) = \\ & = \sqrt{\underline{q}_2 - a}\sqrt{\underline{q}_2 - a + K} - \frac{\left(2\underline{q}_2 - 2a + K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} = \\ & = \frac{\left(2\underline{q}_2 - 2a + 2K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} - \frac{\left(2\underline{q}_2 - 2a + K\right)\sqrt{\underline{q}_2 - a}}{2\sqrt{\underline{q}_2 - a + K}} > 0. \end{aligned}$$

This completes the proof. ■

Proof of Lemma 5. The lemma can be proven by analyzing the profit functions of the firms in a duopoly and two cases of a monopoly. Profit maximization based on the system of demands (17) with the optimal quality schedule of Firm 1 (48) yields the following Stackelberg profits of Firm 1 and Firm 2, denoted by π_1 and π_2, respectively:

$$\pi_1 = \frac{1}{1-\rho^2}\left(\sqrt{x} - \sqrt{\varphi}\right)^2, \tag{83}$$

$$\pi_2 = \frac{1}{1-\rho^2}\left(\frac{-\rho\sqrt{\varphi}}{\kappa}x^{1.5} + \frac{c_0\psi}{\kappa}x + \rho\sqrt{\varphi}x^{0.5} - c_0\psi\right). \tag{84}$$

Here, φ, ψ and κ are functions of q_2, which is chosen at the beginning of the game (the quality chosen by Firm 2 is fixed at the moment of undertaking investment). Since π_2 is concave and decreasing for sufficiently large x, it can be shown that for $x > \varkappa$ it holds that $\pi_2 = n_2 = 0$. In the same fashion in can be shown that $\pi_1 = 0$ for $x < \varphi$. What remains to be proven is that $\varphi < \varkappa$. It can be seen upon manipulating (49) and (50) that

$$\varkappa - \varphi = c_0\frac{\left(\overline{q}_1 - a\right)^2 - \left(\underline{q}_1 - a\right)^2}{\left(\underline{q}_1 - a\right)} > 0 \Leftrightarrow \overline{q}_1 - \underline{q}_1 > 0.$$

The latter inequality is proven directly by observing that

$$\begin{aligned} \overline{q}_i - \underline{q}_i = 0 \text{ for } \rho &= 1, \text{ and} \\ \frac{\partial\left(\overline{q}_i - \underline{q}_i\right)}{\partial\rho} &< 0. \end{aligned}$$

This completes the proof. ■

Derivation of Firm 2's optimal investment threshold. First, we derive the value of the Firm 2. Denote the value of the Firm 2 after entering the market, by V_2. V_2 satisfies the following Bellman equation

$$0.5\sigma^2 x_t^2 V_2'' + \alpha x V_2' + \pi_2 = rV_2, \tag{85}$$

where

$$\pi_2 = \begin{cases} \frac{c_0\psi - \rho\varphi}{(1-\rho^2)\kappa}(x-\kappa) & \text{for} \quad x < \varphi, \\ \frac{1}{1-\rho^2}\left(\frac{-\rho\sqrt{\varphi}}{\kappa}x^{1.5} + \frac{c_0\psi}{\kappa}x + \rho\sqrt{\varphi}x^{0.5} - c_0\psi\right) & \text{for} \quad \varphi < x < \varkappa, \\ 0 & \text{for} \quad x > \varkappa, \end{cases} \tag{86}$$

and

$$\kappa = c_0 (q_2 - a). \tag{87}$$

The value of (86) for $x > \varkappa$ is zero since for high demand, Firm 1 captures the entire market share (cf. Lemma 5). For the result corresponds to (84), whereas for $x < \varphi$ Firm 2 achieves monopoly profit (cf. (22)) since Firm 1 remains idle. Solving (85) with the value matching and smooth pasting conditions satisfied for realizations φ and $\varkappa$ yields

$$V_2 = \begin{cases} (B_2 + B_4)x^{\beta_2} + C_0^M + C_2^M x & \text{for} \quad x < \varphi, \\ B_1 x^{\beta_1} + B_2 x^{\beta_2} + C_0^D + C_1^D x^{0.5} + C_2^D x + C_3^D x^{1.5} & \text{for} \quad \varphi < x < \varkappa, \\ (B_1 + B_3)x^{\beta_1} & \text{for} \quad x > \varkappa, \end{cases}$$

where

$$
\begin{aligned}
B_1 &\equiv C_3^D \frac{\varphi^{1.5-\beta_1}(\beta_2-1.5)}{\beta_1-\beta_2} + C_2^D \frac{\varphi^{1-\beta_1}(\beta_2-1)}{\beta_1-\beta_2}\frac{\rho\varphi}{c_0\psi} \\
&\quad + C_1^D \frac{\varphi^{0.5-\beta_1}(\beta_2-0.5)}{\beta_1-\beta_2} + C_0^D \frac{\varphi^{-\beta_1}\beta_2}{\beta_1-\beta_2}\frac{\rho\varphi}{c_0\psi} \\
B_2 &\equiv -C_3^D \frac{\varkappa^{1.5-\beta_2}(\beta_1-1.5)}{\beta_1-\beta_2} - C_2^D \frac{\varkappa^{1-\beta_2}(\beta_1-1)}{\beta_1-\beta_2} \\
&\quad - C_1^D \frac{\varkappa^{0.5-\beta_2}(\beta_1-0.5)}{\beta_1-\beta_2} - C_0^D \frac{\varphi^{-\beta_2}\beta_1}{\beta_1-\beta_2} \\
B_3 &\equiv -C_3^D \frac{\varkappa^{1.5-\beta_1}(\beta_2-1.5)}{\beta_1-\beta_2} - C_2^D \frac{\varkappa^{1-\beta_1}(\beta_2-1)}{\beta_1-\beta_2} \\
&\quad - C_1^D \frac{\varkappa^{0.5-\beta_1}(\beta_2-0.5)}{\beta_1-\beta_2} - C_0^D \frac{\varkappa^{-\beta_1}\beta_2}{\beta_1-\beta_2} \\
B_4 &\equiv C_3^D \frac{\varphi^{1.5-\beta_2}(\beta_1-1.5)}{\beta_1-\beta_2} + C_2^D \frac{\varphi^{1-\beta_2}(\beta_1-1)}{\beta_1-\beta_2}\frac{\rho\varphi}{c_0\psi} \\
&\quad + C_1^D \frac{\varphi^{0.5-\beta_2}(\beta_1-0.5)}{\beta_1-\beta_2} + C_0^D \frac{\varphi^{-\beta_2}\beta_1}{\beta_1-\beta_2}\frac{\rho\varphi}{c_0\psi} \\
C_0^M &\equiv \frac{-(c_0\psi-\rho\varphi)c_0 a}{r} \\
C_2^M &\equiv \frac{c_0\psi-\rho\varphi}{r-\alpha} \\
C_0^D &\equiv \frac{-1}{1-\rho^2}\frac{c_0\psi}{r} \\
C_1^D &\equiv \frac{1}{1-\rho^2}\frac{\alpha\sqrt{\varphi}}{r-0.5\alpha+0.125\sigma^2} \\
C_2^D &\equiv \frac{1}{\kappa(1-\rho^2)}\frac{c_0\psi}{r-\alpha} \\
C_3^D &\equiv \frac{-1}{\kappa(1-\rho^2)}\frac{\alpha\sqrt{\varphi}}{r-1.5\alpha-0.375\sigma^2}
\end{aligned}
$$

Despite the fact that the expressions for the value of Firm 2 differ across the regimes, calculating the option value of the investment opportunity of Firm 2 represents no additional difficulty comparing to the traditional analysis. It can be shown that the value is negative under the first regime, reaches the peak under the second regime and tends asymptotically to zero under the third regime. Therefore, it cannot be optimal for Firm 2 to invest under regimes one and three. Here, I is assumed to be not excessively high so that such x_t exists for which the net present value of Firm 2's investment is positive. Consequently, the value of Firm's 2 option to invest can be calculated on the basis of the value-matching and smooth-pasting conditions corresponding to the second regime

$$\begin{aligned} A_2 x^{\beta_2} &= B_1 x^{\beta_1} + B_2 x^{\beta_2} + C_0 + C_1 x^{0.5} + C_2 x + C_3 x^{1.5} - I \\ \beta_2 A_2 x^{\beta_2 - 1} &= \beta_1 B_1 x^{\beta_1 - 1} + \beta_2 B_2 x^{\beta_2 - 1} + 0.5 C_1 x^{-0.5} + C_2 + 1.5 C_3 x^{0.5}. \end{aligned}$$

The optimal investment threshold of Firm 2, x_2^{**}, is implicitly defined by

$$\begin{aligned} &(\beta_2 - \beta_1) B_1 (x_2^{**})^{\beta_1} + \beta_2 (C_0 - I) + (\beta_2 - 0.5) C_1 (x_2^{**})^{0.5} \\ &+ (\beta_2 - 0.5) C_2 x_2^{**} x + (\beta_2 - 0.5) C_3 (x_2^{**})^{1.5} = 0 \end{aligned}$$

Moreover, the value of the investment opportunity of Firm 2 is

$$F_2 = A_2 x^{\beta_2},$$

where

$$A_2 \equiv \max_{q_2} \frac{B_1 (x_2^{**})^{\beta_1} + B_2 (x_2^{**})^{\beta_2} + C_0 + C_1 (x_2^{**})^{0.5} + C_2 x_2^{**} + C_3 (x_2^{**})^{1.5} - I}{(x_2^{**})^{\beta_2}}.$$

■

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