# The Timing of Initial Public Offerings: A Real Option Approach

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**Abstract**

This paper analyzes the timing of IPOs by treating the going-public decision as a real option. Investors value the private firm using publicly observed market prices of firms from the same industry. With stochastically evolving market values firm insiders want to leave open the option of taking the firm public later, after a positive price shock. Going public exercises this option, which must be viewed as a cost of undertaking an IPO. Optimal exercising of the option dictates that IPOs should only occur after price run-ups. Firms never go public in a down market because the value of waiting is too great. These incentives can lead to clustering of IPOs near market peaks. The results generalize to seasoned issues. Each equity offering is for a unique fraction of the total ownership claim for the firm and is associated with its own timing option. Each equity issue is independent of all the others, and therefore the timing of an offering is unaffected by subsequent issues. Each equity issue is likely to follow an abnormal price increase.

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# 1. Introduction

The recent frenzy over Internet-related IPOs brought wide-spread public attention to the market for new issues. This was a direct consequence of the phenomenal increase in the valuation of these companies prior to and during the frenzy. More recently the share prices of these firms have fallen back considerably from their highs, as have the number of new issues. The Internet industry is not alone in its experience.[1] IPOs in general have been found to occur after abnormal run-ups in their industry market index.[2] Further evidence suggests that IPOs, on average, perform poorly post-issue. Combined, these two observations have led researchers to tentatively conclude that firm insiders are able to exploit their superior information to time the market by issuing during a temporary "window of opportunity".

Two assumptions are implicit in the claim that firm insiders are able to successfully time the market. One, firms actually have the flexibility of deciding when to go public and, two, investors use the free public information of market valuations for similar firms to price the private firm. The validity of these two assumptions is critical to the market timing claim. Yet while their reasonableness is not in question, no formal analysis has been undertaken to discern what impact they should have on the timing of IPOs.

The objectives of this paper are two-fold. The first is to develop a dynamic model for the timing of IPOs that relies exclusively on the two aforementioned assumptions. Asymmetric information between the firm and investors is ruled out to isolate the role played by timing flexibility and market valuations on the timing of IPOs. The optimal timing policy is the solution to the firm's decision problem, which involves no strategic interaction. From this framework predictions for when IPOs might occur, and the market conditions likely to precede the offering, are possible. The second objective is to use the optimal timing decision for an individual firm to propose an explanation for the clustering of IPOs near peaks in the industry market index.

The going-public decision is modeled as a real option. An entrepreneur can decide at any date to take the firm public, but doing so exercises the timing option. Investors value the firm conditional on both public and firm-specific in-

[1] 'Hot Issue' markets for IPOs have been occuring for at least the last 40 years. The first to document this phenomenon was Ibbotson (1975) and Ibbotson and Jaffe (1975).

[2] The empirical evidence on the performance of the industry prior to the IPO is discussed later in the introduction.

formation. In an efficient market all public information is embodied in the market prices of similar firms. The proceeds from an IPO, and therefore its value to an entrepreneur, will depend on market conditions. Over time the set of public information evolves randomly. This will in turn lead to a stochastic path for the valuation of the private firm. The uncertainty over the future value makes the timing option valuable because the entrepreneur can always wait for improved market conditions before going public.

Multiple factors can influence the decision to take a firm public, far more than can be incorporated in any one model.[3] In the absence of any strategic issues the decision to go public is driven by the difference between the public and private valuations of the firm. The entrepreneur is assumed to have a higher discount rate than the market, and therefore value the firm less. The firm will go public when the proceeds from the IPO exceed the value of remaining private. When the firm is private the entrepreneur has claim to the dividend stream and possesses the timing option to go public at any time. By going public this option is exercised and its value to the entrepreneur is lost. Consequently the option value must be considered a cost of going public. IPOs will occur only when the market valuation is sufficiently larger than the private equity valuation to cover this additional cost.

The implication of including the timing option in the going-public decision on the likely market conditions prior to the IPO is immediate. The entrepreneur recognizes that market valuations fluctuate randomly through time and is willing to wait for positive price shocks before taking the firm public. As a consequnce of optimally exercising the timing option IPOs should occur after abnormal price increases. The option value increases with the volatility of the industry index, implying price run-ups prior to IPOs should be greater in high volatility industries. If the firm has not already gone public and the market value turns down the entrepreneur will wait because the timing option is too valuable. The clustering of IPOs near market valuation peaks could simply be the result of firms waiting for a price run-up and not issuing when prices fall. This result does not rely on the entrepreneur exploiting asymmetric information by issuing during a temporary window of opportunity, but simply exploiting the fact that market values evolve stochastically.

[3]Suggested reasons for an IPO include diversifying the portfolio of firm insiders; gaining access to a larger pool of capital; acquiring greater bargaining power with banks (Rajan (1992)); achieving better monitoring and incentives through the market for corporate control (Pagano and Roell (1998)); raising firm value by increasing investor recognition (Merton (1987)); increasing the liquidity in the market for shares in the firm; preparing for a change of control (Zingales (1995)); exploiting windows of opportunity when valuations are too high (Ritter (1991)).

The timing of seasoned equity issues can also be modeled as a real option. Each equity issue is a sale of a unique subset of the total number of shares in the firm. The entrepreneur has only one opportunity to sell a particular subset, which is associated with its own timing option. The option to conduct a seasoned offering is only acquired once the firm exercises the option to go public. The set of shares in the two offerings are independent, as are the two options. Consequently, the timing of an equity issue is unaffected by the possibility of subsequent issues. Price run-ups, either in the industry index or the firm's own share price, should precede any equity offering because of the loss in value from exercising the timing option for those shares issued. If the entrepreneur anticipates selling some fixed, total number of shares it is always optimal to issue them all at once in the IPO, as opposed to multiple equity issues, to minimize the issue costs.

A generalization of the model to allow the entrepreneur to withdraw the offering does not alter any of the conclusions. From the initial preparation to the actual offering the IPO process will take time. During this period market valuations can fall from their earlier highs making an offering much less desirable. Firms pay significant fixed costs to prepare for the IPO. Along with a loss to their reputation resulting from withdrawing the offering firms have a strong incentive to go public, even when the market turns averse. This extension allows for the possibility of firms going public after the market index has started to fall. Firms that have not started the IPO process will continue to wait. The timing of the IPO is largely unaffected because the entrepreneur attaches low value to the option to withdraw.

### 1.1. Empirical Evidence and Related Literature

A number of papers have documented that IPOs occur following price run-ups. A study by Lerner (1994) of the decision by venture capitalists to take biotechnology firms public found that in the 60 trading days immediately prior to the IPO date the industry index raw return was 9.9 percent. Using a sample of Italian firms Pagano, Panetta and Zingales (1998) showed that high market valuations of similar firms had the most significant impact on the probability of a firm undertaking an IPO. A one standard deviation increase in the industry market-to-book ratio raised the odds of an IPO by 25 percent. Chen and Hansen (2000) documented that industry abnormal returns reached a peak one year before the IPO before falling or remaining flat post-issue. Rajan and Servaes (1995) found that IPOs usually occur near the peaks in valuations of firms from the same industry. Similar evidence was found for seasoned equity offerings. Using a sample of 3702 seasoned offerings from 1970 to 1990 Loughran and Ritter (1995) found that in the year prior to the issue the average firm experienced a total return of 72 percent.[4]

The timing of IPOs has been explored in other recent papers. Chemmanur and Fulghieri (1999) compare the benefits of public versus private financing. They suggest firms will go public when the cost arising from the duplication of information production by public investors is less than the risk premium demanded by venture capitalists. Benveniste, Busaba, and Wilhelm (1997) examine a setting where firm insiders must learn about the value from outsiders. Firms that go public later can free ride on the information produced by early issuing firms, creating an incentive to delay the offering. Maksimovic and Pichler (2000) explore the timing issue of firms in technologically changing industries. Firms that go public early can invest first in productive activities, but this produces information that later arrivals to the industry can utilize for their own investment. The trade-off between these two factors will determine when the firm goes public.[5]

In all three papers information externalities play a critical role in both the timing for an individual IPO and the clustering of IPOs in a hot market. In the current paper information externalities also factor in by assuming market indexes can be used to value private firms. The distinction here is that the information production process is exogenous to the firm and its issuing decision. Further, the notion of timing refers to when the firm will issue relative to the general market conditions, and not at what stage in its life the firm will go public.

The rest of the paper proceeds as follows. Section 2 describes the basic structure of the model. The timing problem is examined in section 3. The maximization problem for the entrepreneur is defined. The value of the timing option and the critical value for an IPO are derived. A discussion of comparative static results ensues along with simple numerical solutions for the model. Section 4 extends the model to allow for seasoned equity issues. The affect of allowing firms to withdraw the IPO is considered in Section 5. A discussion of the results and possible extensions follows in section 6. Section 7 concludes.

[4]Loughran and Ritter report that approximately half of the 72 percent was due to market run-ups and half is due to issuers outperforming the market.

[5]Another recent paper by Subrahmanyam and Titman (1999) explored linkages between financial markets and the going-public decision. Public financing is desirable if costly information is diverse and investors acquire it without cost. They conclude that large public markets meet this criteria by having market prices aggregate disparate information, and this will induce more public offerings. The current paper implicitly assumes these conditions hold as investors use market indexes to value private firms.

## 2. The Model

A firm is started by a risk averse entrepreneur at time $t = 0$. Time evolves continuously. A start-up cost $I_0$ is required at the firm's inception. The funds are provided by the entrepreneur. The firm then generates an instantaneous profit flow $\pi_t^p$ with initial profit level $\pi_0^p$. A superscript $p$ on any variable or parameter indicates it is specific to the private entrepreneur. The value of the firm is the expected present discounted value of this profit stream.

At any time after $t = 0$ the entrepreneur can sell an equity stake in the firm by taking it public. Aside from the start-up cost no further investment, beyond a simple operating cost implicit in $\pi_t^p$, is necessary. The potential equity issue is not motivated by capital requirements. The fraction $0 < \alpha < 1$ of the shares to be sold to public investors is exogenously determined.[6] A sale of equity to outside investors does not introduce any distortions that affect the profit flow.

The entrepreneur's decision to take the firm public is determined by a comparison of the private value of the shares with the expected proceeds from the IPO. Both the public and private values are determined by the information available to each about the firm's profit flow, the uncertainty over future profits, and the discount rates of the entrepreneur and investors.

### 2.1. Information Structure

Calculation of the firm value requires knowledge of the current and future profits. When the firm is private investors can not observe its profit, only the entrepreneur knows $\pi_t^p$. The future profit flow will depend on two factors: one, the unique characteristics of the firm; two, the effect of external factors including, but not limited to, the likely growth in demand for the product, the potential threat of competition, and the development of new products by competitors.

[6] The actual choice of $\alpha$ can be determined endogenously. Following Leland and Pyle (1977) the entrepreneur might try to signal the firm's type by the fraction of the shares he retains following the offering. The entrepreneur of a good firm can signal the type by retaining a larger fraction of the equity, or equivalently choosing a small $\alpha$. His willingness to do this stems from the fact that for a given level of risk he receives a higher profit flow from the firm than does the entrepreneur of a bad firm. The fraction $\alpha$ can also be influenced by the risk aversion of the entrepreneur. Independent of the firm type an entrepreneur with greater risk aversion will retain less of the firm's shares, increasing $\alpha$. The choice of $\alpha$ might also affected by control considerations. Zingales (1995) argued that by strategically selling a specific fraction of the firm in an IPO the entrepreneur can extract higher proceeds in a subsequent sale of the firm to a third party.

Firms from the same industry will be affected by the same external factors listed above and should have profits that are highly correlated with the private firm. By examining the profits of publicly traded firms from that industry investors will have some initial information about the private firm's profit level. To minimize the effect of individual firm shocks an industry average profit is calculated, equal to $\pi_t$.

The private information of the entrepreneur consists of the true type of the firm, which is fixed. For simplicity the firm can be either good or bad. The profit level at any $t$ will be a multiplicative constant times the average industry profit $\pi_t$. Formally,

$$\pi_t^p \in \{l\pi_t, h\pi_t\}, \tag{2.1}$$

where

$$0 < l < 1 < h < \infty \tag{2.2}$$

and $(h + l)/2 = 1$. The firm profits are perfectly correlated with the industry profit.[7] The unique firm profits are also symmetrically distributed around the industry average. Assuming an equal proportion of good and bad firms the average profit will be $\pi_t$.

When the firm goes public the entrepreneur must provide investors with information about the financial status of the company in the prospectus. This information is a signal investors will use to update their beliefs about the firm. The signal investors receive when the firm goes public is for the current profit level. To eliminate any potential strategic interaction between the entrepreneur and investors the signal is assumed to be perfectly revealing. This is equivalent to assuming that investors know ex ante the type of the firm. The optimal timing of the IPO is then strictly a decision problem for the entrepreneur.[8]

To value the firm current and expected future profits are needed. The entrepreneur and investors share the same uncertainty over future profits. Both the

[7] This assumption can be relaxed to allow the firm-specific value to vary over time as well. The firm profit level will then be less than perfectly correlated with the industry profits.

[8] There is no loss to the main intuition of the paper from assuming perfectly revealing signals. If, instead, that was not the case it would be necessary to determine whether a pooling or separating equilibrium for the issue dates of the two types existed. If the types separate out investors learn their true value, which is equivalent to having perfectly revealing signals. The possibility of a pooling equilibrium will be contingent on the signal structure assumed. If a pooling equilibrium exists it will only change the profit level at which the firm goes public, but not the relative importance of waiting for favorable market conditions. By abstracting from the signalliing game the model can isolate specifically the effect the real option has on the timing of an IPO.

firm and industry profits are expected to grow at the instantaneous rate $\mu$. The actual realization of the profits will be affected by random disturbances in the market. Any unexpected developments in the external factors discussed earlier (shocks to demand, new entrants, and technological development) will be publicly observable to all and affect the profits of both the private and public firms in equal proportions. The stochastic evolution of the industry profits will follow the geometric Brownian motion:

$$\frac{d\pi_t}{\pi_t} = \mu dt + \sigma dz, \tag{2.3}$$

where $z$ is the standard Brownian motion.[9] Equation (2.3) applies to either type of firm, which can be seen by substituting in $h\pi_t$ or $l\pi_t$ for $\pi_t$. The private firm profit level, not the industry profits, will determine when the firm goes public. Any reference to the profit flow will apply specifically to the firm profits $\pi_t^p$. For notational convenience the superscript $p$ on the profit will be dropped.

Conditional on an initial profit level $\pi_0$ investors will know the profit level $\pi_t$ for the private firm for any $t$ simply by using (2.3) over the period 0 to $t$. In the case of no uncertainty, $\sigma = 0$, the profit level at $t$ will be

$$\pi_t = \pi_0 e^{\mu t}. \tag{2.4}$$

With uncertainty the conditional expectation for the time $t$ value of $\pi$ is:

$$E[\pi_t | \pi_0] = \pi_0 e^{\mu t}. \tag{2.5}$$

The effect of new public information, implicit in $\sigma dz$, on the valuations will depend on the size of $\sigma$. In emerging industries new information will have a significant affect on market values and should be associated with a large $\sigma$. The opposite is likely to hold for firms in mature industries. This distinction will be relevant when making predictions about the timing of IPOs.

[9] To eliminate the perfect correlation between the firm and industry profits an additional term can be added to (2.3). A firm-specific shock $\sigma_f dz_f$ can capture firm profit fluctuations around the industry average. This is the continuous type analog to the discrete type space assumed above. Again it would be assumed that the profit signal perfectly reveals the firm type. The inclusion of this term will produce value associated with waiting before going public because of potential firm-specific profit shocks.

## 2.2. Risk and Valuation

The public equity market is composed of identical risk averse investors, each of whom has access to the same information. Public investors are well diversified and discount future cash flows at the appropriate risk-adjusted rate $\rho^m$, which is exogenously specified. The entrepreneur has all his wealth concentrated in the firm and can not access financial markets. He must bear the cost of industry-wide idosyncratic risk. As a result of this additional risk he will discount the firm profits at rate $\rho^p > \rho^m > \mu > r$, where $r$ is the risk free rate.[10]

All profits will be paid out as dividends to the shareholders. The expected value of the firm can be computed using either discount rate. Conditional on $\pi_t$ the value is[11]

$$v^i(\pi_t) = E\left[\int_t^{\infty} \pi_s e^{-\rho^i(s-t)} ds \,|\pi_t\right] = \frac{\pi_t}{\rho^i - \mu}, \qquad i \in \{m, p\}. \tag{2.6}$$

Define $\delta^i = \rho^i - \mu$ and substituting equals

$$v^i(\pi_t) = \frac{\pi_t}{\delta^i}. \tag{2.7}$$

The economic interpretation of $\delta$ is that it is equivalent to an implicit dividend yield from investing in the firm. Financial market equilibrium requires that the return from investing in the firm, the profit flow and capital gain, equal the required rate of return $\rho^m$.

To solve the timing problem the entrepreneur will compare the valuation of the firm by the market with his own valuation. The market valuation for the firm

[10] The assumption of a higher private discount rate can be justified on both empirical and theoretical grounds. Sahlman (1990) found that when venture capitalists value a company they apply discount rates that range from 60 percent for initial start-ups to 25 percent for firms receiving a third or fourth round of financing. Using a model of capital market equilibrium, in which each investor is only aware of a subset of all securities, Merton (1987) showed that increasing the relative size of the firm's investor base will reduce the firm's cost of capital and increase the market value of the firm. Taking the firm public and increasing its marketability should also increase its value. Merton also shows that a larger firm-specific variance will lead to a larger cost of capital. Along similar lines Mauer and Senbet (1992) suggest that the combination of incomplete spanning of the firm's risk and a restricted set of investors will lead to lower private valuations.

[11] This calculation is done by first defining $y = \ln \pi$, and deriving the Brownian motion for $y$ to be $dy = (\mu - \frac{1}{2}\sigma^2)dt + \sigma dz$. Taking the expectation of $\pi$ is equivalent to calculating $E[e^y|y_0 = y]$, which will equal $\exp y\left((\mu - \frac{1}{2}\sigma^2) + \frac{1}{2}\sigma^2\right)$. Discounting this last expression with $e^{-\rho t}$ produces equation (2.6), with $\pi$ substituted back in for $e^y$.

is

$$v_t^m(\pi_t) = \frac{\pi_t}{\delta^m}, \tag{2.8}$$

and the entrepreneur's private valuation is

$$v_t^p(\pi_t) = \frac{\pi_t}{\delta^p}. \tag{2.9}$$

The market valuation will always be higher than that of the entrepreneur because $\delta^m < \delta^p$.[12]

The dynamics for the firm value can be derived from the profit flow. Applying Ito's Lemma to the relation in (2.7) gives

$$dv^i(\pi_t) = v_t^{i\prime} d\pi_t + \frac{1}{2} v_t^{i\prime\prime} (d\pi_t)^2 \tag{2.10}$$

$$= \frac{1}{\delta^i}(\mu\pi_t dt + \sigma\pi_t dz). \tag{2.11}$$

Substituting in for $\pi_t = v_t^i \delta^i$ yields

$$\frac{dv_t^i}{v_t^i} = \mu dt + \sigma dz. \tag{2.12}$$

The value of the firm, for both the entrepreneur and investors, has the same stochastic properties as the profit flow.

The uncertainty over future profits arises strictly from changing market conditions. The fixed value of $h$ or $l$ rules out firm-specific shocks. The uncertainty in the market conditions, represented by $\sigma dz$, can be effectively decomposed into two parts, market risk and industry-specific shocks. While investors diversify away the idosyncratic industry risk and price only the systematic risk of the firm, the entrepreneur must bear both types.

## 3. Timing Decision

The entrepreneur's only action is to decide when to take the firm public. Waiting for the optimal time to issue makes the decision an optimal stopping problem.

[12] The lower private valuation relative to the market value could also arise for liquidity reasons. A study by Williamette Mangement Associates (see Pratt 1989) examined the discounts of private share transaction prices relative to the IPO price during the three years prior to the IPO. It reported discounts ranging from 60 to 80 percent, adjusted for industry price changes, over the years 1975 to 1985. Blackwell and Pavlik (1996) found similar results for IPOs during 1989-1990, with private market prices on average 75 percent lower than the public market prices.

A necessary assumption to treat the going-public decision in this manner is that the IPO is irreversible. While such a constraint is not true in practice, as public firms are taken private, the option of taking the firm private again has relatively low value to the entrepreneur and should not affect the timing decision.[13] The optimal time is derived under the assumption that the IPO can be completed instantaneously.

The decision to go public is determined by a comparison of the proceeds from the IPO with the value of the shares if held privately. The timing decision is resolved when the IPO is worth more to the enterpreneur than retaining the shares.[14] To illustrate how the optimal timing policy is determined a two step procedure is followed. The timing of the IPO is first found in the deterministic case. Next, calculation of the timing option value and critical value for the IPO is done with stochastic profit flows. Before proceding the maximization problem of the entrepreneur is defined.

### 3.1. The Entrepreneur's Decision Problem

The objective of the entrepreneur is to maximize the present value of the cash flows he expects to receive from the $\alpha$ shares. The cash flows come from the dividend stream produced by the shares when the firm is still private and the proceeds from the IPO when the firm finally goes public. The timing of the IPO

[13] This assumption can be justified on a number of grounds. First, the actual number of going-private transactions relative to IPOs is small. Over the period 1979 to 1986 Kaplan (1991) found there to be 183 leveraged buyouts worth $100 million or more. Over the same period Loughran and Ritter (1995) documented 2683 IPOs. Over this time period the going-private versus going-public ratio is approximately 7 percent. Many of these buyouts were completed by non-management bidders. The actual number of going-private transactions completed by the management team that took it public would be sufficiently small as to suggest that the option to go private at the time of the IPO is worth very little. The fraction of the ownership claim retained by the original owners and management was shown to fall over the five to ten year period after the IPO by Mikkelson, Partch and Shah (1997), suggesting that insiders are more interested in diversifying their positions as opposed to retaining the option to repurchase the firm. Finally, a significant percentage of IPO firms see an outright change in control in the five years after the IPO. Mikkelson, Partch and Shah report a 29 percent turnover in control following the IPO for established firms, and 13 percent for younger start-ups.

[14] Institutional constraints make the timing issue more complex. Exchanges set minimum requirements that the firm must meet before it can be listed. The age since incorporation, a history of a sufficient level of revenues and profits, or constraints on the losses, and minimum market capitalization are all constraints that may cause a firm to delay an offering than it otherwise would like. These institutional features are not incorporated into the model.

is chosen to maximize the expected present value of these cash flows.

When the entrepreneur takes the firm public he will incur issuing costs. These costs take two forms, direct expenses and underwriting fees. Direct expenses include filing fees, legal expenses, and other administrative costs. The underwriting fee is a percentage of the issue proceeds. A study by Lee, Lockhead, Ritter and Zhao (1996) estimated the combined cost as a percentage of the issue proceeds and found that they ranged from a high of 17 percent for the smallest issues to six percent for the largest, indicating substantial economies of scale in issuance costs. These two costs are included through $\lambda$, the underwriting spread where $0 < \lambda < 1$, and $C$, the fixed direct expenses. Not specifically included, but easily added on to $C$, is the indirect expense of managerial time and effort devoted to the IPO. This cost could be considerably more than $C$ itself.[15] The net proceeds from the IPO are defined as $\Omega(\pi_t)$, where

$$\Omega(\pi_t) = \alpha \frac{\pi_t}{\delta^m}(1-\lambda) - C. \tag{3.1}$$

The underwriter receives the fraction $\lambda$ of the issue proceeds, leaving $(1-\lambda)$ for the entrepreneur. In addition, underpricing of the offering does not occur.

The value from owning the $\alpha$ shares, or equivalently the value of being private, consists of the dividend stream and the IPO proceeds. This value is defined as $F(\pi_t)$ and equals

$$F(\pi_t) = E\left[\int_t^{t+T(\pi^*)} \alpha \pi_s e^{-\rho^p(s-t)} ds + e^{-\rho^p T(\pi^*)} \Omega(\pi^*) \,|\pi_t\right]. \tag{3.2}$$

The critical profit level at which the firm goes public is $\pi^*$ and $T(\pi^*)$ is the first time the process for $\pi_t$ reaches $\pi^*$, where $T(\pi^*)$ is a random variable given the initial information. The first term in the expectation in (3.2) is the present value of the dividend stream accumulating to the entrepreneur until the IPO and the second term is the present value of net IPO proceeds. The objective of the entrepreneur is to choose a timing strategy that maximizes $F(\pi_t)$.

If $T(\pi^*) > dt > 0$ the entrepreneur will wait some positive amount of time before taking the firm public. The value of being private $F(\pi_t)$ can then be

[15] An additional component to the cost $C$ is the expenditures investors must incur to become informed about the firm. Investors need to be compensated for these costs in the form of a lower issue price. The costs are ultimately borne by the issuing firm. This cost of information production was used by Chemmanur and Fulghieri (1999) to determine the trade-off between public and private equity and the decision to go public.

broken into two parts, the immediate dividend plus the discounted value of being private. Equation (3.2) can be re-written as

$$F(\pi_t) = \alpha\pi_t dt + \frac{1}{(1+\rho^p dt)} E\left[F(\pi_t + d\pi_t)\,|\pi_t\right]. \tag{3.3}$$

If instead $T(\pi^*) = 0$ the entrepreneur will take the firm public immediately. In this case

$$F(\pi^*) = \Omega(\pi^*). \tag{3.4}$$

With uncertainty the entrepreneur can not specify an optimal time for the IPO ex ante. Instead, the timing strategy for the IPO consists of finding a profit level, $\pi^*$, at which the IPO proceeds equal the value of being private. Since the entrepreneur is free to take the firm public at any time he is choosing between remaining private and receiving the value in (3.3) and going public to get the payoff in (3.4). The choice of ownership structure is the one which maximizes the expected present value of the entrepreneur's cash flows. Formally,

$$F(\pi_t) = \max\left\{\Omega(\pi_t), \alpha\pi_t dt + \frac{1}{(1+\rho^p dt)} E\left[F(\pi_t + d\pi_t)\,|\pi_t\right]\right\}. \tag{3.5}$$

The IPO will occur when $\Omega(\pi^*)$ is greater than the continuation payoff in (3.3). To find $\pi^*$ a value for the continuation payoff is needed, which requires an expression for $F(\pi_t)$. The derivation of this function is examined in the next two sections.

### 3.2. Deterministic Profit Flow

As a first step to solving the optimal timing policy the case of a naive, or myopic, entrepreneur is considered. The entrepreneur will take the firm public as soon as the net IPO proceeds are greater than the private valuation of the shares, assuming they are held forever. Any benefit associated with waiting is disregarded. This criteria to determine when the firm goes public is simply that the net present value at the critical profit level equal zero. The critical profit level can be found by setting (3.1) equal to the private value of the shares:

$$\alpha\frac{\pi^n}{\delta^m}(1-\lambda) - C = \alpha\frac{\pi^n}{\delta^p}. \tag{3.6}$$

Solving for $\pi^n$ yields the following naive critical profit level

$$\pi^n = \frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)}. \tag{3.7}$$

For $\pi^n$ to have a positive value the following constraint is imposed on the parameter values.

**Assumption 1**:

$$1 > \frac{\delta^m}{(1-\lambda)\delta^p} \tag{3.8}$$

This is a necessary condition for the market value of the firm to be larger than the private value once the underwriting spread is taken into account. This condition guarantees the monotonic increase in the difference between the market and private values as the profit level increases. This ensures that a unique solution to the stopping problem exists, and that the firm will go public at some point.[16] A second assumption is imposed on the parameters values.

**Assumption 2**: The critical profit level $\pi^n$ is greater than the initial profit $\pi_0$.

The assumption ensures that the firm will not issue immediately, but would rather wait for the profit level to rise before going public. Were this not the case examining the timing decision would be pointless because the firm would never wait to issue.

When profits evolve deterministically the entrepreneur knows the profit level at $t+T$ will be $\pi_{t+T}$, given the time $t$ profit. Assuming the IPO occurs at $t+T$ the present value of the cash flows accruing to the entrepreneur can be found from equation (3.2). Substituting in for $\pi_s$ and $\Omega(\pi)$ yields

$$F(\pi_t) = \int_t^{t+T} \alpha \pi_t e^{\mu(s-t)} e^{-\rho^p(s-t)} ds + e^{-\rho^p T}\left(\alpha \frac{\pi_t e^{\mu T}}{\delta^m}(1-\lambda) - C\right). \tag{3.9}$$

The IPO will be timed to maximize (3.9). To find the profit level at which the IPO will occur, and with deterministic profits the issue date as well, (3.9) is maximized with respect to $T$. The solution is given in the next proposition.

[16] The restriction this assumption places on the possible discount rates is rather weak. A simple numerical example will illustrate this claim. Set $\lambda = 0.07$, $\mu = 0.1$, and $\rho^m = 0.12$. Given these parameter values the inequality in (3.8) will be satisfied if $\rho^p > 0.1215$. If the public discount rate is 12 percent then the entrepreneur discount rate only has to be greater than 12.15 percent for the firm to go public at some point.

**Proposition 3.1.** *When the profit flow is deterministic the entrepreneur will take the firm public when the profit level* $\pi_t$ *reaches*

$$\pi^d = \left(\frac{\rho^p}{\rho^p - \mu}\right) \frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} = \frac{\rho^p}{\rho^p - \mu}\pi^n, \tag{3.10}$$

*where* $\rho^p/(\rho^p - \mu) > 1$.

**Proof.** See Appendix.

A comparison of (3.10) with (3.7) shows there is value in waiting even in the deterministic case. The entrepreneur will wait to take the firm public because of the difference in the effective rates at which future profits and the issue cost are discounted. The value of the shares grow at rate $\mu$, but future share values are discounted at $\rho^p > \mu$. By delaying the offering the issue cost $C$ falls in present value terms at rate $\rho^p$. By waiting the shares lose less of their value, in present value terms, than the issue cost. At the critical value $\pi^d$ the benefit of waiting to incur the issue cost is outweighed by the discrepancy in the public versus private valuation of the shares. Comparing $\Omega\left(\pi^d\right)$ with the private share value at $\pi^d$ shows that the entrepreneur is willing to forgoe positive value from an IPO until the offering is made at $\pi^d$:

$$\alpha\frac{\pi^d}{\delta^m}(1-\lambda) - C - \alpha\frac{\pi^d}{\delta^p} = \frac{\mu}{\rho^p - \mu}C > 0. \tag{3.11}$$

An important caveat to the result in the proposistion is the interpretation of the discount rate $\rho^p$. With deterministic profit flows there is no uncertainty and the appropriate discount rate is the risk free rate, that is $\rho^p = r$.[17] The growth rate $\mu$ must also adjust for the condition $\mu < \rho^p$ to hold. For notational convenience define

$$K^d = \frac{\rho^p}{\rho^p - \mu}, \tag{3.12}$$

which allows $\pi^d$ to be written as

$$\pi^d = K^d\pi^n. \tag{3.13}$$

[17] If both $\rho^p$ and $\rho^m$ converge to $r$ when there is no uncertainty the entrepreneur will not take the firm public because there is no difference in the valuations. Assuming a higher impatience parameter for the entrepreneur would overcome this problem. As the limiting case that is unlikely to happen in practice this issue will not be pursued any further here.

With fixed issue costs and no uncertainty the entrepreneur will wait to take the firm public. However, it would not be correct to suggest that he is strategically exercising his timing option of when to go public. Without any uncertainty the entrepreneur knows at the firm's inception when the firm will go public. Even if he has the option of going public at any time he will always exercise it when $\pi_t = \pi^d$. Furthermore, with the deterministic evolution of the profit flow it can not be said that the entrepreneur is timing the market with the IPO. Market values will be growing at the same constant rate before and after the IPO.

### 3.3. Stochastic Profit Flow

When there is uncertainty in the profit flow having the flexibility of deciding when to take the firm public can have considerable value. The possibility of a sequence of positive profit shocks over a short period of time makes waiting an attractive option. Unlike the deterministic case where the entrepreneur knows exactly when he will take the firm public, with uncertainty the issue date is unknown ex ante. However, it is still possible to determine a critical profit level that will trigger an IPO. In order to do so a functional form is needed for $F(\pi_t)$.

The objective function for the entrepreneur was given in equation (3.5), which is re-stated here:

$$F(\pi) = \max\left\{\Omega(\pi), \alpha\pi dt + \frac{1}{(1+\rho^p dt)}E[F(\pi + d\pi)|\pi]\right\}$$

Time plays no role in the analysis so the $t$ subscript on $\pi$ will be dropped for convenience. In the continuation region for $\pi$ equation (3.5) can be re-written as

$$(1+\rho^p dt)F(\pi) = \alpha\pi dt(1+\rho^p dt) + E[F(\pi + d\pi) - F(\pi) + F(\pi)|\pi]. \qquad (3.14)$$

Dropping terms of order $dt^2$ gives

$$\rho^p F(\pi)dt = \alpha\pi dt + E[dF(\pi)|\pi]. \qquad (3.15)$$

Equation (3.15) is an equilibrium condition. The expected total return from holding the asset $F(\pi)$ over a small time interval $dt$, the right hand side of (3.15), must equal the required return from owning the asset, the left hand side. The total expected return consists of the immediate dividend $\alpha\pi dt$ plus the expected capital appreciation of the asset. The uncertainty in the asset $F(\pi)$ is spanned by the uncertainty in $\pi$ and must have an expected return equal to $\rho^m$ in financial markets. Since, by assumption, $\rho^p > \rho^m$ the entrepreneur is sure to take the firm public because owning the $\alpha$ shares does not produce a return equal to the cost of holding them. The difference between $\rho^p$ and $\rho^m$ represents an opportunity cost of keeping the shares private. The difference in the valuations, $v^m(1-\lambda) - v^p$, grows proportionally with $\pi$. As profits grow larger the cost of remaining private increases.

Expanding $dF(\pi)$ using Ito's Lemma in equation (3.15), taking the expectation and canceling the $dt$'s gives the following second-order differential equation

$$\frac{1}{2}\sigma^2\pi^2 F''(\pi) + (\rho^p - \delta^p)\pi F'(\pi) - \rho^p F(\pi) + \alpha\pi = 0. \tag{3.16}$$

To find the critical value to induce an IPO when there is uncertainty the function $F(\pi)$ that solves this equation must also satisfy boundary conditions. They are:

$$F(0) = 0, \tag{3.17}$$

$$F(\pi^*) = \alpha\frac{\pi^*}{\delta^m}(1-\lambda) - C, \tag{3.18}$$

$$F'(\pi^*) = \alpha\frac{1-\lambda}{\delta^m}. \tag{3.19}$$

The first condtion (3.17) states that if the firm value equals zero the option to go public is worthless. Conditions (3.18) and (3.19) are the value-matching and smooth-pasting conditions, respectively, at the critical value $\pi^*$. The solution to this problem is given in the next proposition.

**Proposition 3.2.** *The value of the firm to the entrepreneur is*

$$F(\pi) = A_1\pi^{\beta_1} + \alpha\frac{\pi}{\delta^p}, \tag{3.20}$$

*where the timing option of when to go public is worth $A_1\pi^{\beta_1}$. The critical value of $\pi$ to induce an IPO is*

$$\pi^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta p}\right)} = \left(\frac{\beta_1}{\beta_1 - 1}\right)\pi^n, \tag{3.21}$$

*where $\beta_1$ equals*

$$\beta_1 = \frac{1}{2} - \frac{(\rho^P - \delta^p)}{\sigma^2} + \sqrt{\left(\frac{(\rho^P - \delta^p)}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho^P}{\sigma^2}} > 1. \tag{3.22}$$

*The value of $A_1$ is given in the appendix.*

**Proof.** See Appendix

The substitution $K^s = \beta_1/(\beta_1 - 1)$ is again made for notational convenience and $\pi^* = K^s \pi^n$. The effect of the uncertainty on the timing decision is best seen by examining the limiting case when $\sigma$ converges to 0. Substituting expression (3.20) into (3.16) and canceling terms will produce the quadratic equation

$$\frac{1}{2}\sigma^2 \beta_1 (\beta_1 - 1) + (\rho^p - \delta^p)\beta_1 - \rho^p = 0. \tag{3.23}$$

Setting $\sigma = 0$ gives

$$\beta_1 = \frac{\rho^p}{\rho^p - \delta^p}, \tag{3.24}$$

which implies that

$$K_s = \frac{\rho^p}{\rho^p - \mu}. \tag{3.25}$$

Assuming that $\rho^p$ conveges to $r$ when the uncertainty is eliminated and $\delta^p$ is a fixed constant $K^s$ converges to $K^d$. For any $\sigma > 0$ $K^s > K^d$ and $\pi^* > \pi^d$. With a stochastic profit flow the entrepreneur has additional incentive to wait before taking the firm public.

By going public the entrepreneur sacrifices the opportunity of going public later when market conditions may be more favorable. To compensate for this loss the entrepreneur must receive a benefit from going public that is significantly larger than the private valuation of the shares. The critical value for going public is equivalent to a strike price of the timing option.

Introducing the timing option creates an additional cost of going public. Along with issuing costs, underwriting fees, and underpricing going public exercises this valuable option and it must be included as an opportunity cost in the issuing decision. The timing decision is resolved by comparing the proceeds from the IPO with the value of remaining private, which is the sum of the private share value and the timing option. Note that the timing option includes the value from waiting to reduce relative issue costs. With a naive entrepreneur the firm went public as soon as the IPO proceeds exceeded the private valuation of the dividend stream. With the timing option the firm will go public when the following constraint is satisfied:

$$\alpha\frac{\pi}{\delta^m}(1 - \lambda) - C - \frac{\alpha\pi}{\delta^p} - A_1\pi^{\beta_1} > 0. \tag{3.26}$$

Evaluated at $\pi^*$ (3.26) will hold as an equality.

From assumption 1 the market valuation is greater than the private value of the shares for any positive profit level. The delay in going public is caused by the fixed cost $C$. Adding the timing option further delays the IPO. But, as the next corollary shows, the value of being private is still less than the market value.

**Corollary 3.3.** *The option value $A_1\pi^{\beta_1}$ equals*

$$A_1\pi^{\beta_1} = \frac{1}{\beta_1}\left(\alpha\frac{\pi}{\delta^m}(1-\lambda) - \frac{\alpha\pi}{\delta^p}\right). \tag{3.27}$$

Expression (3.27) can be derived by substituting for $F(\pi)$ in the smooth pasting condition (3.19). The option value is less than the difference between the market and private share valuations. Without having to pay $C$ the entrepreneur would take the firm public immediately. However, this does not change the fact that the timing option will have a significant affect on when the IPO occurs. To find the magnitude of this effect numerical calculations of the critical value were be done. Before preceding the comparative static properties of the parameters on the critical value are summarized in the following corollary.

**Corollary 3.4.** *Holding constant all other parameter values, the critical value $\pi^*$ is:*

*1) increasing in the costs of going public, $\lambda$ and $C$;*
*2) decreasing in the issue size $\alpha$;*
*3) decreasing in the growth rate $\mu$;*
*4) increasing in the public discount rate $\rho^m$ and decreasing in the private discount rate $\rho^p$;*
*5) increasing in the volatility $\sigma$.*

**Proof.** See Appendix.

The effect of the issuing costs $\lambda$ and $C$ on $\pi^*$ is straight forward. They create a greater deterent to going public, requiring a larger firm value before issuing. The firm will only go public if the market value of the fraction of shares $\alpha$ exceeds the private value by some critical amount. The difference in values is an opportunity cost of remaining private. An increase in $\alpha$ will increase the total cost of being private for any given profit level. Consequently larger issues will be completed sooner. If $\alpha$ is partly determined by the need to raise capital for investment it would suggest that capital intensive firms are likely to go public earlier.

The impact of a higher growth rate $\mu$ on $\pi^*$ has two countervailing forces. For fixed $\rho$'s a larger $\mu$ will increase both the public and private valuation. However, the public valuation will increase faster because $\delta^m < \delta^p$, both of which decrease in $\mu$, and the valuation formula $\pi/\delta$ is convex in $\delta$. This will lower $\pi^*$. A higher $\mu$ will decrease $\beta_1$, which in turn increases $K^s$ and $\pi^*$. Future IPO proceeds will be higher, so the timing option is worth more. It can be shown numerically that the net effect is such that the increased market value dominates the increase in the timing option value.

The discount rates have a similar effect as $\mu$. A higher public discount rate $\rho^m$ lowers the market valuation and reduces the incentive to go public. The risk pricing benefits of public markets are mitigated in this case. The opposite is true when $\rho^p$ increases. The relative market valuation increases and it leads to earlier issues. In addition a higher $\rho^p$ increases $\beta_1$ which further decreases $\pi^*$. The option of going public later is worth less when future proceeds are discounted at a higher rate.

The standard option pricing result that the option value increases with volatility holds. With larger value fluctuations there is greater incentive to delay the offering. The effect of $\sigma$ on $\pi^*$ is independent of the risk preferences of the entrepreneur and the decomposition of $\sigma$ into systematic and unsystematic risk components.

### 3.4. Numerical Examples

To understand how the parameters interact to determine the absolute level of $\pi^*$ and how important the timing option might be in the decision to go public numerical calculations were done. In these calculations the following parameter values were held constant throughout: $\alpha = 0.2$, $\lambda = 0.07$, and $C = 2$. The entrepreneur is selling 20 percent of the shares to the public, the underwriting spread is seven percent, and the fixed cost of going public is two million dollars.[18] Changing these values will affect $\pi^n$, and in turn $\pi^*$, but not the relative importance of the timing option.

Caution should be used when interpreting the numerical results. Changes in the parameter values are not likely to be independent of one another. An increase in $\sigma$ should also increase both $\rho^m$ and $\rho^p$. Similarly as $\sigma$ goes to 0 the discount

[18] In a recent study Chen and Ritter (2000) found that more than 90 percent of deals raising \$20-80 million had gross underwriting spreads of exactly seven percent. The majority of IPOs were of this issuing size, while larger offerings had spreads that were at most one to two percent lower.

rates should start to converge. The entrepreneur will have less incentive to take the firm public, independent of the timing option.

The effect of volatility and the private discount rate on $\pi^*$ is shown in Figures 1 and 2. The value of $\pi^*$ is measured in millions of dollars. In both figures $\mu = 0.1$ and $\rho^m = 0.12$. Altering these values does not affect the qualitative results. The responsiveness of $\pi^*$ to $\sigma$ for three different levels of $\rho^p$ $(0.13, 0.15, 0.2)$ is shown in Figure 1. Increasing the volatility, measured as the annual standard deviation of the industry index, raises $\pi^*$, and at an increasing rate. Likewise, decreasing $\rho^p$ for any level of $\sigma$ will raise $\pi^*$. As $\rho^p$ converges to $\rho^m$ the price of risk advantage for the market dissipates. The effect of this decline in the market pricing advantage is most dramatic when $\rho^p$ and $\rho^m$ get closer, as $\pi^*$ will increase exponentially. If $\rho^m$ and $\rho^p$ are functions of $\sigma$ then the actual relationship between $\pi^*$ and $\sigma$ may actually be negative, unlike the positive relationship in the diagram. Firms from low volatility industries may wait longer to go public because of the small differential in discount rates. The opposite would hold for high volatility industries.

The effect of allowing the entrepreneur to behave non-myopically and recognize the value to waiting is evident in Figure 2, where $K^s$ is plotted against $\sigma$. $K^s$ measures the factor by which the naive critical value $\pi^n$ must increase to reach $\pi^*$. $K^s$ increases with $\sigma$ and decreases with $\rho^p$, future profits are worth less, giving the firm less incentive to wait to go public. For a standard deviation in the range of 20 to 50 percent the critical value $\pi^*$ must be two to eight times larger than the naive critical level $\pi^n$. Ignoring the timing option will lead to substantially different predictions for when a company might go public. Note that $K^s$ is not a function of $\rho^m$. The entrepreneur's willingness to wait is driven by the volatility in the market and the difference in the effective rates at which the fixed cost $C$ and future profit flows are discounted.

Figure 3 shows the impact of $\mu$ on $\pi^*$. The parameters $\rho^m = 0.12$ and $\sigma = 0.3$ are fixed, while $\rho^p$ again varies among the three values. Increasing the growth rate lowers $\pi^*$. As $\mu$ gets closer to $\rho^m$ the price of risk advantage of the market increases, inducing earlier issues. Similar to the case where $\rho^p$ is decreasing for a fixed $\mu$, increasing $\mu$ with $\rho^p$ fixed will increase $K^s$. The payoff from waiting is greater. For a large range of parameter values the risk pricing benefit was found to dominate the increased option value.[19]

[19] While not shown in any figure the effect of increasing the market discount rate $\rho^m$, holding everything else fixed, is equivalent to either a lower $\mu$ or a lower $\rho^p$. The market price of risk benefits decrease as $\rho^m$ increases and firms wait longer to go public.

### 3.5. Market Conditions and Issuing Patterns

Starting from an initial level $\pi_0$ profits must increase to $\pi^*$ before an issue will occur. These profit levels can easily be mapped into valuations, and even per share prices. There is nothing in the model though that restricts the path $\pi$ can follow to reach $\pi^*$. This is a direct result of the geometric Brownian motion assumption for $\pi$. Even though a precise prediction about the pre-issue return is not possible, general predictions about issuing patterns for entire industries are. Before that can be done a formal definition of an information event is required.

**Definition 3.5.** *A negative (positive) public information event has occurred if* $d\pi < 0$ ($d\pi > 0$).[20]

Using this definition the following corollary results.

**Corollary 3.6.** *A firm will never go public following a negative public information event.*[21]

This corollary follows as a logical consequence of proposition 3.3. For a firm to go public following a negative information event requires that it was private in the moment before. Since the value $\pi$ was actually higher in that preceding moment and the firm still did not go public, it certainly will not after negative information.

While this result is simply stating the obvious it does suggest likely issuing patterns for an entire industry. Each private firm in a given industry will have its own unique characteristics, implying a range of critical values that will trigger an IPO. However, common among these critical values is the same timing option. Only once the industry market index has risen significantly will firms start to exercise their option to go public. The result will be an increasing volume of IPOs as the index continues to rise, but if the index declines following a negative

[20] A more precise definition of a negative (positive) information event would be if $dz < 0$ ($dz > 0$). Using this definition will alter the subsequent proposition slightly. For values of $\pi_t$ for which $\pi_t < \pi_a^* < \pi_t + \mu dt$ it is possible for a negative information event to occur, provided it is sufficiently small, and the firm will still issue. To avoid this extreme case the alternative definition is used.

[21] It should be noted that this result is true even without including any uncertainty in the model. As shown in the preceding section adding uncertainty can more than double the critical profit level. For a large fraction of the possible profit levels up to the critical value the delay in going public is caused by the timing option.

information event all issuing will stop. Absent any other information it would appear that firms successfully timed the market. The observed phenomenon of IPOs clustering near industry valuation peaks could simply result from an entrepreneur following his optimal timing strategy.

The conclusion also contradicts the conventional wisdom on the termination of hot markets. It is suggested that following a decline in prices the market demand for new offerings dries up. Implicit in that claim is firms want to go public but no one will buy their shares. The model suggests that firms voluntarily choose not to go public to preserve their option of going public later in more favorable conditions. If it so desired the firm could go public after negative information, investors will still buy the offering, but only at a loss in value. Under this scenario issuing fads end voluntarily by the firms.

### 3.6. Investment and the Option to Go Public

The option of taking the firm public at any time clearly affects when the IPO will occur. In addition, the timing option can also play an important role in the decision to start the firm. The entrepreneur will invest $I_0$ only if starting the firm is a positive net present value project. The value of the firm to the entrepreneur when it is started is $F(\pi_0)$. The entrepreneur will invest if[22]

$$I_0 < F(\pi_0) = \frac{\pi_0}{\delta^p} + A_1 \pi_0^{\beta_1}. \tag{3.28}$$

Failure to consider the option of going public can lead to rejection of projects that should be accepted. This can happen if

$$\frac{\pi_0}{\delta^p} < I_0 < \frac{\pi_0}{\delta^p} + A_1 \pi_0^{\beta_1}. \tag{3.29}$$

As additional firms enter the industry the profit flow will no longer be exogenous. If condition (3.28) holds as a strict inequality firms will enter, driving the profit level for each firm down until the (3.28) holds as an equality.

The option of going public is only valuable if the market assigns greater value to the firm than the entrepreneur. By assumption 1 this is true because of the different discount rates. More generally, if the entrepreneur anticipates the possibility of selling the firm to the market for more than he thinks it is worth then

[22] In (3.28) the value of the timing option $A_1 \pi_0^{\beta_1}$ will depend on the number of shares the entrepreneur is willing to sell in the IPO.

starting the firm is an attractive option purely on speculative grounds. Furthermore, since the option value increases in the industry volatility firms in such an industry may be started as much for their possible future market valuations as for the certain initial cash flows.

## 4. Multiple Equity Issues

Constraining the entrepreneur to sell equity only during the IPO simplified the optimal timing problem, but ignored the fact that the firm can issue further equity at any time once it is public. The obvious question to ask is how the timing of the IPO is affected by allowing for seasoned equity offerings (SEO). Under the assumptions of the model seasoned issues are not undertaken to raise capital for investment, but rather for the entrepreneur to liquidate some of his holdings in the firm. As with the IPO the SEO will consist entirely of a secondary offering of shares.[23]

The model is generalized to allow for the option of a single SEO following the IPO. The intuition easily extends to the case of $n$ SEOs. The profit stream of the firm is fully observable when it is public. Investors need only look at the share price to learn the firm value.[24] This contrasts with having to study the prospectus to learn the value before the IPO. This lowers the information production costs for a SEO. In addition the direct issuing expenses for SEOs are lower than for IPOs.[25] The fixed cost $C$ of going public is relabed $C_1$ and the fixed cost for the SEO is $C_2$, with $C_1 > C_2$. The underwriting spread $\lambda$ is unchanged.

The entrepreneur knows when the firm is started how large an equity stake he will eventually sell. Unanticipated changes to the issue sizes or the number of issues do not affect the general timing problem. When the entrepreneur decides to make a change to his original issue plan the problem can be reformulated by defining the current profit level as the initial value and solving for the new optimal timing strategy given the desired equity issue plan. The total fraction of shares sold will again be $\alpha$, but now it is split over two equity issues. The fraction $\alpha_1$

[23]There is no loss in generality in assuming that both the IPO and SEOs consist of a secondary offering of shares and not a primary offering. What is needed is that some fraction of the outstanding shares, post-issue, were sold in the offering.

[24]Throughout the paper the assumption of efficient markets is implicit.

[25]Lee, Lockhead, Ritter, and Zhao (1996) found that direct issuing expenses in SEOs were on average about half of those for equivalent issue-size IPOs and the SEO underwriting spread was approximately two percent less than IPOs spreads.

will be sold in the IPO and $\alpha_2$ in the SEO, with $\alpha_1 + \alpha_2 = \alpha$.

The solution to the optimal timing of these two equity issues can be found recursively. First, the optimal timing of the SEO is determined when the firm is already public; second, the timing of the IPO is calculated conditional on the SEO timing strategy.[26]

### 4.1. Timing of the SEO

With no further equity issues planned after the SEO the timing decision reduces to the same single equity issue problem examined earlier in the case of the IPO. The entrepreneur will compare the proceeds he will receive from the SEO with the value of keeping the shares himself. The private value of the shares consists of the claim to the dividend stream plus the option of selling them to the investors. The entrepreneur discount rate is assumed to be unchanged from $\rho^p$ after the firm goes public. This is not a necessary assumption, the only requirement is that his discount rate remains larger than $\rho^m$ by some small amount. An objective function similar to (3.5) can be derived. The value associated with retaining the $\alpha_2$ shares is denoted $F_2(\pi)$ and will equal

$$F_2(\pi) = D_2 \pi^{\beta_1} + \alpha_2 \frac{\pi}{\delta^p}. \tag{4.1}$$

To find the critical value to induce the SEO and parameter value $D_2$ the following boundary conditions must hold:

$$F_2(0) = 0, \tag{4.2}$$

$$F_2(\pi_2^*) = \alpha_2 \frac{\pi_2^*}{\delta^m}(1 - \lambda) - C_2, \tag{4.3}$$

[26] In footnote 13 the optimal stopping characterization of the going-public decision was justified on the grounds that IPO firms attach little value to the option to go private. With firms now allowed to conduct multiple equity issues it is reasonable to ask whether the option to repurchase some of the outstanding shares will affect the timing decision of an equity issue. In the model the entrepreneur would never want to repurchase shares and the timing decision is unaffected by this option. While the use of share repurchases have grown considerably in the past decade (see Ikenberry, Lakonishok and Vermalien (2000) and Fama and French (1999)) the main motivation appears to be as a preferred alternative to dividends in returning cash to shareholders. In addition share repurchases are necessary to fund the stock option positions of the firm's employees. The option to repurchase does not appear to be a strategic consideration of firms undertaking an equity issue.

$$F_2^{'}(\pi_2^*) = \alpha_2 \frac{(1-\lambda)}{\delta^m}. \tag{4.4}$$

The problem in equations (4.1) to (4.4) is the same one analyzed in proposition 3.2. Using the results of that proposition the critical value will be

$$\pi_2^* = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C_2}{\alpha_2 \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)}. \tag{4.5}$$

The value of the parameter $D_2$ is

$$D_2 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^{\beta_1} \frac{\alpha_2^{\beta_1}}{C_2^{\beta_1 - 1}}. \tag{4.6}$$

### 4.2. Timing of the IPO

Using both $F_2(\pi)$ and $\pi_2^*$ the value of remaining private and the timing of the IPO can be determined. Following the same procedure as above the value of keeping the shares private prior to the IPO is

$$F_1(\pi) = D_1 \pi^{\beta_1} + \alpha \frac{\pi}{\delta^p}. \tag{4.7}$$

In (4.7) the private value consists of the option to issue the $\alpha_1$ shares and the present value of the dividend flow to both the $\alpha_1$ and $\alpha_2$ shares. When the firm goes public, but before the SEO, the entrepreneur will continue to receive dividends for the $\alpha_2$ shares.

Boundary conditions are again necessary to find the critical value $\pi_1^*$ and $D_1$. Now when the firm goes public the entrepreneur receives the proceeds from taking the firm public and the option to conduct a SEO for the $\alpha_2$ shares. The boundary conditions are

$$F_1(0) = 0, \tag{4.8}$$

$$F_1(\pi_1^*) = F_2(\pi_1^*) + \alpha_1 \frac{\pi_1^*}{\delta^m}(1-\lambda) - C_1, \tag{4.9}$$

$$F_1^{'}(\pi_1^*) = F_2^{'}(\pi_1^*) + \alpha_1 \frac{(1-\lambda)}{\delta^m}. \tag{4.10}$$

The exact expression for conditions (4.9) and (4.10) will depend on the value of $F_2(\pi_1^*)$. However, as the next proposition shows the entrepreneur will find it optimal to issue the $\alpha_2$ shares as part of the IPO, independent of the value of $F_2(\pi_1^*)$.

**Proposition 4.1.** *The optimal policy for the entrepreneur is to issue all the shares, $\alpha_1 + \alpha_2$, in the IPO. The critical profit level for the IPO is*

$$\pi_1^* = \pi^* = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C_1}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} \tag{4.11}$$

This proposition is a direct consequence of corollary 3.3. The entrepreneur would always prefer to sell the shares to the market rather than keep them in the absence of an issue cost $C$. When the IPO is conducted the $\alpha_2$ shares can be added to the offering at no additional cost. Any shares the entrepreneur plans to sell should be included in the IPO.

The result of the proposition is consistent with the observation that firms rarely undertake SEOs. This fact is usually justified by the presence of asymmetric information between firm insiders and investors. Firms only want to issue equity when it is overvalued. Investors recognize this incentive and react negatively to equity issues. The proposition above suggests that firms do not undertake many SEOs simply because they want to avoid the costs associated with multiple equity issues.

The entrepreneur may prefer to have two separate equity issues to reduce his ownership stake gradually, even if it means having to pay additional issue costs. When the firm goes public the entrepreneur acquires the option to undertake a subsequent SEO for the $\alpha_2$ shares. In this case $F_2(\pi_1^*)$ will have the form

$$F_2(\pi_1^*) = D_2 \pi_1^{*\beta_1} + \alpha_2 \frac{\pi_1^*}{\delta^p}.$$

By plugging this expression for $F_2(\pi_1^*)$ into (4.9) and (4.10) $\pi_1^*$ and $D_1$ can be determined.

**Proposition 4.2.** *The timing of the IPO is unaffected by a subsequent SEO. The critical value to induce an IPO will be*

$$\pi_1^* = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C_1}{\alpha_1\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)}. \tag{4.12}$$

*The coefficient $D_1$ equals*

$$D_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^{\beta_1} \left(\frac{\alpha_1^{\beta_1}}{C_1^{\beta_1 - 1}} + \frac{\alpha_2^{\beta_1}}{C_2^{\beta_1 - 1}}\right). \tag{4.13}$$

**Proof.** See Appendix.

A comparison of $\pi_1^*$ with $\pi^*$ shows that the critical value for the IPO is unaffected by the possibility of a SEO. The exact values of $\pi_1^*$ and $\pi^*$ will differ because $\alpha_1 < \alpha$. The firm anticipating a SEO will wait longer to go public because of the smaller issue size. Two identical firms contemplating an IPO with equal issue sizes will go public at the same time, even if one firm expects to undertake a SEO. This result leads to the conclusion that subsequent equity issues will not affect the timing of the current equity issue.[27]

The entrepreneur has the initial option of deciding when to take the firm public. After the IPO he acquires the option to conduct a SEO at any time. These options do not cancel each other out because they apply to a different set of shares. The entrepreneur will sell the fraction $\alpha_1$ in the IPO and $\alpha_2$ in the SEO. These equity stakes can only be sold once, and each offering can effectively be treated independent of all others. All equity issues should occur only when the net issue proceeds exceed both the private share valuation and the timing option for that specific set of shares. Price run-ups should occur prior to all equity issues; in the industry market index before the IPO and in the firm's own share price before a SEO.

The timing option for the IPO, $D_1\pi^{\beta_1}$, is larger than the option for the SEO, $D_2\pi^{\beta_1}$, because $D_1 > D_2$. This occurs because the option to go public entitles the entrepreneur to the IPO proceeds and the option to conduct a SEO. With no further equity issues after the SEO the second option only gives the entrepreneur entitlement to the SEO proceeds. With a larger option value the entrepreneur has greater incentive to remain private, but the payoff from going public has also increased, making the IPO more attractive. The value of $D_1$ in (4.13) includes the value for $D_2$ in (4.6). The increase in the timing option for the IPO corresponds exactly to the value of the timing option for the SEO. These two values cancel out, leaving the IPO critical value unaffected.

[27] The independence of the IPO and SEO timing is, in part, a consequence of the abstraction from the operating side of the firm, the information structure and the motivations for going public. For a firm investing heavily a SEO may occur shortly after the IPO to ensure that the firm has sufficient capital. If there is asymmetric information about the firm type the SEO may occur only after information about the firm's true value has been revealed. Welch (1995) used this idea to explain why underpricing of the IPO occurs and the timing of the SEO relative to the IPO.

## 5. Timing and the Option to Withdraw

The assumption that firms can decide to go public and complete the IPO instantaneously is unrealistic. The time elapsed from when the firm files the registration statement with the SEC until the offering is complete has averaged in the past about two months (Hanley (1993)).[28] Market conditions can change considerably during this interval, with adverse information making an offering much less desirable. Negative information will have a different effect on firms contemplating an offering with those that have already begun the IPO process but have not yet completed the issue.

The model is modified to allow for a separation of length $T$ between the date of filing and issue date. If at $t$ the entrepreneur decides to take the firm public the offering is completed at $t + T$, when the proceeds are received. No further equity issues are allowed. The fixed cost $C$ is paid at $t$, while the underwriting spread comes out of the proceeds. $C$ is sunk, equivalent to a cost of entry. At $t+T$ the entrepreneur can withdraw the offering if the market valuation has fallen sufficiently to make remaining private the preferable alternative.[29] Otherwise the firm will go public.

The firm must pay a cost equal to $W$ if it withdraws the IPO. This is not a result of direct expenditures associated with withdrawal, but the intangible cost to the firm's reputation. Investors typically view a withdrawal, even in adverse market conditions, as a signal of the firm's quality. Attempting to go public, and then not, will make subsequent efforts at an IPO that much more difficult.[30] If withdrawn the entrepreneur has to pay $C$ when he tries to take the firm public again.

Allowing the entrepreneur to withdraw the offering means he now has two options, the withdrawal option and the option of going public. From the previous

[28] This does not include the considerable amount of time managers must work with investment bankers preparing for the offering. In total the IPO process can last over six months.

[29] Withdrawn IPOs may not be strictly voluntary. Insufficient interest from investors may make a withdrawal the only viable option.

[30] Dunbar (1998) provides indirect evidence associated with the cost of withdrawing an offering. Based on a sample of unsuccessful IPOs from 1979 to 1982 he finds that only nine percent of these firms ever go public. Those which eventually go public do so on average about two years after the initial offering. Dunbar does not report on the fate of the remaining unsuccessful offerings, but the results do suggest that investors may view unsuccessful firms unfavorably. He also finds that offerings that are less likely to succeed must pay higher compensation to investment banks. In the current paper these additional costs would be reflected in either $C$ or $\lambda$ and not $W$.

analysis the going-public option is only exercised when $\pi$ reaches the critical level $\pi^*$. Intuition suggests that there should now be two critical values. One will induce the firm to go public, $\overline{\pi}$, and one will trigger a withdrawal, $\underline{\pi}$. Using the same methodology as before each option value is found first, and then by invoking boundary conditions the critical values can be derived.

The option of deciding when to go public is only alive when the firm is still private. The value of the firm to the entrepreneur when it is private was derived in proposition 3.2. A similar value will hold in the case when offerings can be withdrawn, the difference being the coefficient values will change because the issuing procedure is different. The value of being private in this case is denoted $V_0(\pi)$, which equals

$$V_0(\pi) = B_1 \pi^{\beta_1} + \alpha \frac{\pi}{\delta^p}. \tag{5.1}$$

The option to withdraw is alive only after the firm announces it is going public. The value of the firm during the IPO process, $V_1(\pi)$, is the sum of three parts. At the announcement date $t$ the entrepreneur has an expectation for the IPO proceeds at $t+T$, has the option of withdrawing the IPO, and will receive the profit flow for length $T$. With no actions allowed between $t$ and $t+T$ the value $V_1(\pi)$ need only be considered at those two dates.

At $t$ the value of a firm starting an IPO will be

$$V_1(\pi_t) = P(\pi_t, \underline{\pi}, W) + \alpha \frac{\pi_t e^{\mu T}}{\delta^m} (1-\lambda) e^{-\rho^p T} + \alpha \frac{\pi_t}{\delta^p} (1 - e^{-\delta^p T}). \tag{5.2}$$

The first term is the value of the option to withdraw, the second term is the present value of the expected future proceeds of the IPO at $t+T$, and the third term is the present value of the expected flow of profits from $t$ to $t+T$. The option is a put, with a cost of exercising equal to $W$. The option is exercised only for a profit level below the lower trigger $\underline{\pi}$. The entrepreneur of a potential IPO firm is effectively selling the firm back to himself as a private entrepreneur and reaping the difference in the two values.

At $t+T$ $V_1(\pi)$ has the simpler form

$$V_1(\pi_{t+T}) = P(\pi_{t+T}, \underline{\pi}, W) + H(\pi_{t+T}), \tag{5.3}$$

where

$$H(\pi_{t+T}) = \alpha \frac{\pi_{t+T}}{\delta^m} (1-\lambda). \tag{5.4}$$

The discounted expected IPO proceeds have been replaced with the actual value. The put option has the form

$$P\left(\pi_{t+T},\underline{\pi},W\right)=\max\left[V_0\left(\pi_{t+T}\right)-H\left(\pi_{t+T}\right)-W,0\right]. \tag{5.5}$$

The withdrawal option will pay off if the value of being private is greater than the IPO proceeds and the cost of withdrawl.

To find the critical values $\overline{\pi}$ and $\underline{\pi}$, the parameter $B_1$, and the withdrawal option value, boundary conditions are necessary. For the entrepreneur to take the firm public the following must hold:

$$V_0\left(\overline{\pi}\right) = V_1\left(\overline{\pi}\right)-C, \tag{5.6}$$
$$V_0^{'}\left(\overline{\pi}\right) = V_1^{'}\left(\overline{\pi}\right). \tag{5.7}$$

These are the value-matching and smooth-pasting conditions, respectively. For the offering to be withdrawn at the critical value $\underline{\pi}$ it must be that

$$H\left(\underline{\pi}\right) = V_0\left(\underline{\pi}\right)-W, \tag{5.8}$$
$$H^{'}\left(\underline{\pi}\right) = V_0^{'}\left(\underline{\pi}\right). \tag{5.9}$$

At $\pi=\underline{\pi}$ the entrepreneur is indifferent between taking the firm public and keeping it private so the option value equals 0, and $V_1\left(\underline{\pi}\right)$ will equal $H\left(\underline{\pi}\right)$. The next proposition summarizes the entry and exit critical values for an IPO.

**Proposition 5.1.** *With separation of the announcement and issue dates the critical value to induce an IPO will equal*

$$\overline{\pi}=\left(\frac{\beta_1}{\beta_1-1}\right)\frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m}-\frac{1}{\delta p}\right)e^{-\delta^p T}}. \tag{5.10}$$

*The entrepreneur will never withdraw an IPO once the process has started,* $\underline{\pi}=0$. *The option to withdraw* $P\left(\pi,\underline{\pi},W\right)$ *is worthless. The value of* $B_1$ *is given in the appendix.*

**Proof.** See Appendix.

The extreme result that an IPO is never withdrawn is a consequence of corollary 3.3. That result guaranteed that the firm will go public for any positive profit level if there was no cost of entry. Because $C$ was paid and is sunk the entrepreneur will always prefer to proceed with the IPO. This does not mean the firm will go public immediately. It still must pay $C$ at the start of the IPO process, and that will delay the offering until the firm is sufficiently large. With no desire to ever cancel an IPO the option to withdraw is worthless.[31] Finding the profit level at which the firm will go public proceeds as before.

A comparison of $\overline{\pi}$ and $\pi^*$ shows that the former is larger than the latter by the term $e^{\delta^p T}$. The proceeds from the offering are not received for $T$ units of time. The proceeds are expected to grow at the rate $\mu$, but they are discounted at rate $\rho^p$. The expected present value of the proceeds are lower when there is a delay, which will raise $\pi^*$. During the selling process the entrepreneur continues to receive the profit flow, which raises the value from going public, and lowers $\pi^*$. But since profits are discounted faster than they grow the interim profit flow is insufficient compensation for the delay in completing the offering.[32]

A more general model that relaxes the assumption that a firm will always prefer public over private ownership, absent the fixed cost, can lead to the result $\underline{\pi} > 0$. There are valuations at which the firm will withdraw the offering. This in turn makes the option to withdraw $P(\pi, \underline{\pi}, W)$ valuable. As this option value increases it makes going public more attractive relative to remaining private because the IPO process is no longer irreversible. The critical entry value $\overline{\pi}$ will fall as a result. The general conclusion that there is a separation in the entry and exit values for IPOs will continue to hold.

The effect of introducing a lengthy selling procedure on the timing of IPOs is to allow for firms issuing in down markets. If an IPO was started while the market was rising, it will be completed even if valuations start to fall. The separation of the announcement and issue dates will also mean there is a lag in IPOs in response to the market index. Offerings will appear both before and after market peaks.

[31] Another reason why the option to withdraw may not have much value was proposed by Chemmanur and Fulghieri (1994). Being the underwriter for an unsuccessful offering can damage the bank's credibility with investors in its ability to certify the quality of an issuing firm. Having the option to withdraw the IPO will have little value to either the underwriter or firm. Evidence consistent with this argument was found by Dunbar (2000), who found that banks associated with unsuccessful offerings lose market share.

[32] The effect of a separation between the filing and offer dates on the issuing decision also provides some insights into how lock-up provisions will affect when the firm goes public. The original investors of the firm usually sign a provision with the investment bank to not sell their shares for a pre-specified period of time. By viewing time $t$ as the offer date and $t + T$ as the date at which insiders can sell their positions, the use of a lock-up provision will increase the critical value.

## 6. Discussion

The results of the model suggest that the clustering of IPOs near market valuation peaks could be a consequence of the entrepreneur optimally exercising the option to go public. While the paper provides a rational explanation for the price run-up prior to the IPOs, nothing can be said about the decline in valuations that follows the issuing fad. The poor performance of IPOs post-issue was first documented by Ritter (1991). He found that issuing firms have stock returns that underperform relative to comparative benchmark firms over a three to five year time period. Similar results have been documented by other researchers.[33] A large literature has emerged on this topic of expected returns following equity issues, and the question of whether IPO firms actually underperform is a subject of much debate.[34]

Loughran and Ritter (1995) contend that the underperformance is a result of firm insiders exploiting their information advantage over investors to issue when the firm is overvalued. As evidence of this Teoh, Welch and Wong (1998) found that issuing firms which manipulate their earnings through legal accounting procedures produce worse long-run returns than firms engaging less in such discretionary activities. Nor is post-issue underperformance limited to stock returns. Jain and Kini (1994) and Mikkelson, Partch and Shah (1997) both documented a decline in the operating performance of IPO firms following their issue.

Collectively the empirical evidence suggests that asymmetric information is playing an important role in the timing of IPOs. To explain the more general phenomenon that IPOs cluster near valuation peaks would require that all the firm insiders know that the industry as a whole is overvalued. This is certainly a much stronger condition, one that suggests that markets are not entirely efficient. Even if this is true it still can not explain why the clustering near peaks occurs. If a firm is overvalued it should go public, whether or not it is near a peak.

As demonstrated by the model the entrepreneur will wait to take the firm public, even when the net IPO proceeds exceed the private share valuation. The

[33]See also Peavy (1990), Loughran, Ritter, and Rydqvist (1994), Loughran and Ritter (1995), Levis (1995), Rajan and Servaes (1997), Baker and Wurgler (2000), and Hansen (2000) for other evidence on the poor post-issue returns for IPOs.

[34]Fama (1998) provides a review of the evidence that contradicts the claim that IPOs underperform in the long-run. He also suggests that it is difficult in principle to even test this claim. Any test for market efficiency jointly tests the null hypothesis that the assumed model for expected returns is correct. One can not be certain if market efficiency should be rejected or the model is misspecified.

difference in the valuations was assumed because of the different discount rates, but it shows more generally that by itself a higher market value is not sufficient to induce an IPO. Assuming market prices, whether they are rational or not, evolve stochastically is enough to ensure that price run-ups should always occur prior to an IPO. A complete explanation for IPO issuing patterns should include both asymmetric information and the timing option to go public.

The assumption of an exogenous information process may be reasonable when looking at a single firm, but it is restrictive when considering the issuing patterns of an entire industry. IPOs are informative events for investors, not only about the issuing firm but for all firms in the industry. A positively received IPO can result in a discrete jump in the valuations attached to the remaining private firms. Such an effect was documented by Rajan and Servaes (1997). They examined how analyst's forecasts of the growth prospects for recent IPOs affected the decision of firms to engage in their own IPOs. High growth forecasts, or equivalently high valuations, were more likely to lead to subsequent IPOs.[35]

Endogenizing the information production process would allow the price run-ups and clustering of IPOs to be jointly determined. By doing so it may be possible to show how a hot market arises endogenously as a result of positive information produced by early issuers. Investors revise their expected valuations upward, triggering a flood of new IPOs. It could also provide an explanation for the price decline after the issuing fad. If investors are trying to learn the value of the industry the early positive information may lead to overvaluation, relative to the true unknown value. Only after a number of issues do investors learn the true value, at which point the prices fall from their earlier highs.

One final extension of the model that may prove useful is to allow firms to issue alternative securities other than equity. These additional choices would only be necessary if the capital raised was to be used for investment purposes, and not simply as a means for insiders to liquidate their holdings in the firm. Implicit in this approach is the interaction between the real investment side of the firm and the financing decisions.

[35] Further evidence on the positive feedback of well received IPOs on subsequent issues was offered by Loughran and Ritter (2000). They found that the average first-day returns by month have a first order autocorrelation of 0.5 (based on a sample from March 1991 to August 1998). They also find that the average monthly price revision from the offer price range midpoint to the final offer price has a first order autocorrelation of 0.61.

## 7. Conclusion

This paper explicitly modeled the effect that timing flexibility and valuing firms with market indexes has on the decision to go public. It was shown that when there is uncertainty over future valuations the option of deciding when to go public can have considerable value. Going public exercises this option, which must be viewed as a cost of an IPO. Firms are willing to wait for a possible price run-up before issuing to cover the cost of exercising this option. The model also suggests that the observed clustering of IPOs near peaks in industry market valuations could be a result of firms optimally exercising their timing option. The results for price run-ups prior to IPOs generalizes to seasoned equity offerings. Each equity sale is unique and will occur only after the payoff is sufficiently high.

The paper made a simple assumption about why firms go public, but the intuition is entirely general. Any firm contemplating an IPO should factor the timing option into the decision. A firm trying to raise capital for new investment might find that public equity is cheaper than private equity. When the timing option is factored in the entrepreneur may prefer to continue financing with private funds in order to delay the IPO to a more favorable time. If an IPO is motivated by the desire to increase the liquidity in the shares of the firm the benefit should be greater than the value of the timing option when the firm goes public. In all cases the benefit from an IPO should exceed the value of the alternative option to issuing shares by the amount of the timing option.

## 8. Appendix

*Proof of Proposition 3.1:* From equation (3.9) the value of $F(\pi_t)$ will equal

$$F(\pi_t) = \alpha\frac{\pi_t}{\delta^p}\left(1 - e^{-\delta^p T}\right) + \alpha\frac{\pi_t e^{-\delta^p T}}{\delta^m}(1-\lambda) - Ce^{-\rho^p T}. \tag{8.1}$$

Maximizing $F(\pi_t)$ with respect to $T$ gives the first order condition

$$F_T(\pi_t) = \delta^p\alpha\frac{\pi_t}{\delta^p}e^{-\delta^p T} - \delta^p\alpha\frac{\pi_t e^{-\delta^p T}}{\delta^m}(1-\lambda) + \rho^p Ce^{-\rho^p T}. \tag{8.2}$$

Setting this expression equal to zero and solving for $T^*$ gives the optimal amount of time to wait before going public:

$$T^* = \frac{1}{\mu}\ln\left[\frac{\rho^p}{\delta^p}\frac{C}{\alpha\pi_t\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta p}\right)}\right]. \tag{8.3}$$

$T^*$ can not be negative. If the difference between the market and private valuations is not too much larger than $C$ then $T^* > 0$, otherwise $T^* = 0$. It can be shown that the second order condition to (8.1) is negative at $T^*$, so $T^*$ is a maximum. The critical value to induce the IPO occurs when there is no further benefit to waiting, or $T^* = 0$. Setting (8.3) equal to 0 and solving for $\pi$ yields

$$\pi^d = \left(\frac{\rho^p}{\rho^p - \mu}\right)\frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta p}\right)}. \tag{8.4}$$

*Proof of Proposition 3.2*: (The proof follows from Dixit and Pindyck (1994)) The solution to the second-order differential equation (3.16) will have the form

$$F(\pi) = A_1\pi^{\beta_1} + A_2\pi^{\beta_2} + \frac{\alpha\pi}{\delta^p}. \tag{8.5}$$

The first two terms in (8.5) are solutions to the homogeneous part of (3.16) and the third term is the particular integral to the whole equation. Plugging in $A\pi^\beta$ for $F(\pi)$ into (3.16) will yield the quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + (\rho^p - \delta^p)\beta - \rho^p = 0. \tag{8.6}$$

with the roots

$$\begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \frac{1}{2} - \frac{(\rho^p - \delta^p)}{\sigma^2} \pm \sqrt{\left[\frac{(\rho^p - \delta^p)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2\rho^p}{\sigma^2}} \gtrless \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{8.7}$$

Since $\beta_2 < 0$ and the boundary condition $F(0) = 0$, it must be that $A_2 = 0$. If this was not the case the option value would explode as $\pi$ went to 0, which violates the boundary condition. Then

$$F(\pi) = A_1 \pi^{\beta_1} + \frac{\alpha\pi}{\delta^p}. \tag{8.8}$$

From the smooth pasting condition (3.19)

$$A_1 \pi^{\beta_1} = \left(\frac{\pi^*}{\beta_1}\right) \alpha \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right). \tag{8.9}$$

Substituting this into the value matching condition (3.18) gives

$$\left(\frac{\pi^*}{\beta_1}\right) \alpha \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right) + \frac{\alpha\pi^*}{\delta^p} = \alpha\pi^* \frac{1-\lambda}{\delta^m} - C,$$

$$\pi^* \alpha \left(1 - \frac{1}{\beta_1}\right) \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right) = C,$$

$$\pi^* = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)}.$$

The value of $A_1$ is

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^{\beta_1} \frac{\alpha^{\beta_1}}{C^{\beta_1 - 1}}.$$

*Proof of Corollary 3.3:* Equation (3.27) is the smooth pasting condition (8.9) from above. $\beta_1 > 1$ so the option value is less than the difference between the market and private share valuations.

*Proof of Corollary 3.4:* The proof for each part consists of deriving the first order condition.

(1)

$$\begin{aligned}\frac{\partial \pi^*}{\partial C} &= \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{1}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} > 0, \\ \frac{\partial \pi^*}{\partial \lambda} &= \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C\frac{1}{\delta^m}}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^2} > 0.\end{aligned}$$

(2)

$$\frac{\partial \pi^*}{\partial \alpha} = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{-C}{\alpha^2\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} < 0.$$

(3) The effect of $\mu$ on $\pi^*$ can be found by totally differentiating (3.21) with respect to $\mu$:

$$\frac{d\pi^*}{d\mu} = \frac{\partial \pi^*}{\partial \beta_1}\frac{\partial \beta_1}{\partial \mu} + \frac{\partial \pi^*}{\partial \mu}. \tag{8.10}$$

The three derivatives are

$$\begin{aligned}\frac{\partial \pi^*}{\partial \beta_1} &= \frac{-1}{(\beta_1 - 1)^2}\frac{C}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} < 0, \qquad (8.11) \\ \frac{\partial \beta_1}{\partial \mu} &= \frac{1}{\sigma^2}\left(-1 + \left(\frac{\mu}{\sigma} - \frac{1}{2}\right)\left(\left(\frac{\mu}{\sigma} - \frac{1}{2}\right)^2 + \frac{2\rho^p}{\sigma^2}\right)^{-\frac{1}{2}}\right) < 0, \\ \frac{\partial \pi^*}{\partial \mu} &= \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{-C\left(\frac{1-\lambda}{(\rho^m-\mu)^2} - \frac{1}{(\rho^p-\mu)^2}\right)}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^2} < 0.\end{aligned}$$

Substituting these three derivatives into (8.10) will yield an expression whose sign is contingent on the parameter values. For the parameter values of interest it will be that (8.10) is always negative.

(4)

$$\frac{\partial \pi^*}{\partial \rho^m} = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{C\left(\frac{1-\lambda}{(\rho^m-\mu)^2}\right)}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^2} > 0. \tag{8.12}$$

To find the effect of $\rho^p$ on $\pi^*$ totally differentiate (3.21) with respect to $\rho^p$:

$$\frac{d\pi^*}{d\rho^p} = \frac{\partial \pi^*}{\partial \beta_1}\frac{\partial \beta_1}{\partial \rho^p} + \frac{\partial \pi^*}{\partial \rho^p}. \tag{8.13}$$

The first derivative is derived in (8.11) and the two remaining derivatives are

$$\frac{\partial\beta_1}{\partial\rho^p} = \frac{1}{\sigma^2}\left(\left(\frac{\mu}{\sigma}-\frac{1}{2}\right)^2+\frac{2\rho^p}{\sigma^2}\right)^{-\frac{1}{2}} > 0,$$

$$\frac{\partial\pi^*}{\partial\rho^p} = \left(\frac{\beta_1}{\beta_1-1}\right)\frac{-C\left(\frac{h}{(\rho^p-\mu)^2}\right)}{\alpha\left(\frac{1-\lambda}{\delta^m}-\frac{1}{\delta^p}\right)^2} < 0.$$

By substituting these derivatives into (8.13) it is easy to see that $\pi^*$ is decreasing in $\rho^p$.

(5) $\pi^*$ depends on $\sigma$ implicitly through $\beta_1$ so

$$\frac{\partial\pi^*}{\partial\sigma} = \frac{\partial\pi^*}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma}.$$

From (8.11) $\frac{\partial\pi^*}{\partial\beta_1}$ is negative. The proof to show $\frac{d\beta_1}{d\sigma} < 0$ follows that in Dixit and Pindyck (1994). Define equation (8.6) above as $Q$. Totally differentiating $Q$ with respect to $\sigma$ gives

$$\frac{\partial Q}{\partial\beta_1}\frac{\partial\beta_1}{\partial\sigma}+\frac{\partial Q}{\partial\sigma} = 0. \tag{8.14}$$

The coefficient on $\beta_1^2$ in (8.6) is positive so $Q$ will be upward pointing. The two roots are less than zero and greater than one. At $\beta_1$ $Q$ will be upward sloping, $\partial Q/\partial\beta_1 > 0$. Also proves $\partial Q/\partial\sigma = \sigma\beta(\beta-1) > 0$. Equation (8.14) can hold only if $\partial\beta_1/\partial\sigma < 0$. Together with (8.11) implies that $\pi^*$ is increasing in $\sigma$.

*Proof of Proposition 3.7:* The three boundary conditions are

$$F_1(0) = 0, \tag{8.15}$$

$$D_1\pi_1^{*\beta_1}+\alpha\frac{\pi_1^*}{\delta^p} = D_2\pi_1^{*\beta_1}+\alpha_2\frac{\pi_1^*}{\delta^p}+\alpha_1\frac{\pi_1^*}{\delta^m}(1-\lambda)-C_1, \tag{8.16}$$

$$D_1\pi_1^{*\beta_1}\frac{\beta_1}{\pi_1^*}+\frac{\alpha}{\delta^p} = D_2\pi_1^{*\beta_1}\frac{\beta_1}{\pi_1^*}+\frac{\alpha_2}{\delta^p}+\frac{\alpha_1}{\delta^m}(1-\lambda) \tag{8.17}$$

Solving for $(D_1-D_1)\,\pi_1^{*\beta_1}$ in (8.17) gives

$$(D_1-D_2)\,\pi_1^{*\beta_1} = \alpha_1\frac{\pi_1^*}{\beta_1}\left(\frac{1-\lambda}{\delta^m}-\frac{1}{\delta^p}\right). \tag{8.18}$$

Substituting into (8.16) solves for $\pi_1^*$:

$$\pi_1^* = \left(\frac{\beta_1}{\beta_1 - 1}\right) \frac{C_1}{\alpha_1 \left(\frac{(1-\lambda)}{\delta^m} - \frac{1}{\delta p}\right)}. \tag{8.19}$$

The value of $D_1$ equals

$$D_1 = D_2 + \alpha_1 \left(\frac{\pi_1^*}{\delta^m}(1-\lambda) - \frac{\pi_1^*}{\delta p}\right) - C_1. \tag{8.20}$$

Substituting for $D_2$ and $\pi_1^*$ gives

$$D_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)^{\beta_1} \left(\frac{\alpha_1^{\beta_1}}{C_1^{\beta_1 - 1}} + \frac{\alpha_2^{\beta_1}}{C_2^{\beta_1 - 1}}\right). \tag{8.21}$$

*Proof of Proposition 3.8:* At $\underline{\pi}$ the boundary conditions (5.8) and (5.9) must hold:

$$\begin{aligned} \alpha\frac{\underline{\pi}}{\delta^m}(1-\lambda) &= B_1\underline{\pi}^{\beta_1} + \alpha\frac{\underline{\pi}}{\delta^p} - W, \\ \alpha\frac{(1-\lambda)}{\delta^m} &= B_1\underline{\pi}^{\beta_1}\frac{\beta_1}{\underline{\pi}} + \frac{\alpha}{\delta^p}. \end{aligned}$$

Solving for $\underline{\pi}$ will yield

$$\underline{\pi} = \frac{-W}{\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)} < 0.$$

Profits can never be negative so the critical value to terminate an IPO is $\underline{\pi} = 0$. Once the firm starts the IPO process it will never withdraw the offering. The option to withdraw $P(\pi, \underline{\pi}, W)$ will never be used and is worthless. The critical value $\overline{\pi}$ to induce an IPO is found by using the boundary conditions (5.6) and (5.7)

$$B_1\overline{\pi}^{\beta_1} + \alpha\frac{\overline{\pi}}{\delta^p} = \alpha\overline{\pi}\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)e^{-\delta^p T} + \alpha\frac{\overline{\pi}}{\delta^p} - C, \tag{8.22}$$

$$B_1\overline{\pi}^{\beta_1}\frac{\beta_1}{\overline{\pi}} + \frac{\alpha}{\delta^p}. = \alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)e^{-\delta^p T} + \frac{\alpha}{\delta^p}. \tag{8.23}$$

Solving for $B_1\overline{\pi}^{\beta_1}$ in (8.23) gives

$$B_1\overline{\pi}^{\beta_1} = \frac{\overline{\pi}}{\beta_1}\alpha\left(\frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p}\right)e^{-\delta^p T}. \tag{8.24}$$

Substituting into (8.22) solves for $\overline{\pi}$:

$$\overline{\pi} = \left( \frac{\beta_1}{\beta_1 - 1} \right) \frac{C}{\alpha \left( \frac{(1-\lambda)}{\delta^m} - \frac{1}{\delta p} \right) e^{-\delta^p T}}. \tag{8.25}$$

The value of $B_1$ is

$$B_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{\beta_1^{\beta_1}} \left( \frac{1-\lambda}{\delta^m} - \frac{1}{\delta^p} \right)^{\beta_1} \frac{\alpha^{\beta_1} e^{-\beta_1 \delta^p T}}{C^{\beta_1 - 1}}.$$

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Figure 1: The Effect of Volatility on the Critical Value
$\pi^*$ is measured in millions of dollars
( $\mu$ = 0.10 and $\rho^m$ = 0.12 )

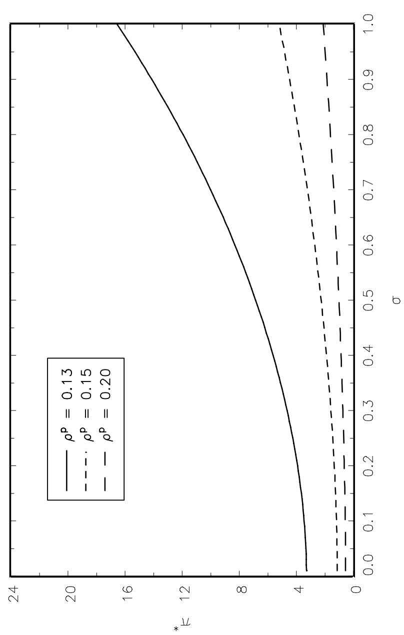


Figure 2: The K–Factor Increase in the Critical Value from Waiting
( $\mu$ = 0.10 and $\rho^m$ = 0.12 )

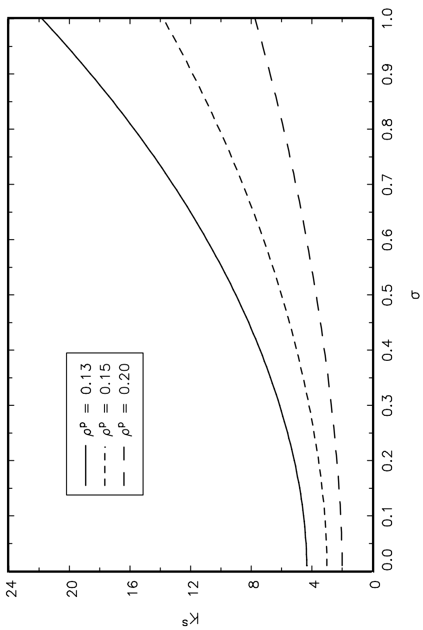


Figure 3: The Effect of the Growth Rate on the Critical Value
( $\rho^m$ = 0.12 and $\sigma$ = 0.3 )