An Analytical Theory of Project Investment
A Comparison with Real Option Theory

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First draft: September, 2002
Current draft: June, 2003

We thank David Carter, Steven Clark, Alex Ng, Tie Su and participants of EFA for helpful comments.

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Abstract

The real option theory offers great insights about project investment by capturing its analogy with financial options. However, at present, most articles on this subject either use stylized numerical examples or adopt a purely conceptual approach to describe how option pricing can be used in capital budgeting. In this work, we observe a fundamental difference between the problems of project investment and financial option, that project investment is a forward problem while option pricing is a backward problem. From this simple observation, we develop an analytical theory of project investment, which, for the first time in economic literature, provides an analytic formula that explicitly represents the relation among fixed costs, variable costs, uncertainty of the environment and the duration of a project, which is the core concern in most business projects and other economic decisions. While empirical research has lagged considerably behind the conceptual and theoretical contributions in the real option framework, this analytical theory captures the universal empirical regularity uncovered in many different fields.

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1. Introduction

Many projects that are expected to offer a high rate of return, such as in pharmaceutical industry, IT industry, or exploration of natural resources, often require significant outlay of funds before products can be brought to the markets. If market conditions change unexpectedly, the sunk investment is generally worth very little. So it is of great importance to develop an analytical theory to understand how the fixed investment of a project may influence the future earnings under different environment. Many articles apply the option pricing theory to offer insight about project investment. (Brennan and Schwartz, 1985; McDonald and Siegel, 1985; Dixit and Pindyck, 1994) The real option theory captures some of the basic conditions in project investment, "namely irreversibility, ongoing uncertainty and some leeway in timing" (Dixit and Pindyck, 1994, p.23). "It seems clear that the incorporation of contingent claims analysis into capital budgeting decision-making promises to revolutionize the way corporations organize and assess their investment programs" (Megginson, 1997, p. 292). However, "at present, most articles on this subject either use stylized numerical examples or adopt a purely conceptual approach to describing how option pricing can be used in capital budgeting" (Megginson, 1997, p. 292). What, then, is missing in making the promise of a revolution into a reality?

The answer lies at the fundamental difference between the problems of project investment and financial option. The problem of option pricing is to estimate option price when the payoff structure at the end of a contract is given. The problem of project investment is to estimate variable cost in production when irreversible fixed cost is invested or committed at the beginning of a project. So project investment is an initial value problem while option pricing is an end value problem. This simple observation helps us to develop an analytical theory of project investment. The basic ideas are the following.

The evolution of the value of an economic commodity can be represented by lognormal processes. A business project that is designed to produce such commodity involves fixed cost and variable cost. Irreversible fixed cost is spent or determined at the beginning of a project. Variable costs, on the other hand, are influenced by the state of output. For example, if consumer taste about the product of a project changes very fast, it will increase the variable cost in production. Since variable cost is a function of the output of a project, from Feynman-Kac Formula, it satisfies a partial differential equation very similar to Black-Scholes equation. (Øksendal, 1998) The difference is that this equation is a forward equation while the Black-Scholes equation is a backward equation. Applying non-arbitrage argument, we obtain the initial condition for this equation.

With fixed cost and duration of the project determined, we derive an analytic formula of variable cost as a function of fixed cost, duration of the project and uncertainty of environment. In a special case, this formula takes the same form as the well-known Black-Scholes formula for European call options. This similarity explains why many insights from option theory can be applied to investment problems. With this analytical formula, the theory provides a framework that directly models the cash flows in business projects. It enables us to make detailed estimation of returns of different types of investment under different kinds of environment.

The main insight from real option theory is that companies have options to wait on projects. The higher the uncertainty, the higher is the value to wait. This analytical theory offers a clear understanding of this option value. From this theory, it can be derived that as fixed costs of a project increase, variable costs decrease rapidly in a low uncertainty environment and decrease very little in a high uncertainty environment. So the option to wait is not only related to environmental uncertainty, but also to the level of fixed cost of projects. If the fixed costs of the projects can be low, small companies will enter the fields early. So the option value to wait mainly concentrates on projects with high fixed costs. For example, projects in natural resource exploration generally require high initial capital investment because of natural environment and projects in pharmaceutical industry require high cost in research and development because of regulatory environment. This explains why it is in the industries of high entry barriers that real option theory is often applied.

Real option theory states that options to wait arise from the earlier investment made by companies. (Dixit and Pindyck, 1994) The know-how, brand recognition and other company specific resources often allowed established companies to delay irreversible investment until the expected rate of return over a project is much higher than the cost of capital. On the other hand, statistics show small firms account for a disproportionately high share of innovative activity. (Acs and Audretsch, 1990) Despite the huge financial resources of large firms, they often are unable to compete effectively against small companies in emerging industries with great profit potential. This analytical theory provides reconciliation of the apparent inconsistency between established companies' options to wait in some cases and their inability to compete effectively in other cases. By necessity, large firms adopt more rigorous and systematic approach in evaluating and developing projects than small firms. Projects undertaken by large companies often need higher fixed cost than those by small companies. Since higher fixed cost projects enjoy advantage in significantly lower variable cost in production only when the uncertainty is low, large and established companies enjoy options to wait on projects in their own fields and are less effective in entering new markets with high uncertainty. This means that option to wait in one field can be detriment to entry in another field.

Authors in real option theories often believe that this theory can be potential applied to very broad areas. However, empirical research has lagged considerably behind these conceptual and theoretical contributions. (Moel and Tufano, 2002; Bulan, 2003) This sharp contrast indicates that better theoretical framework is needed to understand empirical regularities. The analytical theory of project investment developed in this paper shows that the level of fixed cost is the most fundamental property of a system. Indeed, biological species are classified as low fixed cost r-strategists and high fixed cost K-strategists. (MacArthur and Wilson, 1967) Cultures are classified as high context (fixed cost) cultures and low context cultures (Hall, 1977) Firms are classified as highly competitive large (high fixed cost) firms in mature markets and highly innovative small (low fixed cost) firms in new markets. (Acs and Audretsch, 1990) High fixed cost large banks concentrate on standard financial products with high volumes, such as credit card business, while low fixed cost community banks concentrate on small business loans that need individual judgment case by case. (DeYoung, Hunter and Udell, 2003) The chief concern about migration and trade is between high fixed cost North and low fixed cost South. (Hamilton and Whalley, 1984; Krugman and Venables, 1995) Population theory studies the fertility rate according to the level of living standard (fixed cost). (Dasgupta, 1995) The list can go on and on. Empirical studies in many different fields classify the systems by the level of their fixed costs, although different terminologies are employed in different fields. The universality of empirical regularity demonstrates the universality of the analytical theory, which will enable the insights developed from well established fields, such as biology, to be easily applied to newer fields. This has been done in other works. (Chen, 2003) In this paper, we will concentrate on the technical aspect of the analytical theory and its relation with the real option theory.

The paper is organized as follows. Section 2 derives the analytical results of project investment. Section 3 discusses the return of various projects under different environments and explains how this analytical theory can refine the results of the real option theory. Section 4 concludes.

2. Basic theory

Suppose *S* represents economic value of a commodity, *r*, the expected rate of change of value and σ, the rate of uncertainty. Then the process of *S* can be represented by the lognormal process

$$\frac{dS}{S} = rdt + \sigma dz \,. \tag{1}$$

The production of the commodity involves fixed cost, *K*, and variable cost, *C*, which is a function of *S*, the value of the commodity. If the discount rate of a firm is *q*, from Feymann-Kac formula, (Øksendal, 1998, p. 135) the variable cost, *C*, as a function of *S*, satisfies the following equation

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - qC \tag{2}$$

with the initial condition

$$C(S,0) = f(S) \qquad (3)$$

where *f(S)* is the variable cost of a project with a duration that is infinitesimally small. When the duration of a project is infinitesimal small, it has only enough time to produce one piece of product. In this situation, if the fixed cost is lower than the value of the product, the variable cost should be the difference between the value of the product and the fixed cost to avoid arbitrage opportunity. If the fixed cost is higher than the value of the product, there should be no extra variable cost needed for this product. Mathematically, the initial condition for the variable cost is the following

$$C(S,0) = \max(S - K, 0) \qquad (4)$$

where S is the value of the commodity and K is the fixed cost of a project. When the duration of a project is T, solving equation (2) with the initial condition (4) yields the following solution

$$C = Se^{(r-q)T} N(d_1) - Ke^{-qT} N(d_2) \qquad (5)$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function *N(x)* is the cumulative probability distribution function for a standardized normal random variable. When the discount rate of the firm is equal to r, the rate of change of the commodity value, formula (5) takes the same form as the well-known Black-Scholes (1973) formula for European call options

$$C = SN(d_1) - Ke^{-rT} N(d_2) \qquad (6)$$

This result is easy to understand. When the discount rate, q, is equal to r, Equation (2) becomes

$$\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{7}$$

while the Black-Scholes equation is

$$-\frac{\partial C}{\partial t} = rS\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} - rC \tag{8}$$

The only difference between the two equations is the direction of time. Both the evolution of share prices and value of economic commodities are represented by lognormal processes. For a financial option, the strike price at the end of the contract is known. The problem in option theory is to estimate the option price when the strike price, as well as several other parameters, is given. For a business project, irreversible fixed investment is determined at the beginning of a project. The problem in project investment is to estimate variable costs when fixed costs, as well as other factors, are given. Mathematically speaking, option theory solves a backward equation derived from a lognormal process for option prices with a known end condition, the strike price; the problem in project investment is to solve a forward equation derived from a lognormal process for variable costs with a known initial condition, the fixed investment. The similarity between these two problems explains why the option theory becomes so important in understanding project investment and other economic problems.

This theory, for the first time in economic literature, provides an analytic theory that explicitly represents the relation among fixed costs, variable costs, uncertainty of the environment and the duration of a project, which is the core concern in most business projects.

A new theory is ultimately justified by its implications. We will look at the properties and implications of this theory. For simplicity, we will only examine formula (6), the special case when the discount rate is equal to r. Several properties can be derived from (6). First, when the fixed costs, K, are higher, the variable costs, C, are lower. Second, for the same amount of fixed cost, when the duration of a project, T, is longer, the variable cost is higher. Third, when uncertainty, σ, increases, the variable cost increases. Fourth, when the fixed cost approaches zero, the variable cost will approach to the value of the product. Fifth, when the value of the product approaches zero, the variable cost will approach zero as well. All these properties are consistent with our intuitive understanding of production processes.

3. The detailed analysis of performance of business projects

An analytical framework enables us to make detailed estimation of returns of different projects under different kinds of environment. First, we examine the relation between fixed cost and variable cost with different level of uncertainty. The variable cost of a production mode is an increasing function of uncertainty. As fixed costs are increased, variable costs, calculated from (6), decrease rapidly in a low uncertainty environment and decreases slowly in a high uncertainty environment. (Figure 1)

Then we discuss the returns of investment on different projects with respect to the volume of output. For simplicity, we assume the duration of the project is one year. *K* is the fixed cost of production and *C(K, σ)* is the variable cost. Suppose the volume of output is Q, which is bound by production capacity or market size. Then the total value of the products and the total cost of production are

$$SQ \text{ and } C(K,\sigma)Q + K. \tag{9}$$

respectively. The return that this producer earns is

$$\ln\left(\frac{SQ}{C(K,\sigma)Q + K}\right) \tag{10}$$

Figure 2 is the graphic representation of (10) for different levels of fixed costs. From Figure 2, we can observe that, higher fixed cost investments, which have lower variable costs in production, need higher output volume to breakeven.

From the above discussion, the level of fixed investment in a project depends on the expectation of the level of uncertainty of production technology and the size of the market. When the outlook is stable, projects with high fixed investment are more efficient and when the outlook is uncertain, projects with low fixed cost are more flexible. When the market size is big, higher fixed cost projects, with their lower variable costs, are more competitive.

At firm level, higher fixed cost large firms have lower variable costs and hence benefit more from increasing return. Lower fixed cost small firms are better at handling uncertain situations. So firms of different sizes will choose different types of markets. For example, large banks, as high fixed cost systems, concentrate on standard financial products with high volumes, such as credit card business. Small community banks, as low fixed cost systems, concentrate on small business loans that need individual judgment case by case. (DeYoung, Hunter and Udell, 2003)

The real option theory shows that firms often require the rate of return on investment to be substantially higher than the cost of capital. The analytical theory provides clear understanding about the nature of this option and further shows how different firms make entry decisions differently.

The option to wait comes from the previous fixed investment. (Dixit and Pindyck, 1994, p. 9) Large firms with high level of fixed assets have more options to wait on projects that are along their own technological trajectories, where uncertainty is low for them. From (6), firms with high fixed assets enjoy low variable costs, which make them highly competitive in the market. The low variable cost is the source of the option to wait. How long a firm can wait depends on the level of its variable cost, which is a function of fixed assets and uncertainty. Take Microsoft for example. From time to time, Microsoft develops its own application softwares to replace popular application softwares from other vendors, such as Word to WordPerfect, Excel to Lotus 123 and IE to Netscape. Why Microsoft can have this option to wait until other companies have developed popular products? It is because Microsoft has huge amount fixed asset in its dominant operating systems. Application softwares are developed on top of the operating systems, which are upgraded from time to time. Compared with outsider developers, software development inside Microsoft faces less uncertainty to upgrade application softwares to take advantage of the improvement from the new versions of operating systems. Microsoft also has huge marketing network for its operating systems. Since Microsoft bundle an application software together with the operating system, its variable cost for distributing an application software is lower than other software makers, who have to market their products separately. Because of this option to wait, Microsoft saves a lot of money on R&D and marketing research to develop softwares that have huge market potential. It can wait until other companies develop highly profitable application softwares before it decides to enter the market and internalize a popular product into its window system.

While high fixed cost large firms are highly competitive and enjoy options to wait on projects in their own fields, they are less effective in entering new markets with high uncertainty. From Figure 1, 2, high fixed cost systems need large market size to break even and are more sensitive to uncertainty. Small firms, however, can explore niche markets easily because their low fixed costs make them more flexible. This is why small firms account for a disproportionately high share of innovative activity. (Acs and Audretsch, 1990) Empirical evidences show that small firms are less sensitive to uncertainty than large firms. (Bulan, 2003) Despite the huge financial resources of large firms, it is usually the small firms that pioneer in emerging industries.

From the above discussion, we find that different firms require different rate of return for their investment. Small firms requires lower rate of return for they are more flexible and hence suffer less when environment changes. At the same time, they have less option to wait because higher profit potential will trigger the entry of more competitive large firms. Large firms require higher rate of return for their investment. They can wait longer than small firms but have less freedom to explore new territories, where uncertainty is high.

Can cash rich large firms diversify into new industries so they can hold leading positions in potentially highly profitable ventures? Statistics show that firms that are more focused generate higher returns than those more diversified ones. (Lang and Stulz, 1994; Comment and Jarrell, 1995) As high fixed cost systems are highly optimized structures in producing particular products, innovation, with its inevitable uncertainty, is often very disruptive in high fixed cost systems. Only innovative ideas that are compatible with the current architecture and that have a large expected market, can be adopted by high fixed cost large firms with little internal frictions. That is why it is often difficult for successful firms to diversify into new and potentially highly profitable industries. The tradeoff between the efficiency of high fixed cost systems and flexibility in low fixed cost systems, which is an important result from real option theory, becomes very clear in this analytical theory of project investment.

Production capacity or market size is closely related to fixed costs and variable costs. When the market demand is up, firms generally increase production capacity by offering overtime pay, which increase variable costs, before hiring more permanent staff, which increase fixed costs. (Dixit and Pindyck, 1994) The following example illustrates the relation between fixed cost, variable cost and market size. If a software is targeted to sophisticated users, its interface can be simple, which reduce development cost and its sales effort can be small, which reduce variable cost. If the software developer considers increasing the market size by targeting general users, the interface of the software needs to be very intuitive with many help facilities, which increase development cost and its sales effort and after sales service can be substantial for less sophisticated users, which increase variable cost. Since the increase of market size often involve both the increase of variable cost and fixed cost, most projects are designed that the marginal cost to be much lower than the product value.

4. Concluding remarks

In this work, we develop an analytical theory of project investment by observing the fundamental difference between project investment and financial options. It extends and clarifies many ideas developed from real option theory, which captures the analogies between project investment and financial options. Since most problems in social and biological systems can be understood as project investment in various forms, this analytical theory offers a simple and unified understanding to a broad range of problems.

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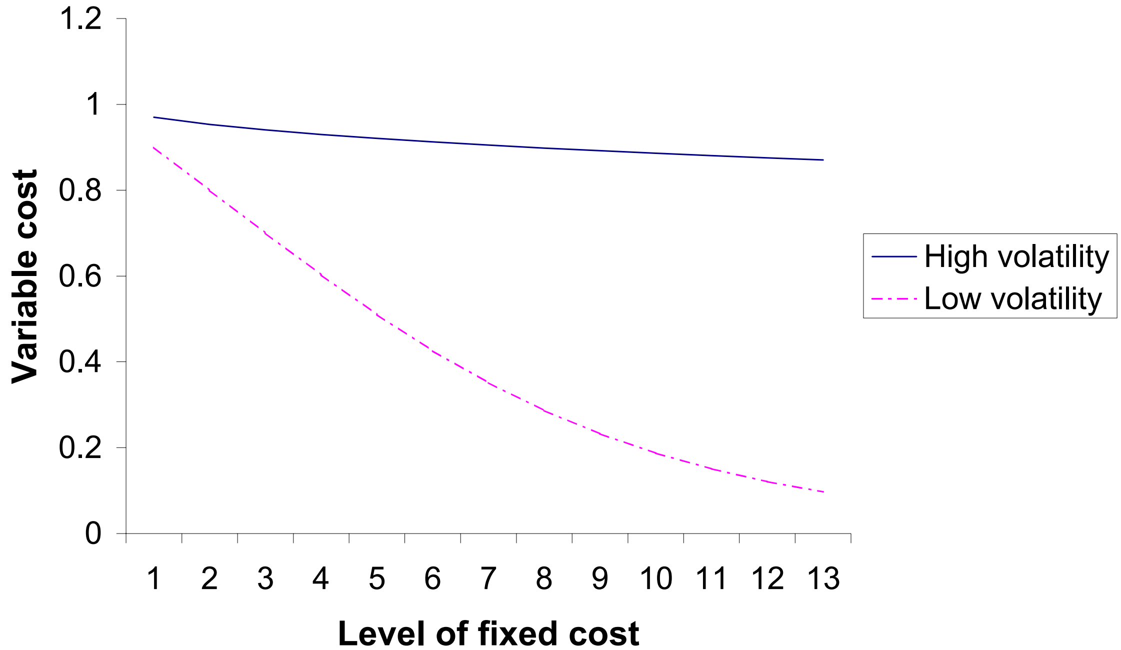
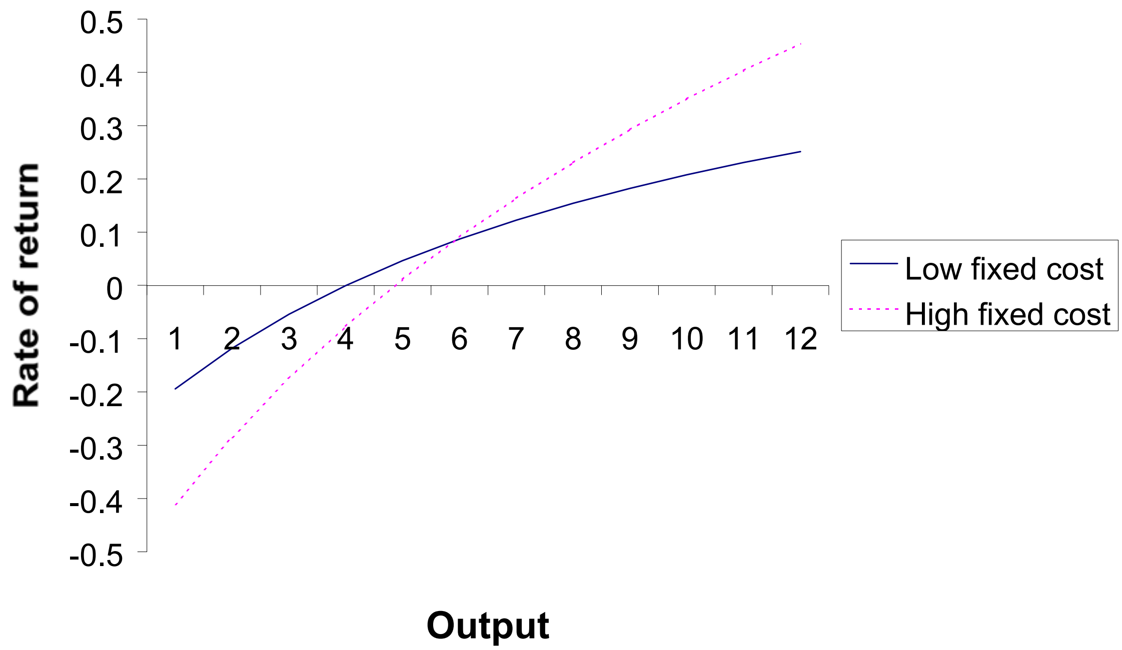
Figure captions

Figure 1. Uncertainty and variable cost
Figure 2. Output and return with different levels of fixed costs


1.2
1
0.8
0.6
0.4
0.2
0
Variable cost
1
2
3
4
5
6
7
8
9
10
11
12
13
Level of fixed cost
High volatility
Low volatility


0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
Rate of return
1
2
3
4
5
6
7
8
9
10
11
12
Output
Low fixed cost
High fixed cost