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Abstract

This paper provides a model of optimal investment timing in a decentralized firm under conditions of agency and asymmetric information. Using a real options approach, we show that an underlying option to invest can be decomposed into two components: a manager's option and an owner's option. The terms of the manager's option are determined by an optimal contracting model, and provide an incentive for the manager to both extend effort and truthfully reveal his private information. The implied investment behavior differs significantly from that of standard real options models. In particular, there will be greater inertia in investment, in that the model leads to the manager having an even greater "option to wait" than the owner. The interplay between the twin forces of hidden information and hidden action leads to markedly different investment outcomes than when only one of the two forces is at work.

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1 Introduction

One of the most important topics in corporate finance is the formulation of the optimal investment strategies of firms. There are two components of the investment decision: how much to invest and when to invest. The first is the capital allocation decision and the second is the investment timing decision. The standard textbook prescription for the capital allocation decision is that firms should only invest in projects if their net present values (NPV) are positive. Similarly, a standard framework for the investment timing decision is the real options approach. The real options approach posits that the opportunity to invest in a project is analogous to an American call option on the investment project, and the timing of investment is economically equivalent to the optimal exercise decision for an option. The real options approach is well summarized in Dixit and Pindyck (1994) and Trigeorgis (1996).[1]

However, both the simple NPV rule and the standard real options approach fail to account for the incentive problems that confront decentralized firms. In most modern corporations, shareholders delegate the investment decision to managers, taking advantage of their special skills and expertise. In such decentralized settings, there are likely to be both information asymmetries (e.g., managers are better informed than owners about projected cash flows) and agency issues (e.g., unobserved managerial effort, perquisite consumption, empire building). A number of papers in the corporate finance literature provide models of capital budgeting under asymmetric information and agency.[2] The focus of this literature is on the first piece of the investment decision: the amount of capital allocated to managers for investment. Thus, this literature provides predictions on whether firms over- or under-invest relative to the benchmark of perfect information and no agency costs. The focus of this paper is on the second piece of the investment decision: the timing of investment. We extend the real options framework to account for the issues of information and agency in a decentralized firm. Analogous to the notions of over- or under-investment, our paper provides results on hurried or delayed investment.

[1] The application of the real options approach to investment is quite broad. Brennan and Schwartz (1985) use an option pricing approach to analyze investment in natural resources. McDonald and Siegel (1986) provided the standard continuous-time framework for analysis of a firm's investment in a single project. Majd and Pindyck (1987) enrich the analysis with a time-to-build feature. Dixit (1989) uses the real option approach to examine entry and exit from a productive activity. Triantis and Hodder (1990) analyze manufacturing flexibility as an option. Titman (1985) and Williams (1991) use the real options approach to analyze real estate development.

[2] See Stein (2001) for a useful summary.

In the standard real options paradigm, there are no agency conflicts as it is assumed that the option's owner makes the exercise decision.[3] However, in this paper, an owner delegates the option exercise decision to a manager. Thus, the timing of investment is determined by the manager. The owner's problem is to design an optimal compensation contract under both hidden action and hidden information. The true quality of the underlying project can be high or low. The hidden action problem is that the manager can influence the likelihood that the quality of the project is high. An optimal contract will have the property that the manager will be induced into providing costly (but unverifiable) effort.[4] The hidden information problem is that the underlying project's future cash flow contains a component that is only privately observable to the manager. Absent any mechanism that induces truth-telling, the manager may have an incentive to lie about the true quality of the project and divert value for his private interests. For example, the manager could divert privately observed cash flows by consuming excessive perquisites, building empires, or by working less hard to manage the project. An optimal contract (in accordance with the revelation principle) will induce the manager to truthfully reveal his private information about project quality, and thus no actual value diversion will take place in equilibrium.

Importantly, we show that the underlying option can be decomposed into two components: a "manager's option" and an "owner's option." The manager's option has a payout upon exercise that is a function of the contingent compensation contract. Based on this contractual payout, the manager determines the exercise time. The owner's option has a payout, received at the manager's chosen exercise time, equal to the payoff from the underlying option minus the manager's compensation. The model provides the solution for the optimal compensation contract that comes as close as possible to the first-best no-agency solution.

The model implies investment behavior that differs substantially from that of the standard real options approach with no agency problems. In general, managers will display greater inertia in their investment behavior, in that they will invest later than implied by the first-best solution. In essence, this is a result of the manager (even in an optimal contract) not

[3] While our paper focuses on the agency issues that provide a disconnect between the interests of owners and shareholders, similar issues exist between stockholders and bondholders. Mello and Parsons (1992), Mauer and Triantis (1994), Leland (1998), Mauer and Ott (2000), and Morellec (2001, 2003) examine the impact of agency conflicts on firm value using the real options approach.

[4] As is standard in problems of moral hazard, we only consider parameter ranges under which exerting effort is desirable for the owner. Outside of this parameter region, the problem reduces to one of hidden information only.

having a full ownership stake in the option payoff. This less than full ownership interest implies that the manager has a greater "option to wait" than the owner.

An important aspect of the model is the interaction of hidden action and hidden information. In fact, we find that the nature of the optimal contract depends explicitly on the relative importance of these two forces. While we focus on the economically most interesting case in which both forces play a role in the optimal contract, it is instructive to consider two extremes. If the cost benefit ratio of inducing effort (a measure of the strength of the hidden effort component) is very low, then the hidden action component disappears from the optimal contract terms. Thus, if the nature of the underlying option is such that inducing effort is sufficiently inexpensive, then we are left with a simple problem of hidden information and the contract will simply reward the manager with informational rent. This is the setting of Maeland (2002), which considers a real options problem with only hidden information about the exercise cost.[5] Conversely, as the cost benefit ratio of inducing effort becomes very high, then the hidden action component dominates the optimal contract. The cost of inducing effort is so high as to no longer necessitate the payment of informational rents. When the cost benefit ratio of inducing effort is in an intermediate range, both forces are in effect, and the optimal compensation contract must induce effort and truthful revelation of private information. Interestingly, the interplay between hidden information and hidden action may actually reduce the inefficiency in investment timing, compared with the setting in which hidden information is the only friction. This is because the manager's additional option to exert effort makes his incentives more closely aligned with those of the owner.

In a later section, we generalize the model to allow for managers to display greater impatience than owners. There are several potential justifications for such an assumption. First, there are various models of managerial myopia that attempt to explain managers' preference for choosing projects with quicker pay-backs, even in the face of eschewing more valuable long-term opportunities.[6] Such models are based on information asymmetries and agency problems. Second, in our "investment timing" setting, greater impatience can represent the manager's preference for empire building or greater perquisite consumption and reputation

[5] Bjerksund and Stensland (2000) provide a similar model to Maeland (2002), where a principal delegates an investment decision to an agent who holds private information about the investment's cost. Brennan (1990) considers a setting in which managers attempt to signal the true quality of "latent" assets to investors through converting them into observable assets (e.g., exercising real options).

[6] See Narayanan (1985), Stein (1989) and Bebchuk and Stole (1993).

that comes from running a larger business sooner rather than later. Third, managers may simply have a shorter horizon (due to job loss, alternative job offers, death, etc.). Phrased in real options terms, managerial impatience leads to a smaller option to wait for the manager, potentially reversing the previously mentioned larger option to wait that appears in the base case model. While the base case model predicts that investment will never occur sooner than the first-best case, in this generalized setting investment can occur earlier or later than the first-best case.

The setting of our paper is most similar to that of Bernardo et al. (2001). In a decentralized firm under asymmetric information and moral hazard, they examine the capital allocation decision, while we examine the investment timing decision. In their model, the firm's headquarters delegates the investment decision to a manager, who possesses private information about project quality. The manager can improve project quality through the exertion of effort, which is costly to the manager but unverifiable by headquarters. These two assumptions mirror our framework. In addition, managers have preferences for "empire building" in that they derive utility from overseeing large investment projects. This assumption is addressed in the generalized version of our model that appears in Section 5. Absent any explicit incentive mechanism, managers will always claim that all projects are of high quality and worthy of funding, and then provide the minimal amount of effort. As in our paper, they use an optimal contracting approach to jointly derive the optimal investment and compensation policies. An incentive contract is derived that induces truth-telling and minimizes the costs of agency. In equilibrium, they find that there will be under-investment in all states of the world. Our model provides an intertemporal analogy to their equilibrium: in our base case model, we find that in equilibrium there will be delayed investment due to the information asymmetries and agency costs.[7]

While our paper derives an optimal compensation contract that best aligns the incentives of owners and managers, other papers in the corporate finance literature analyze the capital

[7] In a different setting, Holmstrom and Ricart i Costa (1986) provide a model that combines an optimal wage contract with capital rationing. In their model, the manager and the market learn about managerial talent over time by observing investment outcomes. A conflict of interest arises because the manager wants to choose investment to maximize the value of his human capital while the shareholders want to maximize firm value. The optimal wage contract has the option feature that insures the manager against the possibility that an investment reveals his ability to be of low quality, but allows the manager to captures the gains if he is revealed to be of high quality. This option feature of the wage contract encourages the manager to take on excessive risks. Rationing capital mitigates the manager's incentive to over-invest. As a result, in equilibrium both under- and over-investment are possible.

budgeting problem under information asymmetry and agency using other control mechanisms. Harris et al. (1982) consider the case of capital allocation in a decentralized firm with multiple division managers. Managers have private information about project values. In addition, managers have private interests in overstating investment requirements, and then diverting the excess cash flows in order to minimize effort or to consume greater perquisites. They focus on the role of transfer prices in allocating capital. Firms offer managers a menu of allocation/transfer price combinations. In equilibrium, truth-telling is achieved, and there can be both under- and over-investment.[8] Harris and Raviv (1996) use a very similar framework, but focus on a random auditing technology. By combining probabilistic auditing with a capital restriction, headquarters is able to learn the true project quality from the manager. In equilibrium there will be both regions of under- and over-investment. Stulz (1990) considers a decentralized investment framework in which the manager has private information about investment quality and a preference for empire building. Absent any controls, the manager would always overstate the investment opportunities and invest all available cash. The owners of the firm use debt as a mechanism of aligning the interests of managers and shareholders. By increasing the required debt payment, managers have less free cash flow to spend on investment projects. The optimal level of debt is chosen to trade off the benefits of preventing managers from investing in negative NPV projects when investment opportunities are poor with the costs of rationing managers away from taking positive NPV projects when investment opportunities are good. Again, in equilibrium there will be both under- and over-investment.

The remainder of the paper is organized as follows. Section 2 describes the setup of the model and simplifies the optimization program. Section 3 solves for the optimal contracts. In Section 4, we analyze the implications of the model in terms of the stock price's reaction to investment, equilibrium investment lags, and the erosion of the option value due to the agency problem. Section 5 generalizes the model to allow for managers to display greater impatience than owners. Section 6 concludes. Appendices contain the solution details of the optimal contracts.

[8] Antle and Eppen (1985) provide a model that is very similar to that of Harris et al (1982).

2 Model

In this section, we begin with a description of the model. We then, as a useful benchmark, provide the solution to the first-best no-agency investment problem. Finally, we present and simplify the full principal-agent optimization problem faced by the owner.

2.1 Setup

The principal owns an option to invest in a single project. We assume that the principal (owner) delegates the exercise decision to an agent (manager). Once investment takes place, the project provides a stream of cash flows. A portion of the cash flows is observable to both owner and manager, while another portion is privately observed only by the manager. Let $P(t)$ represent the present value of the observable component of the project's cash flows, and θ the present value of the privately observed component of cash flows. Thus, the total value of the project is $P(t) + \theta$.[9]

The assumption that a portion of cash flows is only known by the manager and not verifiable by the owner is quite common in the capital budgeting literature. This information asymmetry invites a host of agency issues. Harris et al. (1982) posit that managers have incentives to understate project payoffs and to divert the free cash flow to themselves. In their model, such value diversion takes the form of managers reducing their level of effort. Stulz (1990), Harris and Raviv (1996) and Bernardo et al. (2001) model managers as having preferences for perquisite consumption or empire building. In these models, managers have incentives to divert free cash flows to inefficient investments or to excessive perquisites. In all of these models, mechanisms are used by firms (i.e., incentive contracts, auditing, required debt payments) to mitigate such value diversion.

In a standard call option setting, exercise yields the difference between the observable value $P(t)$ of the underlying asset and the exercise price, K. Thus, the payoff from exercise is typically $P(t) - K$. However, in the present model, the payoff from exercise also includes a privately observed random variable, θ, whose realization directly impacts the option payoff.

[9] For ease of presentation, we model the process $P(t)$ for the present value of observable cash flows. We could back up a step, and begin with an underlying process for observable cash flows. However, if observable cash flows follow a geometric Brownian motion, then the present value of expected future observable cash flows will also follow a geometric Brownian motion as above. Similarly, rather than modeling θ as the present value of unobservable cash flows, we could begin with an underlying process for the unobservable cash flows themselves.

Thus, in this model the net payoff from exercise is $P(t) + \theta - K$. Note that the problem could be equivalently formulated as one in which the total value of the project is $P(t)$ and the effective cost of exercising the option is $K - \theta$.

Let the value $P(t)$ of the observable component of the underlying project evolve as a geometric Brownian motion:

$$dP(t) = \alpha P(t)\, dt + \sigma P(t)\, dz(t), \tag{1}$$

where α is the instantaneous conditional expected percentage change in $P(t)$ per unit time, σ is the instantaneous conditional standard deviation per unit time, and dz is the increment of a standard Wiener process. Let P_0 equal the value of the project at time zero, in that $P_0 = P(0)$. Both the owner and the manager are risk neutral, with the risk-free rate of interest denoted by r.[10] For convergence, we assume that $r > \alpha$.

The private component of value, θ, may take on two possible values: θ_1 or θ_2, with $\theta_1 > \theta_2$.[11] We denote $\Delta\theta = \theta_1 - \theta_2 > 0$. One may interpret a draw of θ_1 as a "higher quality" project and a draw of θ_2 as a "lower quality" project. The effort of the manager plays an important role in determining the likelihood of obtaining a higher quality project. The manager may affect the likelihood of drawing θ_1 by exerting a one-time effort, at time zero. If the manager exerts no effort,[12] the probability of drawing a higher quality project θ_1 equals q_L. However, if the manager exerts effort, he incurs a cost $\xi > 0$ at time zero, but increases the likelihood of drawing a higher quality project θ_1 from q_L to q_H. Immediately after his exerting effort, the manager observes the private component of project quality. In order to ensure a positive "net" exercise price, we restrict $\theta_1 < K$.

The owner does not observe the private component of value, θ, and thus cannot contract on θ. However, the manager can contract on the observable component of value, $P(t)$. Contingent on the level of $P(t)$ at exercise, the manager is paid a wage.[13] The revelation

[10] We rule out the time-zero selling-the-firm contract between the owner and the manager. This may be justified, for example, if the manager is liquidity constrained and cannot obtain financing.

[11] In Section 3.3 we generalize the model to allow θ to have continuous distributions.

[12] Without loss of generality, we may normalize the manager's lower effort level to zero.

[13] In order for the compensation contract to provide any incentive to the agent, wages cannot be paid prior to the moment of exercise. However, the efficacy of paying wages after the moment of exercise depends on the information structure of the model. If θ remains perpetually unobservable and/or not enforceable in courts (as we assume in the model), then there is no benefit to paying wages after exercise; future wages would simply be discounted back to the values used in the model. However, if θ becomes partially (or fully) revealed at some future date, then the adverse selection problem (but not the moral hazard problem) can be mitigated through delaying the payment of wages until a future date.

principle will ensure that the private component of value will be truthfully revealed to the owner at exercise. The manager has limited liability and is always free to walk away.[14] In addition, renegotiation is not allowed. While commitment leads to inefficiency in investment timing *ex-post*, it increases the value of the project *ex-ante*.

In summary, the owner faces a problem with both hidden information (the owner does not observe the true realization of θ) and hidden action (the owner cannot verify the manager's effort level). The owner needs to provide compensation incentive to both (i) induce the agent exert effort at time zero and (ii) to have the agent reveal his type voluntarily and truthfully. Before analyzing the optimal contract, we first briefly review the first-best no-agency solution that is used as the benchmark.

2.2 First-Best Benchmark (The Standard Real Options Case)

As a benchmark, we consider the case in which there is no delegation of the exercise decision and the owner observes the true value of θ. Equivalently, this first-best solution can be achieved in a principal-agent setting, provided that θ is both publicly observable and contractible. Let $W(P;\theta)$ denote the value of the owner's option, in a world where θ is a known parameter and P is the current level of $P(t)$. Using standard arguments [i.e., Dixit and Pindyck (1994)], $W(P;\theta)$ must solve the following differential equation:

$$0 = \frac{1}{2}\sigma^2 P^2 W_{PP} + \alpha P W_P - rW. \tag{2}$$

Differential equation (2) must be solved subject to appropriate boundary conditions. These boundary conditions serve to ensure that an optimal exercise strategy is chosen:

$$\begin{aligned} W\left(P^*(\theta),\theta\right) &= P^*(\theta) + \theta - K, & (3)\\ W_P\left(P^*(\theta),\theta\right) &= 1, & (4)\\ W(0,\theta) &= 0. & (5) \end{aligned}$$

Here, $P^*(\theta)$ is the value of $P(t)$ that triggers entry. The first boundary condition is the value-matching condition. It simply states that at the moment the option is exercised, the

[14]The limited-liability condition is essential in delivering the investment inefficiency result in this context. Otherwise, with risk-neutrality assumptions for both the owner and the manager, and no limited liability, the first-best optimal investment timing may be achieved even in the presence of hidden information and hidden action. For a related discussion of limited liability, see Innes (1990). An alternative mechanism of generating investment inefficiency in an agency context is to assume managerial risk aversion.

payoff is $P^*(\theta)+\theta-K$. The second boundary condition is the smooth-pasting or high-contact condition.[15] This condition ensures that the exercise trigger is chosen so as to maximize the value of the option. The third boundary condition reflects the fact that zero is an absorbing barrier for $P(t)$.

Closed-form solutions for the value of the owner's option at time zero, $W(P_0, \theta)$, and the exercise trigger $P^*(\theta)$ are easily obtained. The value of the first-best option value and exercise trigger can be written as:

$$W(P_0;\theta) = \begin{cases} \left(\frac{P_0}{P^*(\theta)}\right)^{\beta} (P^*(\theta) + \theta - K\,)\,, & \text{for } P_0 < P^*(\theta), \\ \\ P_0 + \theta - K, & \text{for } P_0 \geq P^*(\theta), \end{cases} \tag{6}$$

where

$$P^*(\theta) = \frac{\beta}{\beta - 1}\,(K - \theta)\,, \tag{7}$$

and

$$\beta = \frac{1}{\sigma^2}\left[-\left(\alpha - \frac{\sigma^2}{2}\right) + \sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2r\sigma^2}\,\right] > 1. \tag{8}$$

Since the realized value of θ can be either θ_1 or θ_2, we denote $P^*(\theta_1) = P_1^*$ and $P^*(\theta_2) = P_2^*$. We shall always assume the initial value of the project is less than the lower trigger, $P_0 < P_1^*$, to ensure that there is always some positive option value inherent in the project.

The *ex-ante* value of the owner's option in the first-best no agency setting is $q_H W(P_0;\theta_1) + (1 - q_H)W(P_0;\theta_2)$. We can therefore write this first-best option value, $V^*(P_0)$, as:

$$V^*(P_0) = q_H \left(\frac{P_0}{P_1^*}\right)^{\beta} (P_1^* + \theta_1 - K) + (1 - q_H)\left(\frac{P_0}{P_2^*}\right)^{\beta} (P_2^* + \theta_2 - K)\;. \tag{9}$$

It will prove useful in future calculations to define the present value of one dollar received at the first moment that a specified trigger $\hat{P}$ is reached. Denote this present value operator by the discount function $D(P_0;\hat{P})$. This is simply the solution to differential equation (2) subject to the boundary conditions that $D(\hat{P};\hat{P}) = 1$, and $D(0;\hat{P}) = 0$. The solution can be written as:

$$D(P_0;\hat{P}) = \left(\frac{P_0}{\hat{P}}\right)^{\beta}, \quad P_0 \leq \hat{P}. \tag{10}$$

[15] See Merton (1973) for a discussion of the high-contact condition.

2.3 A Principal-Agent Setting

The owner offers the manager a contract at time zero. The contract specifies a payment made to the manager, paid at the time of exercise. The owner is committed to implementing the contract. The payment can be made contingent on the observable component of the value of the project at the time of exercise. Thus, in principle, for any realized value of $P(t)$ obtained at the time of exercise, $\hat{P}$, a contracted wage $w(\hat{P})$ can be specified, provided that $w(\hat{P}) > 0$.

The principal-agent setting leads to a decomposition of the underlying option into two options: an owner's option and a manager's option. The owner's option has a payoff function of $\hat{P}+\theta-K-w(\hat{P})$, and the manager's option has a payoff function of $w(\hat{P})$. Upon exercise, the owner receives the value of the underlying project $(\hat{P}+\theta)$, after paying the exercise price (K) and the manager's wage $(w(\hat{P}))$. The manager's payoff is the value of the contingent wage, $w(\hat{P})$. Obviously, the sum of these payoff functions equals the payoff of the underlying option. The manager's option is of the tradition American call option variety, since the manager chooses the exercise time to maximize the value of his option. However, in this optimal contracting setting, it is the owner who ultimately controls the timing of exercise through the choice of contract parameters that induces the exercise policy that maximizes the value of its option. In addition, the manager also possesses a compound option, since the manager has the option to exert effort at time zero to increase the total expected surplus. The properties of the manager's option thus are contingent upon this initial effort choice.

Since there are only two possible values of θ, for any $w(\hat{P})$ schedule, there can be at most two wage/exercise trigger pairs that will be chosen by the manager.[16] Thus, the contract need only include two wage/exercise trigger pairs from which the manager can choose: one that will be chosen by a manager when he observes θ_1, and one chosen by a manager when he observes θ_2. Therefore, the owner will offer a contract that promises a wage of w_1 if the option is exercised at P_1 and a wage of w_2 if the option is exercised at P_2. The revelation principle will ensure that a manager who privately observes θ_1 will exercise at the P_1 trigger, and a manager who privately observes θ_2 will exercise at the P_2 trigger.

The owner's option has a payout function of $P_1+\theta_1-K-w_1$, if $\theta=\theta_1$, and $P_2+\theta_2-K-w_2$,

[16] We allow for the possibility of a pooling equilibrium in which only one wage/exercise trigger pair is offered. However, this pooling equilibrium always will be dominated by a separating equilibrium with two wage/exercise trigger pairs.

if $\theta = \theta_2$. Thus, using the discounting function $D(\,\cdot\,;\,\cdot\,)$ derived in (10), conditional on the manager exerting effort, the value of the owner's option, $\pi^o(P_0; w_1, w_2, P_1, P_2)$, can be written as:

$$\pi^o(P_0; w_1, w_2, P_1, P_2) = q_H D(P_0; P_1)(P_1 + \theta_1 - K - w_1) + (1 - q_H) D(P_0; P_2)(P_2 + \theta_2 - K - w_2). \tag{11}$$

The manager's option has a payout function of w_1 if $\theta = \theta_1$ and w_2 if $\theta = \theta_2$. Conditional on the manager exerting effort, the value of the manager's option, $\pi^m(P_0; w_1, w_2, P_1, P_2)$, can be written as:

$$\pi^m(P_0; w_1, w_2, P_1, P_2) = q_H\, D(P_0; P_1) w_1 + (1 - q_H)\, D(P_0; P_2) w_2. \tag{12}$$

For notational simplicity, we sometimes will drop the parameter arguments and simply write the owner's and manager's option values as $\pi^o(P_0)$, and $\pi^m(P_0)$, respectively.

The owner's objective is to maximize its option value through its choice of the contract terms w_1, w_2, P_1, and P_2. Thus, the owner solves the following optimization problem:

$$\max_{w_1, w_2, P_1, P_2} \quad q_H \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K - w_1) + (1 - q_H) \left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K - w_2)\,. \tag{13}$$

This optimization is subject to a variety of constraints induced by the hidden information and hidden action of the manager. The contract must induce the manager to accept the contract, exert effort, exercise at the trigger P_1 if $\theta = \theta_1$, and exercise at the trigger P_2 if $\theta = \theta_2$. It is the specification of these constraints to which we now turn.

There are both *ex-ante* and *ex-post* constraints. The *ex-ante* constraints ensure that the manager exerts effort and that the contract is accepted. These are the standard constraints in a static moral hazard setting. The *ex-post* constraints will ensure truth-telling in accordance with the revelation principle. That is, in equilibrium the manager will exercise at the trigger P_1 when it truly has a high quality project, and at the trigger P_2 otherwise. In addition, there will be *ex-post* constraints on limited liability.

- *ex-ante* incentive constraint:

$$q_H \left(\frac{P_0}{P_1}\right)^\beta w_1 + (1 - q_H) \left(\frac{P_0}{P_2}\right)^\beta w_2 - \xi \geq q_L \left(\frac{P_0}{P_1}\right)^\beta w_1 + (1 - q_L) \left(\frac{P_0}{P_2}\right)^\beta w_2. \tag{14}$$

The left side of this inequality is the value of the manager's option if effort is exerted minus the cost of effort. The right side is the value of the manager's option if no effort is exerted.

This constraint ensures that the manager will exert effort. Re-arranging the *ex-ante* incentive constraint (14) gives

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 - \left(\frac{P_0}{P_2}\right)^{\beta} w_2 \geq \frac{\xi}{\Delta q}, \tag{15}$$

where $\Delta q = q_H - q_L > 0$.

- *ex-ante* participation constraint:

$$q_H \left(\frac{P_0}{P_1}\right)^{\beta} w_1 + (1 - q_H) \left(\frac{P_0}{P_2}\right)^{\beta} w_2 - \xi \geq 0. \tag{16}$$

This constraint ensures that the total value to the manager of accepting the contract is non-negative.

The *ex-post* constraints ensure that managers will not have any incentive to divert value. As discussed at the beginning of Section 2.1, managers with private information have an incentive to misrepresent cash flows and divert free cash flows to themselves. For example, the manager may have an incentive to lie and claim that a higher quality project is a lower quality project, and then divert the difference in cash flows. This could be done by working less hard [as in Harris et al. (1982)], or by spending the free cash flows on perquisites or negative present value projects [as in Stulz (1990), Harris and Raviv (1996) and Bernardo et al. (2001)]. Importantly, these incentive compatibility conditions ensure that this value diversion does not occur; such deception only occurs off the equilibrium path.

- *ex-post* incentive constraints:

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 \geq \left(\frac{P_0}{P_2}\right)^{\beta} (w_2 + \Delta\theta)\,, \tag{17}$$

$$\left(\frac{P_0}{P_1}\right)^{\beta} (w_1 - \Delta\theta) \leq \left(\frac{P_0}{P_2}\right)^{\beta} w_2\,. \tag{18}$$

The second constraint will be shown not to bind, so only constraint (17) is relevant to our discussion. The first inequality ensures that a manager of a higher quality project will choose to exercise at P_1. By truthfully revealing the private quality θ_1 through exercising at P_1, the manager receives the wage w_1. This inequality requires the payoff from truthful revelation to be greater than or equal to the present value of the payoff from misrepresenting the private quality by waiting until the trigger P_2. The payoff from misrepresenting θ_1 as θ_2 is equal to the wage w_2, plus the value of diverting the private component of value $\Delta\theta$.

These truth-telling constraints are common in the literature on moral hazard and asymmetric information. For example, entirely analogous conditions appear in Bolton and Scharfstein (1990) and Harris et al. (1982).

- *ex-post* limited-liability constraints:

$$w_i \geq 0, \quad i = 1,\, 2. \tag{19}$$

Therefore, the owner's problem has a total of six inequality constraints: the *ex-ante* incentive and participation constraints, and each of the two *ex-post* incentive and limited-liability constraints. Fortunately, the following four propositions simplify the problem in that we can reduce the number of constraints to two.

Proposition 1. *The limited-liability condition for a manager of a θ_1-type project does not bind. That is, $w_1 > 0$.*

Proof.

$$w_1 \geq \left(\frac{P_1}{P_2}\right)^{\beta} (w_2 + \Delta\theta) \geq \left(\frac{P_1}{P_2}\right)^{\beta} \Delta\theta > 0,$$

The first and second inequalities follow from (17) and (19), respectively. ∎

In order to motivate the manager to exert effort, we need to reward the manager with an option value larger than zero, which is the manager's reservation value. This leads to the following result.

Proposition 2. *The ex-ante participation constraint (16) does not bind.*

Proof.

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 + \frac{1-q_H}{q_H}\left(\frac{P_0}{P_2}\right)^{\beta} w_2 - \frac{\xi}{q_H} \geq \frac{\xi}{\Delta q} - \frac{\xi}{q_H} > 0,$$

where the first inequality follows from *ex-ante* incentive constraint (15) and the limited liability condition for the type-θ_2 project. ∎

The following result states that there is no rent for the manager of a θ_2-type project.

Proposition 3. *The limited liability for a manager of a θ_2-type project binds, in that $w_2 = 0$.*

We verify the claim of this proposition in the appendix. The intuition is straightforward. Giving the manager of a θ_2-type project any positive rent implies a higher rent for managers of θ_1-type projects in order to meet the truth-telling constraint of the manager of a θ_1-type project. In order to minimize the rents subject to the manager's participation and incentive constraints, the owner shall give the manager of a θ_2-type project zero *ex-post* rent.

Proposition 4 allows us to ignore (18) in the optimization problem.

Proposition 4. *Optimal contracts imply $w_1 \leq \Delta\theta$.*

A formal proof of this proposition appears in the appendix. Intuitively, if $w_1 > \Delta\theta$, then the manager of a θ_2-type project would never accept the equilibrium contract with $w_2 = 0$. This would clearly be inconsistent with Proposition 3.

Propositions 1–4 jointly simplify the owner's optimization problem as follows:

$$\max_{w_1,\, P_1,\, P_2} \quad q_H \left(\frac{P_0}{P_1}\right)^{\beta} (P_1 + \theta_1 - K) \ - q_H \left(\frac{P_0}{P_1}\right)^{\beta} w_1 + (1 - q_H) \left(\frac{P_0}{P_2}\right)^{\beta} (P_2 + \theta_2 - K) \tag{20}$$

subject to

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 \ \geq \ \left(\frac{P_0}{P_2}\right)^{\beta} \Delta\theta\,, \tag{21}$$

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 \ \geq \ \frac{\xi}{\Delta q}. \tag{22}$$

In summary, we now have a simplified optimization problem for the owner. Equation (20) is the owner's option value. Constraint (21) is the simplified *ex-post* incentive constraint for the manager of the θ_1-type project. Constraint (22) ensures that it is in the manager's interest to extend his effort at time zero.

The following proposition demonstrates that at least one of the two constraints binds.

Proposition 5. *At least one of (21) and (22) binds.*

The argument is immediate. If Proposition 5 did not hold, then reducing w_1 will increase the owner's value strictly without violating any of the constraints. Note that the two constraints can be written more succinctly as

$$\left(\frac{P_0}{P_1}\right)^{\beta} w_1 \geq \max\left[\left(\frac{P_0}{P_2}\right)^{\beta} \Delta\theta, \frac{\xi}{\Delta q}\right]. \tag{23}$$

3 Model Solution: Optimal Contracts

In this section, we provide the solution to the optimal contracting problem described in the previous section: maximizing (20) subject to inequality constraints (21) and (22). We find that the nature of the solution depends on the parameter values. In particular, the solution depends explicitly on the magnitude of the cost benefit ratio of inducing the manager's effort. Depending on this magnitude, the optimal contract can take on three possible types: a "pure hidden information" type, a "joint hidden information/hidden action" type, and a "pure hidden action" type.

3.1 General Properties of the Solution

Before we provide the explicit solutions for the three contract regions, we discuss some general properties of contracts that hold for all regions.

The following proposition demonstrates that the manager of the higher quality project will exercise at the first-best level. Intuitively, for any manager's option value that satisfies the constraint (23), the owner will always prefer to choose the first best timing trigger P_1^*, and vary the wage w_1 to achieve the same level of compensation. On the margin, for the good state it is cheaper for the owner to increase the wage than to deviate from optimal timing.

Proposition 6. *The optimal contracts have $P_1 = P_1^*$, for all admissible parameter regions.*

Proof. Consider any candidate optimal contract $\left(\bar{w}_1, \bar{P}_1, \bar{P}_2\right)$ with $\bar{P}_1 \neq P_1^*$. The owner may improve his surplus by proposing an alternative contract $\left(\hat{w}_1, P_1^*, \bar{P}_2\right)$, in which $\hat{w}_1$ is chosen such that the manager's option has the same value as the first contract, in that $(P_0/P_1^*)^\beta \ \hat{w}_1 = \left(P_0/\bar{P}_1\right)^\beta \ \bar{w}_1$. The newly proposed contract is clearly feasible, as it will also satisfy constraints (21) and (22). For all such constant levels of the manager's option value, the owner's objective function (20) is maximized by choosing $P_1 = P_1^* = \arg\max_x \ (P_0/x)^\beta \ (x + \theta_1 - K)$. ∎

As we shall now see, it is less costly for the owner to distort P_2 away from the first-best level than to distort P_1 away from the first-best level in order to provide the appropriate incentives to the manager. The next proposition demonstrates that delay (beyond first-best) for the lower quality project is needed in order to create enough incentives for the manager of a higher-quality project not to imitate the one with a lower-quality project.

Proposition 7. *For all admissible parameter regions, the investment trigger for a manager of a θ_2-type project is (weakly) later than the first-best, in that $P_2 \geq P_2^*$.*

Proof. Suppose $P_2 < P_2^*$. It is simple to show that this contract is dominated by the contract with $P_2 = P_2^*$. we can always increase P_2 without violating constraint (21). Moreover, the objective function (20) is increasing in P_2, for $P_2 < P_2^*$, irrespective of which constraint binds. Thus, any contract with $P_2 < P_2^*$ is dominated by one with $P_2 = P_2^*$. ∎

Intuitively, the necessity of inducing truth-telling leads the manager to display a greater "option to wait" than the first-best solution. In order to dissuade the manager of a θ_1-type project from exercising at the trigger P_2, the contract must sufficiently increase P_2 above P_2^* to make such "lying" unprofitable.

We shall see that the extent to which P_2 exceeds P_2^* depends explicitly on the relative strengths of the forces of hidden information and hidden action. The amount of suboptimal delay will vary across the three regions, and will be discussed in greater detail below.

3.2 Optimal Contracts

We first define the three regions that serve to determine the nature of the optimal contract. As a result of Proposition 5, the solution will depend on which of the two constraints (21) and (22) bind. The key to the contract is the magnitude of the ratio of costs to benefits of inducing the manager's effort, defined by $\xi/\Delta q$. The numerator is the direct cost of extending effort, and the denominator is the change in the likelihood of drawing a higher quality project θ_1 due to effort. The regions are then defined by where this cost benefit ratio falls relative to the present value of receiving a cash flow of $\Delta\theta$ at three particular trigger values: $P_1^* = P^*(\theta_1)$, $P_2^* = P^*(\theta_2)$, and $P_3^* = P^*(\theta_3)$, where

$$\theta_3 = \theta_2 - \frac{q_H}{1 - q_H}\Delta\theta < \theta_2. \tag{24}$$

These present values are ordered by $(P_0 \,/\, P_3^*)^\beta \Delta\theta < (P_0 \,/ P_2^*)^\beta \Delta\theta < (P_0 \,/\, P_1^*)^\beta \Delta\theta$. Note that another potential region in which $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$ exists, however in this range the costs of effort are so high as to no longer justify the exertion of effort in equilibrium. Thus, we do not consider this region.[17]

[17] A proof of this result is available from the authors by request.

Because optimal contracts specify $P_1 = P_1^*$ and $w_2 = 0$ across all three regions, we may focus on P_2 and w_1 when we describe the optimal contracts in each of the three regions. The proofs detailing the solution are provided in Appendix A.

- *Hidden Information Only Region*: $\xi/\Delta q < (P_0/P_3^*)^\beta \, \Delta\theta$

In this region, we have

$$\begin{aligned} P_2 &= P_3^* = P^*(\theta_3) > P_2^*, & (25) \\ w_1 &= \left(\frac{P_1^*}{P_3^*}\right)^\beta \Delta\theta, & (26) \end{aligned}$$

where θ_3 is given in (24).

The net costs of inducing effort are low enough so that there is no need for the firm to have to compensate the manager for extending effort. In this range, the *ex-ante* incentive constraint does not bind, and therefore the cost of effort does not find its way into the optimal contract.[18] The compensation that the manager of the θ_1-type project receives is purely an informational rent that induces the manager to exercise at the first-best trigger P_1^*, in accordance with the revelation principle. Since w_1 is relatively low in this region, the P_2 trigger needs to be high (relative to the first-best trigger P_2^*) in order to dissuade the manager of the θ_1-type project from deviating from the equilibrium first-best trigger P_1^*.

We can use these contract terms to place a value on the owner's and manager's option values. The owner's and manager's option values, $\pi^o(P_0)$ and $\pi^m(P_0)$, respectively, can be written as:

$$\begin{aligned} \pi^o(P_0) &= q_H \left(\frac{P_0}{P_1^*}\right)^\beta (P_1^* + \theta_1 - K) + (1 - q_H) \left(\frac{P_0}{P_3^*}\right)^\beta (P_3^* + \theta_3 - K), & (27) \\ \pi^m(P_0) &= q_H \left(\frac{P_0}{P_3^*}\right)^\beta \Delta\theta. & (28) \end{aligned}$$

It is interesting to note that the solution for the owner's option value is observationally equivalent to the first-best solution in which one substitutes θ_3 for the lower project quality θ_2. In such a setting, the owner will choose to exercise at P_1^* if $\theta = \theta_1$ and at P_3^* if $\theta = \theta_3$. Thus, the impact of the costs of hidden information is fully embodied by a reduction of project quality in the low state.

[18] In a different setting where the hidden information is the cost of exercising, Maeland (2002) shows a similar result.

- *Joint Hidden Information/Hidden Action Region:* $(P_0/P_3^*)^\beta\,\Delta\theta \le \xi/\Delta q \le (P_0/P_2^*)^\beta\,\Delta\theta$

In this region, we have

$$P_2 \;=\; P_J = P_0\left(\frac{\Delta q\Delta\theta}{\xi}\right)^{1/\beta} > P_2^*, \tag{29}$$

$$w_1 \;=\; \left(\frac{P_1^*}{P_J}\right)^\beta \Delta\theta = \frac{\xi}{\Delta q}\left(\frac{P_1^*}{P_0}\right)^\beta. \tag{30}$$

Here, both the *ex-ante* and *ex-post* constraints bind. Since now the manager must be induced into providing effort, w_1 must be high enough to provide enough compensation for the *ex-ante* incentive constraint (22) to bind. This reflects the hidden action component of the contract. In addition, the exercise trigger P_2 must be high enough to dissuade the manager of the θ_1-type project from deviating from the equilibrium first-best trigger P_1^*. Thus, in this region, P_2 is set so that the *ex-post* incentive constraint (21) binds, ensuring that the revelation principle holds. This requires that P_2 be above the full-information trigger P_2^*. This deviation from the full-information trigger reflects the hidden information component of the contract.

Importantly, P_2 is lower in this region than it was in the hidden information only region. This is due to the fact that in this joint region w_1 is now higher in order to induce effort. This higher wage makes it easier to satisfy the truth-telling constraint, and no longer necessitate as great a deviation from P_2^* in order to prevent managers of the θ_1-type project from pretending to have a θ_2-type project. Therefore, perhaps surprisingly, moral hazard serves to increase investment efficiency since the increased share of the firm that must go to compensate the manager leads the manager to more fully internalize the benefits of efficient investment timing.

The owner's and manager's option values, $\pi^o(P_0)$ and $\pi^m(P_0)$, respectively, can be written as:

$$\pi^o(P_0) \;=\; q_H\left(\frac{P_0}{P_1^*}\right)^\beta (P_1^* + \theta_1 - K) + (1-q_H)\left(\frac{P_0}{P_J}\right)^\beta (P_J + \theta_3 - K)\,, \tag{31}$$

$$\pi^m(P_0) \;=\; q_H\frac{\xi}{\Delta q}. \tag{32}$$

The owner's option value deviates from the first-best value, $V^*(P_0)$ in (9) in two ways. First, the hidden information rents effectively make the manager mark down his privately observed component of cash flows from θ_2 to θ_3; similar to that in the pure hidden information region. Second, the exercise trigger for a manager of a θ_2-type project is equal to P_J, which is larger than P_2^*. Note that the only difference between $\pi^o(P_0)$ in this region and in the pure hidden-information region is the different terms for the exercise trigger: P_J versus P_3^*. Here, the trigger is lower due to the hidden action component.

- *Hidden Action Only Region:* $(P_0/P_2^*)^\beta \, \Delta\theta < \xi/\Delta q < (P_0/P_1^*)^\beta \, \Delta\theta$

In this parameter range, we have

$$P_2 \;=\; P_2^*, \tag{33}$$

$$w_1 \;=\; \frac{\xi}{\Delta q}\left(\frac{P_1^*}{P_0}\right)^\beta. \tag{34}$$

The equilibrium triggers equal those of the first-best outcomes. The moral hazard costs are so high that the rent needed for motivating high effort (via the *ex-ante* incentive constraint) is sufficiently large so that the *ex-post* incentive constraints do not demand additional rents. That is, the wage needed to motivate the manager to extend effort ends up being high enough so that the manager of the θ_1-type project no longer needs P_2 to exceed P_2^* in order to dissuade him from deviating from the equilibrium trigger P_1^*. Thus, the contract is entirely driven by the need to motivate effort, as the *ex-post* incentive constraint that reflects hidden information does not bind.

The owner's and manager's option values, $\pi^o(P_0)$ and $\pi^m(P_0)$, respectively, can be written as:

$$\pi^o(P_0) \;=\; V^*(P_0) - q_H \frac{\xi}{\Delta q}, \tag{35}$$

$$\pi^m(P_0) \;=\; q_H \frac{\xi}{\Delta q}. \tag{36}$$

The owner's option value is equal to the first-best solution $V^*(P_0)$ characterized in (9), minus the present value of the rent paid to the manager in order to induce effort.

Figure 1 summarizes the details of the optimal contracts through the three regions. The upper and lower graphs plot the equilibrium trigger strategy P_2 and wage payment w_1 in terms of effort cost ξ, respectively. The upper graph shows that the trigger strategy for the manager of the θ_2-type project is flat and equal to P_3^* for ξ in the pure hidden information region; is decreasing and convex in ξ for the joint hidden action/hidden information region; and is flat and equal to the first-best trigger level P_2^* for ξ in the pure hidden action. The equilibrium trigger P_2 is closer to the first-best level, for higher level of ξ, *ceteris paribus.*

The lower graph plots corresponding wage contracts for a manager of the θ_1-type project. For low levels of ξ (pure hidden information region), he only needs to be compensated with pure informational rents. In the joint hidden information/hidden action region, w_1 increases linearly in ξ. For sufficiently high ξ, the manager of a θ_1-type project is sufficiently rewarded so as to exercise at the first-best level.

3.3 An Extension to Cases with Continuous Distributions of θ

For ease of presentation, our basic model uses a simple two-point distribution for θ. In order to check the robustness of our results, we generalize our model to allow for admissible continuous distributions of θ on $[\underline{\theta}, \bar{\theta}]$ in Appendix B. In this setting, the principal designs the contract such that the manager will find it optimal to exert effort at time zero and then reveal their θ truthfully by choosing the recommended equilibrium strategy $P(\theta)$ and $w(\theta)$. The managers are protected by *ex-post* limited liability in that $w(\theta) \geq 0$ for all θ. Also, the manager's participation is voluntary at time zero. We show that the following key results remain valid:

1. Agency problems (hidden information and hidden action) lead to a delayed investment timing decision, compared with first-best trigger levels;

2. Introducing hidden action into the model at time zero lowers agency costs, because the manager has an option to align his incentives better with the owner by exerting effort at time zero. This leads to an investment timing trigger closer to the first-best level.

In addition, the model predicts that the manager with the lowest present value of privately observed cash flows $\underline{\theta}$ receives no rents, in that $w(\underline{\theta}) = 0$ as in our basic setting.[19] The *ex-ante* participation constraint does not bind, because the limited liability condition for the manager and *ex-ante* incentive constraint together provide enough incentive for the manager with any *ex-post* realized θ to participate, as in our basic setting. For technical convenience, we have assumed that the distribution of θ under effort first-order stochastically dominates that under no effort. Intuitively, the manager is more likely to draw a "better" distribution of θ after exerting effort than not exerting effort. Under those conditions,[20] managers of higher quality

[19] Recall that the manager with θ_2 receives no rents.

[20] See Appendix B for other technical conditions.

projects will exercise at lower equilibrium trigger strategies and receive higher equilibrium wages.

We may further generalize our model by allowing for multiple discrete choices of effort levels. One can solve this problem by following a similar two-step procedure: (*i*) first solving for the optimal contract for each given level of effort; and (*ii*) then choosing the "optimal" level of effort for the owner by searching for the maximum among owner's option value across all effort levels. Subtle technical issues arise when we allow for effort choice to be continuous.[21] However, the basic approach and intuition remain valid.

4 Model Implications

In this section, we analyze several of the more important implications of the model. First, Section 4.1 examines the stock price reaction to investment (or failure to invest). We shall see that the stock price will move by a discrete jump due to the information released at the trigger P_1^*. Investment at P_1^* signals good news about project quality and the stock price jumps upward; failure to invest at P_1^* signals bad news about project quality and the stock price falls downward. Second, a clear prediction of our model is that the principal-agent problem will introduce inertia into a firm's investment behavior, in that investment will on average be delayed beyond first-best. Section 4.2 considers the factors that influence the expected lag in investment. Third, specifically because the timing of investment differs from that of the first-best outcome, the principal-agent problem results in a welfare loss and reduction in the owner's option value. Section 4.3 analyzes the comparative statics of the welfare loss and owner's option value with respect to the key parameters of the model.

In this section, we focus our analysis on the contract that prevails in the joint hidden information/ hidden action region. It is in this region that the incentive problems are the richest and most meaningful. The terms of the contract and resulting option values are displayed in equations (29)–(32).

[21] We need to verify the validity of first-order approach, which refers to the practice of replacing an infinite number of *global* incentive constraints imposed by *ex-ante* incentive to exert effort, with simple *local* incentive constraints as captured by first-order condition associated with the *global* incentive constraints. See Rogerson (1985) and Jewitt (1988) for more on the first-order approach.

4.1 Stock Price Reaction to Investment

In this section, we analyze the stock price reaction to the information released via the manager's investment decision.[22] The manager's investment decision will signal to the market the true value of θ, and the stock price will reflect this information revelation. Importantly, this will allow for the manager's compensation contract to be contingent on the firm's stock price. That is, while in the model we have made the incentive contract's wages to be contingent on the manager's investment decision, the wages can also be made contingent on the stock price.

The equity value of the firm is equal to the value of the owner's option value given in (31). Prior to the point at which $P(t)$ reaches the threshold P_1^*, the market does not know the true value of θ: the market believes that $\theta = \theta_1$ with probability q_H and $\theta = \theta_2$ with probability $1 - q_H$.

Once the process $P(t)$ hits the threshold P_1^*, the manager's unobserved component of cash flows is fully revealed. The manager's investment behavior signals to the market the true value of θ. If the manager exercises the option at P_1^*, then the manager reveals to the market that the privately observed component of cash flows is high. Therefore, the firm's value instantly jumps to S_u, given by

$$S_u = P_1^* + \theta_1 - K - w_1 = P_1^* + \theta_1 - K - \left(\frac{P_1^*}{P_J}\right)^{\beta} \Delta\theta \,. \tag{37}$$

If the manager does not exercise his option at P_2^*, then the market infers that the manager's privately observed component of cash flows is low. Then, the firm's value instantly drops to S_d, given by

$$S_d = \left(\frac{P_1^*}{P_J}\right)^{\beta} \left(P_J + \theta_2 - K\right) \,. \tag{38}$$

Figure 2 plots the stock price S as a function of P, the current value of the process $P(t)$. For all $P < P_1^*$, $S(P) = \pi^o(P)$, where π^o is given in (31). For $P = P_1^*$, $S(P) = S_u$ if investment is undertaken, and $S(P) = S_d$ if investment is not undertaken. The jump in the stock price at P_1^* is a result of the information revealed by the manager's actions.

This result is consistent with the empirical findings in McConnell and Muscarella (1985). They find that announcements of unexpected increases in investment spending lead to increases in stock prices, and vice versa for unexpected decreases.

[22] We thank the referee for suggesting this discussion.

Since the stock price movement at the trigger P_1^* reveals the true value of θ, the manager's incentive contract can be made contingent on the stock price. For example, the manager could be paid a bonus w_1 if the stock price jumps upward to S_u. Since $w_2 = 0$, no bonus is paid if the stock price falls to S_d. Similarly, such a contingent payoff could be implemented through a properly parameterized stock option grant.

4.2 Agency Problems and Investment Lags

In the standard real options setting, investment is triggered at the value maximizing triggers, P_1^* and P_2^*, for the higher and lower project quality outcomes, respectively. However, in our setting, while the trigger for investment in the higher quality state remains at P_1^*, investment in the lower quality state may be triggered at P_J, which is higher than the first-best benchmark level P_2^*.

Let T and T^* be the stopping times at which the option is exercised, in our model and the first-best setting, respectively. We denote $\Gamma = E\left(T - T^*\right)$ as the expected time lag due to the principal-agent problem. A solution for such an expectation can be derived using Harrison (1985, Chapter 3). The expected lag is given by

$$\Gamma = \left(\frac{1-q_H}{\alpha - \sigma^2/2}\right) \ln\left(\frac{P_J}{P_2^*}\right) \tag{39}$$

$$= \left(\frac{1-q_H}{\alpha - \sigma^2/2}\right) \left[\ln\left(\frac{P_0}{K-\theta_2}\right) + \frac{1}{\beta}\ln\left(\frac{\Delta q \Delta\theta}{\xi}\right) - \ln\beta + \ln\left(\beta - 1\right)\right], \tag{40}$$

where we assume that $\alpha > \sigma^2/2$ in order for this expectation to exist.

An important insight from Section 3 is that increases in the cost benefit ratio of inducing effort lead to less distortion in investment timing. That is, as the ratio $\xi/\Delta q$ increases, the equilibrium trigger P_J becomes closer to the first-best trigger P_2^*. This is confirmed by the following comparative static:

$$\frac{\partial\Gamma}{\partial\left(\xi/\Delta q\right)} = -\left(\frac{1-q_H}{\alpha - \sigma^2/2}\right)\frac{\Delta q}{\beta\xi} < 0. \tag{41}$$

An increase in the volatility of the underlying project, σ, has an ambiguous effect on the expected time lag Γ. This can be seen from the following comparative static:

$$\frac{\partial\Gamma}{\partial\sigma} = -\left(\frac{1-q_H}{\alpha-\sigma^2/2}\right)\frac{1}{\beta^2}\left[\ln\left(\frac{\Delta q\Delta\theta}{\xi}\right) - \frac{\beta}{\beta-1}\right]\frac{\partial\beta}{\partial\sigma} + \frac{\left(1-q_H\right)\sigma}{\left(\alpha-\sigma^2/2\right)^2}\ln\left(\frac{P_J}{P_2^*}\right), \tag{42}$$

where $\partial\beta/\partial\sigma < 0$. An increase in σ raises the option value and makes waiting more worthwhile, implying that both P_2^* and P_J are larger, *ceteris paribus*. However, if the marginal cost benefit ratio for exerting effort is relatively high, in that

$$\ln\left(\frac{\xi}{\Delta q}\right) > \frac{\beta-1}{\beta} + \ln(\Delta\theta), \tag{43}$$

then the change of P_J relative to the change in P_2^* is larger. Therefore, under such conditions the expected time lag increases in volatility σ.

An increase in the expected growth rate of the project, α, also has an ambiguous effect on the expected time lag Γ. This can be seen from the following comparative static:

$$\frac{\partial\Gamma}{\partial\alpha} = -\frac{1-q_H}{(\alpha-\sigma^2/2)^2}\left[\ln\left(\frac{P_J}{P_2^*}\right) - \frac{1}{\beta}\left(\ln\left(\frac{\Delta q\Delta\theta}{\xi}\right) - \frac{\beta}{\beta-1}\right)\frac{\alpha-\sigma^2/2}{\sqrt{(\alpha-\sigma^2/2)^2+2r\sigma^2}}\right]. \tag{44}$$

However, if (43) holds, then expected time lag decreases with drift α.

4.3 Welfare Loss and Option Values

Although the owner chooses the value-maximizing contract to provide an incentive for the manager to extend effort, the agency problem ultimately still proves costly. In an owner-managed firm, the manager will extend effort and will exercise the option at the first-best stopping time. However, in firms with delegated management, there will be a welfare loss due to the firm's suboptimal exercise strategy.

By a welfare loss, we are referring to the difference between the values of the first-best option value, $V^*(P_0)$ in (9), and the sum of the owner and manager options, $\pi^o(P_0)$ and $\pi^m(P_0)$ in (31) and (32). Thus, define the welfare loss due to agency issues as L, where $L = V^*(P_0) - [\pi^o(P_0) + \pi^m(P_0)]$. Simplifying, we have:

$$L = (1-q_H)\left[\left(\frac{P_0}{P_2^*}\right)^\beta (P_2^* - K + \theta_2) - \left(\frac{P_0}{P_J}\right)^\beta (P_J - K + \theta_2)\right]. \tag{45}$$

This welfare loss is likely to have economic ramifications on the structure of firms. For firms in industries with potentially large welfare losses due to agency costs, there will be powerful forces that will push them to be privately held, or to be organized in a manner that provides the closest alignment between owners and managers.

There are two effects of a later-than-first-best exercising trigger ($P_J > P_2^*$) on the welfare loss L: (i) a larger cash flow $P_J + \theta_2 - K > P_2^* + \theta_2 - K$ reduces welfare loss, *ceteris paribus*, and (ii) a lower discount factor $[(P_0/P_J)^\beta < (P_0/P_2^*)^\beta]$ increases the welfare loss. The latter dominates the former, because $P_J > P_2^*$ and $P_2^* = \arg\max\ (P_0/x)^\beta\ (P_0 + \theta_2 - K)$. Equation

(45) suggests that welfare loss is driven by the distance of the equilibrium trigger P_J from P_2^*. As previously discussed, the firm's exercise timing becomes less distorted as the net cost benefit ratio of inducing effort increases. That is, as the ratio $\xi/\Delta q$ increases, the equilibrium trigger P_J gets closer to the first-best trigger P_2^*, and thus:

$$\frac{\partial L}{\partial\left(\xi/\Delta q\right)} < 0. \tag{46}$$

In terms of the owner's option value, the incentive problem represents a trade-off between timing efficiency and the surplus that must be paid to the manager to extend effort. One can obtain better intuition on the forces at work in the agency problem through the following decomposition. In the first-best solution, the owner pays the cost of effort ξ and obtains the first-best option value $V^*(P_0)$. In the agency equilibrium, the owner delegates the cost of effort to the manager, but then holds the sub-optimal option value $\pi^o(P_0)$. The loss in the owner's option value due to the incentive problem is therefore given by:

$$\Delta\pi^o(P_0) \equiv V^*(P_0) - \xi - \pi^o(P_0\,) = L + V^m, \tag{47}$$

where L is the total welfare loss given in (45), and V^m is the *ex-ante* expected surplus paid to the manager to exert effort, and is given by:

$$V^m = \pi^m(P_0) - \xi = q_H\frac{\xi}{\Delta q} - \xi = \frac{q_L}{\Delta q}\,\xi. \tag{48}$$

Decomposing the loss in the owner's option value given in (47) into the sum of the timing component (L) and the compensation component (V^m) is useful for providing intuition. When the owner delegates the option exercise decision to the manager, the owner's option value is lowered for two reasons: (*i*) the exercising inefficiency induced by agency and informational asymmetry; and (*ii*) the surplus needed to pay the manager to induce him to extend effort and reveal his private information. The impact of a higher effort cost ξ represents a trade-off in terms of the timing and compensation components. As shown in (46), a higher effort cost results in more efficient investment timing. This must be traded-off against the increased compensation that must be paid to provide appropriate incentives to the manager, as seen in (48). Therefore, the total effect on the loss of owner's option value due to an increase in ξ depends on whether the "timing effect" or "compensation effect" is larger, in that

$$\frac{\partial}{\partial \xi}\Delta\pi^{o}(P_0) = -\left(1-q_H\right)\left(\beta-1\right)\left(\frac{P_0}{P_J}\right)^{\beta}\left(1-\frac{P_2^*}{K_2}\right)\frac{P_J}{\beta\xi}+\frac{q_L}{\Delta q} \tag{49}$$

$$= \frac{\beta-1}{\beta}\frac{1}{\Delta q\Delta\theta}\left[-\left(1-q_H\right)\left(P_J-P_2^*\right)+q_L\left(P_2^*-P_1^*\right)\right]. \tag{50}$$

If the investment trigger P_J is significantly larger than P_2^*, in that

$$\left(1-q_H\right)\left(P_J-P_2^*\right) > q_L\left(P_2^*-P_1^*\right), \tag{51}$$

then an increase in ξ leads to a smaller loss in the owner's option value, as the gain in timing efficiency overshadows the loss due to the manager's increased compensation.[23]

5 Impatient Managers and Early Investment

In the model, both owners and managers value payoffs identically. However, it may be the case that managers are more impatient than owners. There are several potential justifications for such an assumption. First, there are various models of managerial myopia that attempt to explain a manager's preference for choosing projects with quicker pay-backs, even in the face of eschewing more valuable long-term opportunities. For example, Narayanan (1985) and Stein (1989) argue that concerns about either the firm's short-term performance or labor-market reputation may give managers an incentive to take actions that pay off in the near term at the expense of the longer term. Second, in our "investment timing" setting, greater impatience can represent the manager's preference for empire building or greater perquisite consumption and reputation that comes from running a larger business sooner rather than later. Third, managerial short-termism could be the result of the manager facing stochastic termination.[24] This termination, for example, could be due to the manager leaving for a better job elsewhere or being fired. We can model such stochastic termination by supposing that the manager faces an exogenous termination driven by a Poisson process with intensity

[23] Note that the above condition is non-empty. This can be seen as follows. Condition (51) is equivalent to

$$P_J > \frac{1}{1-q_H}\left[(1-q_H)P_2^* + q_L(P_2^*-P_1^*)\right] = P_3^* - \frac{\Delta q}{1-q_H}\left(P_2^*-P_1^*\right).$$

The joint hidden action/hidden information region is characterized by $P_2^* \leq P_J \leq P_3^*$. Therefore, the above condition is met for some P_J.

[24] We assume that the owner can costlessly replace the manager in the event of separation.

ζ. The addition of stochastic termination transforms the manager's option to one in which his discount rate r is elevated to $r + \zeta$ to reflect the stochastic termination.[25]

Phrased in real options terms, managerial impatience leads to a smaller option to wait for the manager, potentially reversing the previously mentioned larger "option to wait" that results from the standard contracting model. Thus, this generalization leads to very different predictions about investment timing. While the basic model predicts that investment will never occur earlier than the first-best case, in this generalized setting investment can occur earlier or later than the first-best case. This is similar to the result found in Stulz (1990) where there is both over- and under-investment in the capital allocation decision, as shareholders use debt to constrain managerial empire-building preferences.

Recall that the owner discounts future cash flows by the discount function $D(P_0; \hat{P}) = \left(P_0/\hat{P}\right)^{\beta}$ for $P_0 < \hat{P}$. We can therefore represent greater managerial impatience by defining a managerial discount function $D^m(P_0; \hat{P}) = \left(P_0/\hat{P}\right)^{\gamma}$, where $\gamma > \beta$ ensures that $D^m(P_0; \hat{P}) < D(P_0; \hat{P})$. That is, a dollar received at the stopping time $\hat{P}$ is worth less to the manager than to the owner.[26]

This generalized problem is basically the same as that of Section 2, with the exception that the constraints all use γ rather than β. In addition, much of the solution is similar. For example, Propositions 1 and 2 apply as before, using the same proof. In addition, Proposition 3 and 4 remain valid, and are demonstrated in Appendix C. Thus, the optimal contracting problem in the generalized setting can be written as:

$$\max_{w_1, P_1, P_2} \quad q_H \left(\frac{P_0}{P_1}\right)^{\beta} (P_1 + \theta_1 - K) - q_H \left(\frac{P_0}{P_1}\right)^{\beta} w_1 + (1 - q_H) \left(\frac{P_0}{P_2}\right)^{\beta} (P_2 + \theta_2 - K), \quad (52)$$

subject to

$$\left(\frac{P_0}{P_1}\right)^{\gamma} w_1 \geq \left(\frac{P_0}{P_2}\right)^{\gamma} \Delta\theta \, , \quad (53)$$

$$\left(\frac{P_0}{P_1}\right)^{\gamma} w_1 \geq \frac{\xi}{\Delta q} \, . \quad (54)$$

Similar to Proposition 5, at least one of (53) and (54) binds. Otherwise, the owner may strictly increases his payoff by lowering the wage payment w_1 without violating any constraints.

[25] We suppose that the manager receives his reservation value (normalized to zero), when the termination occurs. See Yaari (1965), Merton (1971) and Richard (1975) for analogous results on stochastic horizon.

[26] Note that this is also consistent with the interpretation that the manager has a higher discount rate than the owner. Since $\partial\beta/\partial r > 0$, the manager's higher discount rate is embodied by the condition $\gamma > \beta$.

Just as in Section 3, there are three contracting regions: a hidden information region, a joint hidden information/hidden action region, and a hidden action region, depending on the level of cost/benefit ratio $\xi/\Delta q$. In this section, we focus on the joint hidden information/hidden action region.[27]

The joint hidden information/hidden action region is defined by: $(P_0/\hat{P}_3^*)^\gamma \Delta\theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta\theta$, where $\hat{P}_3^*$ is defined in (C.11), and shown to be greater than the trigger P_2^*. In this region the optimal contract can be written as:

$$
\begin{aligned}
P_1 &= \hat{P}_1, && (55)\\
P_2 &= \hat{P}_J = P_0 \left(\frac{\Delta q \Delta\theta}{\xi}\right)^{1/\gamma}, && (56)\\
w_1 &= \left(\frac{\hat{P}_1}{\hat{P}_J}\right)^\gamma \Delta\theta < \Delta\theta, && (57)\\
w_2 &= 0, && (58)
\end{aligned}
$$

where $\hat{P}_1$ is the root of $H(x) = 0$, defined by

$$H(x) = \frac{\beta}{\beta - 1}\left[K - \theta_1 + \left(1 - \frac{\gamma}{\beta}\right)\left(\frac{x}{P_0}\right)^\gamma \frac{\xi}{\Delta q}\right] - x. \tag{59}$$

Unlike the results of the basic model, we now have the possibility of investment occurring before the first-best trigger, in that $P_1 = \hat{P}_1 < P_1^*$. To see this, note that $H(0) = P_1^*$ and

$$H(P_1^*) = \frac{\beta}{\beta - 1}\left(1 - \frac{\gamma}{\beta}\right)\left(\frac{P_1^*}{P_0}\right)^\gamma \frac{\xi}{\Delta q} < 0. \tag{60}$$

The derivative of $H(\cdot)$ is

$$H'(x) = \frac{\beta}{\beta - 1}\gamma\left(1 - \frac{\gamma}{\beta}\right)\left(\frac{x}{P_0}\right)^{\gamma - 1} \frac{1}{P_0}\frac{\xi}{\Delta q} - 1 < 0. \tag{61}$$

Therefore, there exists a unique solution $P_1 = \hat{P}_1 < P_1^*$.

As in the basic model, the trigger strategy for the manager of a θ_2-type project is greater than the first-best trigger, P_2^*. Recall that $\hat{P}_J > P_2^*$ in the region $(P_0/\hat{P}_3^*)^\gamma \Delta\theta < \xi/\Delta q < (P_0/P_2^*)^\gamma \Delta\theta$, where P_3^* is given in (C.11). However, for $\gamma > \beta$, the trigger is closer to the first-best trigger than for the standard case in which $\gamma = \beta$. This is true, since for $\gamma > \beta$,

$$\hat{P}_J = P_0\left(\frac{\Delta q \Delta\theta}{\xi}\right)^{1/\gamma} < P_0\left(\frac{\Delta q \Delta\theta}{\xi}\right)^{1/\beta} = P_J. \tag{62}$$

[27] The derivations for the optimal contracts in the other regions are shown in Appendix C.

Thus, when the manager is more impatient than the owner, equilibrium investment occurs sooner than it does in the standard principal-agent model described earlier in the paper. In particular, investment actually occurs prior to the first-best trigger for the θ_1-type project. The greater impatience on the part of the manager implies that it is in the owner's interest to offer a contract that motivates earlier exercise. This results in both costs and benefits to the owner. By motivating investment for the θ_2-type project earlier than the standard principal-agent model, investment timing moves closer to the first-best. Since the manager receives no surplus for the θ_2-type project, the owner is the sole beneficiary of this timing efficiency. However, investment for the θ_1-type project occurs earlier than that in the model of Section 2, which is the first-best outcome. Therefore, the owner is worse off with respect to the θ_1-type projects for two reasons: investment occurs too early, and the wage paid to the manager in this state must be higher (than in the standard model) in order to motivate earlier investment. The net effect on *ex-ante* owner's option value is ambiguous and is driven by the relative parameter values.

6 Conclusion

This paper extends the real options framework to account for the agency and information issues that are prevalent in many real-world applications. When investment decisions are delegated to managers, contracts must be designed that provide incentives for the manager to both extend effort and to truthfully reveal their private information. This paper provides a model of optimal contracting in a continuous-time principal-agent setting in which there is both moral hazard and adverse selection. The implied investment behavior differs significantly from that of the first-best no-agency solution. In particular, there will be greater inertia in investment, as the model leads to the manager having an even greater option to wait than the owner. The interplay between the twin forces of hidden information and hidden action leads to markedly different investment outcomes than when only one of the two forces is at work. Allowing the manager to have an effort choice that affects the likelihood of getting a high quality project mitigates the investment inefficiency due to informational asymmetry. When the model is generalized to include differing degrees of impatience between owners and managers, we find that investment may occur either earlier or later than optimal.

Some extensions of the model would prove interesting. First, the model could allow for repeated investment decisions. This richer setting would permit owners to update their beliefs over time, and for managers to establish reputations. Second, the model could also be generalized to include competition in both the labor and product markets. As shown by Grenadier (2002), the forces of competition greatly alter the investment behavior implied by standard real options models.

Appendices

A Solution to the Optimal Contracting Problem

This appendix provides a derivation of the optimal contracts detailed in Section 3.

Propositions 1 and 2 allow us to express the owner's objective as maximizing the value of his option, given in (13), subject to (15), (17), (18) and $w_2 \geq 0$. Using the method of Kuhn-Tucker, we form the Lagrangian as follows:

$$\begin{aligned}\mathcal{L} = & \left(\frac{P_0}{P_1}\right)^\beta (P_1 + \theta_1 - K - w_1) + \frac{1-q_H}{q_H}\left(\frac{P_0}{P_2}\right)^\beta (P_2 + \theta_2 - K - w_2) \\ & + \lambda_1 \left[\left(\frac{P_0}{P_1}\right)^\beta w_1 - \left(\frac{P_0}{P_2}\right)^\beta (w_2 + \Delta\theta)\right] + \lambda_2 \left[\left(\frac{P_0}{P_2}\right)^\beta w_2 - \left(\frac{P_0}{P_1}\right)^\beta (w_1 - \Delta\theta)\right] \\ & + \lambda_3 \left[\left(\frac{P_0}{P_1}\right)^\beta w_1 - \left(\frac{P_0}{P_2}\right)^\beta w_2 - \frac{\xi}{\Delta q}\right] + \lambda_4 \, w_2, \end{aligned} \tag{A.1}$$

with corresponding complementary slackness conditions for the four constraints.

The first-order condition with respect to w_1 gives

$$\lambda_1 - \lambda_2 + \lambda_3 = 1\,. \tag{A.2}$$

The first-order condition with respect to w_2 implies

$$\left(-\lambda_1 + \lambda_2 - \lambda_3 - \frac{1-q_H}{q_H}\right)\left(\frac{P_0}{P_2}\right)^\beta + \lambda_4 = 0. \tag{A.3}$$

Simplifying (A.3) gives $\lambda_4 = (P_0/P_2)^\beta / q_H > 0$. Therefore, the complementary slackness condition $\lambda_4 \, w_2 = 0$ implies that $w_2 = 0$. This proves Proposition 3.

The first-order conditions with respect to P_1 and P_2 imply:

$$P_1 = \frac{\beta}{\beta - 1}\left(K - \theta_1 - \lambda_2 \Delta\theta\right), \tag{A.4}$$

$$P_2 = \frac{\beta}{\beta - 1}\left(K - \theta_2 + \frac{q_H}{1-q_H}\lambda_1 \, \Delta\theta\right). \tag{A.5}$$

We conjecture that the *ex-post* incentive constraint for the manager of a θ_2-type project, (18), does not bind, in that $\lambda_2 = 0$. We verify this conjecture, formalized in Proposition 4 for each region. With $\lambda_2 = 0$, then (A.2) may be written as $\lambda_1 + \lambda_3 = 1$. Therefore, it must be the case that at least one of (21) and (22) binds (Proposition 5). Combining the conjecture $\lambda_2 = 0$ with (A.4) gives $P_1 = P_1^*$ (Proposition 6).

Thus, we have shown that provided we verify that (18) does not bind, the owner's optimization problem is summarized in equations (20)-(22). This problem is now solved below.

A.1 The Hidden Information Only Region

Suppose that the *ex-ante* incentive constraint (22) does not bind. Since (21) must hold as an equality and $\lambda_1 = 1$, (A.5) implies $P_2 = P_3^*$ where P_3^* is given in (25) and w_1 is given in (26). The inequality $P_1^* < P_3^*$ implies (18) does not bind, as conjectured. Finally, in order to be consistent with the assumption that (22) does not bind, we require that $\xi/\Delta q < (P_0/P_3^*)^\beta \, \Delta\theta$, the parameter range defining this hidden information only region.

A.2 The Joint Hidden Information/Hidden Action Region

We derive the optimal contract in this region by conjecturing that both (21) and (22) bind. Solving these two equality constraints gives (29) and (30). The inequality $P_J > P_1^*$ confirms that (18) does not bind, as conjectured. The solution for P_2 implies that λ_1 can be written as:

$$\lambda_1 = \frac{\beta - 1}{\beta} \left(P_J - P_2^*\right) \frac{1 - q_H}{q_H \Delta\theta}. \tag{A.6}$$

The only possible region under which both constraints may bind[28] is characterized by

$$\left(\frac{P_0}{P_3^*}\right)^\beta \Delta\theta < \frac{\xi}{\Delta q} < \left(\frac{P_0}{P_2^*}\right)^\beta \Delta\theta\,. \tag{A.7}$$

We now show that *indeed* both (21) and (22) bind throughout this entire region. The region characterized by (A.7) can be equivalently expressed as $P_2^* < P_J < P_3^*$. Because (A.6) implies that λ_1 is monotonically increasing in P_J, therefore, $0 < \lambda_1 < 1$ in this region. Since $\lambda_3 = 1 - \lambda_1$, we also have $0 < \lambda_3 < 1$. By complementary slackness conditions, both (21) and (22) bind in this joint region, confirming the result that (A.7) is the whole region, with both constraints binding.[29]

A.3 The Hidden Action Only Region

Suppose that (21) does not bind and (22) binds, then $\lambda_1 = 0$ by complementary slackness, and $\lambda_3 = 1$. Therefore, $P_2 = P_2^*$ given in (33) and w_1 is given in (34). We need to verify that (21) and (18) do not bind. The constraint (18) is non-binding if and only if $P_1^* < P_J$. The constraint (21) is non-binding if and only if $P_J < P_2^*$. Thus, together these imply that

[28] If $(P_0/P_1^*)^\beta \Delta\theta > \xi/\Delta q > (P_0/P_2^*)^\beta \Delta\theta$, then only the third constraint binds. If $(P_0/P_3^*)^\beta \Delta\theta > \xi/\Delta q$, then only the first constraint binds. If $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$, then supporting high effort is no longer in the owner's interest.

[29] We also need additional technical conditions to ensure that inducing the manager to extend high effort is in the interest of the owner.

$P_1^* < P_J < P_2^*$, which is identical to the condition $(P_0/P_2^*)^\beta \Delta\theta < \xi/\Delta q < (P_0/P_1^*)^\beta \Delta\theta$, that defines this region.

If the parameters do not fall in any of the three regions, namely, $\xi/\Delta q > (P_0/P_1^*)^\beta \Delta\theta$ then it can be shown that the owner will not choose to motivate the manager to exert effort. The cost of effort is so high as to overwhelm any potential benefits of motivating effort. A proof of this result is available from the authors upon request.

B Optimal Contracting with a Continuous Distribution of θ

This appendix contains the derivation of the optimal contracts when the distribution of the project's unobserved component θ of cash flows is continuous.

Denote the manager's time-zero expected utility as $u(\hat{\theta}, \theta)$, if he reports that his privately observed present value of cash flows is $\hat{\theta}$, and the true level of his privately observed value is θ. His time-zero expected utility is then given by

$$u(\hat{\theta}, \theta) = \left(\frac{P_0}{P(\hat{\theta})}\right)^\beta \left(w(\hat{\theta}) + \theta - \hat{\theta}\right). \tag{B.1}$$

We denote $U(\theta)$ as the (truth-telling) manager's utility whose privately observed component of cash flows is θ. That is,

$$U(\theta) = u(\theta, \theta) = \left(\frac{P_0}{P(\theta)}\right)^\beta w(\theta). \tag{B.2}$$

As in Section 2, we denote ξ as the cost of extending effort at time zero. Let $F_H(\theta)$ and $F_L(\theta)$ be the cumulative distribution functions of θ drawn if the manager extends effort and if he does not extend effort, respectively. Using the revelation principle, we may write the principal's optimization problem as follows:

$$\max_{P(\cdot),\, w(\cdot)} \quad \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{P_0}{P(\theta)}\right)^\beta \left(P(\theta) + \theta - K - w(\theta)\right)\, dF_H(\theta)\,, \tag{B.3}$$

subject to:

1. *ex-post* incentive-compatibility condition:

$$U(\theta) \geq u(\hat{\theta}, \theta)\,, \quad \text{for any } \hat{\theta} \text{ and } \theta\,; \tag{B.4}$$

2. limited-liability condition:

$$w(\theta) \geq 0\,, \quad \text{for any } \theta\,; \tag{B.5}$$

3. *ex-ante* incentive compatibility condition:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\, dF_H(\theta) - \xi \geq \int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\, dF_L(\theta)\,; \tag{B.6}$$

4. *ex-ante* participation constraint:

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta)\, dF_H(\theta) - \xi \geq 0. \tag{B.7}$$

Proposition 8. *The* ex-ante *participation constraint (B.7) does not bind.*

Proof. The *ex-ante* incentive constraint (B.6) and the limited-liability condition (B.5) together imply that the *ex-ante* participation constraint (B.7) does not bind. ∎

First, we simplify the *ex-ante* incentive constraint (B.6) using integration by parts. This gives

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta)\, dU(\theta)\ \geq \xi\,, \tag{B.8}$$

where $M(\theta) = -\left(F_H(\theta) - F_L(\theta)\right)$.

Next, we simplify the *ex-post* incentive-compatibility condition (B.4) by totally differentiating $U(\theta)$, the utility for a truth-telling manager of a type-θ project, with respect to θ. This gives

$$\frac{dU(\theta)}{d\theta} = u_1 \frac{d\hat{\theta}}{d\theta} + u_2\,, \tag{B.9}$$

where

$$u_1 = \left.\frac{\partial u(\hat{\theta}, \theta)}{\partial \hat{\theta}}\right|_{\hat{\theta}=\theta}, \quad \text{and} \quad u_2 = \left.\frac{\partial u(\hat{\theta}, \theta)}{\partial \theta}\right|_{\hat{\theta}=\theta}. \tag{B.10}$$

Since the manager optimally reveals his project quality by choosing recommended equilibrium strategy, we have $u_1(\theta, \theta) = 0$. Therefore, we have $U'(\theta) = u_2$. Integration gives

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} u_2(s,\, s)\, ds = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \left(\frac{P_0}{P(s)}\right)^{\beta} ds. \tag{B.11}$$

We note that the Spence-Mirrlees condition is satisfied.[30]

A standard result in the contracting literature with asymmetric information states that the limited-liability condition for a manager of a $\underline{\theta}$-type project binds, in that $U(\underline{\theta}) = w(\underline{\theta}) = 0$.

[30]Details are available upon request.

Therefore, the informational rent $U(\theta)$ that accrues to the manager of a θ-type project is given by

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \left(\frac{P_0}{P(s)} \right)^{\beta} ds \,. \tag{B.12}$$

The relationship between $U(\theta)$ and the equilibrium wage implies that

$$w(\theta) = \left(\frac{P(\theta)}{P_0} \right)^{\beta} U(\theta) = \int_{\underline{\theta}}^{\theta} \left(\frac{P(\theta)}{P(s)} \right)^{\beta} ds. \tag{B.13}$$

Using (B.13), we simplify the present value of expected wage payment as follows:

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{P_0}{P(\theta)} \right)^{\beta} w(\theta)\, dF_H(\theta) \quad &= \quad \int_{\underline{\theta}}^{\bar{\theta}} \left[\int_{\underline{\theta}}^{\theta} \left(\frac{P_0}{P(s)} \right)^{\beta} ds \right] dF_H(\theta) \,, \\
&= \quad \left[\int_{\underline{\theta}}^{\theta} \left(\frac{P_0}{P(s} \right)^{\beta} ds \right] F_H(\theta) \Big|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}} F_H(\theta) \left(\frac{P_0}{P(\theta)} \right)^{\beta} d\theta \,, \\
&= \quad \int_{\underline{\theta}}^{\bar{\theta}} \lambda_H(\theta) \left(\frac{P_0}{P(\theta)} \right)^{\beta} dF_H(\theta),
\end{aligned} \tag{B.14}$$

where

$$\lambda_H(\theta) = \frac{1 - F_H(\theta)}{f_H(\theta)} \tag{B.15}$$

is the inverse of the hazard rate under $F_H(\,\cdot\,)$.

Using (B.14) allows us to simplify the principal's optimization problem as follows:

$$\max_{P(\,\cdot\,)} \quad \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{P_0}{P(\theta)} \right)^{\beta} \left(P(\theta) + \theta - K - \lambda_H(\theta) \right) dF_H(\theta) \,, \tag{B.16}$$

subject to the *ex-ante* incentive constraint (B.8) and *ex-post* limited-liability condition (B.5). The equilibrium wage is then obtained by using (B.13).

Similar to the model with discrete values for θ, optimal contracts, characterized by the pair of trigger strategy and wage payment functions, depend on the region in which effort cost ξ lies. Subsection B.1 solves for the optimal contracts in the region under which the *ex-ante* incentive constraint does not bind. Subsection B.2 solves for the optimal contracts in the region under which both the *ex-ante* incentive constraint and the *ex-post* incentive constraint bind.

B.1 The Hidden Information Only Region

If effort cost ξ is low enough, then no additional rent is needed to induce the manager to extend effort. The following condition ensures that the *ex-ante* incentive constraint (B.8) does not bind.

Condition 1.

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left(\frac{P_0}{P_3^*(\theta)} \right)^{\beta} d\theta \geq \xi, \tag{B.17}$$

where

$$P_3^*(\theta) = \frac{\beta}{\beta - 1} \left[K - \theta + \lambda_H(\theta) \right] . \tag{B.18}$$

Maximizing (B.16) may be done point by point. This gives the candidate optimal trigger level $P(\theta) = P_3^*(\theta)$, where $P_3^*(\theta)$ is given in (B.18). Because $\lambda_H(\theta) > 0$, the exercise trigger is larger than the first-best level, confirming the intuition delivered in Section 3 using the two-point distribution of θ. A verification easily confirms that (B.8) does not bind under Condition 1.

The following condition ensures that the candidate trigger strategy is positive for any θ.

Condition 2. *For all θ on the support, $\theta - \lambda_H(\theta) < K$.*

Finally, we ensure that the candidate trigger strategy decreases in θ by requiring the following condition:

Condition 3. $d\lambda_H(\theta)/d\theta < 1$.

It is straightforward to note that Conditions 2 and 3 also imply that wage is positive and increases in the project quality θ, as seen from (B.13).

B.2 The Joint Hidden Information/Hidden Action Region

When the effort cost is higher, both the *ex-ante* incentive constraint (B.6) and the *ex-post* incentive constraint (B.4) bind. The condition governing the parameters for this region when θ is drawn from a continuous distribution is given as follows.

Condition 4.

$$\int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left(\frac{P_0}{P_3^*(\theta)} \right)^{\beta} d\theta \leq \xi \leq \int_{\underline{\theta}}^{\bar{\theta}} M(\theta) \left(\frac{P_0}{P_2^*(\theta)} \right)^{\beta} d\theta \, , \tag{B.19}$$

where $P_3^*(\theta)$ is given in (B.18) and

$$P_2^*(\theta) = \frac{\beta}{\beta - 1} \left(K - \theta + \frac{1 - F_L(\theta)}{f_H(\theta)} \right) . \tag{B.20}$$

Denote l as the Lagrangian multiplier for (B.8). Then, the candidate equilibrium trigger strategy is given by

$$P(\theta) = P_J(\theta) = \frac{\beta}{\beta - 1}\left(K - \theta + \lambda_H(\theta) - l\,\frac{M(\theta)}{f_H(\theta)}\right). \tag{B.21}$$

The Lagrangian multiplier l is positive under Condition 4. Therefore, the optimal trigger with the exception of the one for the manager of the lowest quality project $\underline{\theta}$ is larger than the first-best $P^*(\theta) = \beta(K - \theta)/(\beta - 1)$. Because (B.8) holds with strict equality, we may combine (B.8), (B.12) and (B.21) in order to obtain

$$\xi = \int_{\underline{\theta}}^{\bar{\theta}} M(\theta)\left(\frac{P_0}{P_J(\theta)}\right)^{\beta} d\theta\,. \tag{B.22}$$

Solving the above equation gives the Lagrangian multiplier l. It is straightforward to show that the Lagrangian multiplier l increases in effort cost ξ, in that

$$\frac{dl}{d\xi} = \frac{P_0}{\beta - 1}\left[\int_{\underline{\theta}}^{\bar{\theta}} \frac{M^2(\theta)}{f_H(\theta)}\left(\frac{\beta}{\beta - 1}\right)^2\left(\frac{P_0}{P_J(\theta)}\right)^{\beta+1} d\theta\right]^{-1}. \tag{B.23}$$

Therefore, as effort cost ξ increases, the optimal trigger $P_J(\theta)$ decreases, as shown below:

$$\frac{dP_J(\theta)}{d\xi} = -\frac{\beta}{\beta - 1}\frac{M(\theta)}{f_H(\theta)}\frac{dl}{d\xi} < 0. \tag{B.24}$$

This is consistent with our intuition and results in Section 3 that a higher effort cost mitigates investment inefficiency by pushing the exercise trigger towards the first-best level.

The following two conditions ensure that the conjectured candidate solutions $P_J(\theta)$ is positive and decreasing in θ.

Condition 5.

$$\frac{d}{d\theta}\left(\theta - \lambda_H(\theta) + l\,\frac{M(\theta)}{f_H(\theta)}\right) > 0\,, \tag{B.25}$$

for $0 \le l \le 1$.

Condition 6. *The distribution* $F_H(\,\cdot\,)$ *first-order stochastically dominates* $F_L(\,\cdot\,)$*, in that*

$$F_H(\theta) \le F_L(\theta), \quad \textit{for all } \theta\,. \tag{B.26}$$

This implies $M(\theta) \ge 0$*, for all* θ*.*

We note that, under Conditions 5 and 6, wage is positive and increasing in θ, in that

$$w(\theta_2) = \int_{\underline{\theta}_2}^{\theta}\left(\frac{P(\theta_2)}{P(s)}\right)^{\beta} ds > \int_{\underline{\theta}}^{\theta_2}\left(\frac{P(\theta_1)}{P(s)}\right)^{\beta} ds > \int_{\underline{\theta}}^{\theta_1}\left(\frac{P(\theta_1)}{P(s)}\right)^{\beta} ds = w(\theta_1)\,, \tag{B.27}$$

for $\theta_2 > \theta_1$. The first inequality follows from the monotonicity of $P(\theta)$.

C Derivations of Optimal Contracts in Section 5

This appendix provides a derivation of the optimal contracts for the generalized model of Section 5. Propositions 1 and 2 apply as in Section 2. Using the method of Kuhn-Tucker, we form the Lagrangian as follows:

$$\begin{aligned}\mathcal{L} = & \left(\frac{P_0}{P_1}\right)^{\beta}(P_1+\theta_1-K-w_1)+\frac{1-q_H}{q_H}\left(\frac{P_0}{P_2}\right)^{\beta}(P_2+\theta_2-K-w_2)\\ & +\lambda_1\left[\left(\frac{P_0}{P_1}\right)^{\gamma}w_1-\left(\frac{P_0}{P_2}\right)^{\gamma}(w_2+\Delta\theta)\right]+\lambda_2\left[\left(\frac{P_0}{P_2}\right)^{\gamma}w_2-\left(\frac{P_0}{P_1}\right)^{\gamma}(w_1-\Delta\theta)\right]\\ & +\lambda_3\left[\left(\frac{P_0}{P_1}\right)^{\gamma}w_1-\left(\frac{P_0}{P_2}\right)^{\gamma}w_2-\frac{\xi}{\Delta q}\right]+\lambda_4\, w_2\,, \qquad \text{(C.1)}\end{aligned}$$

with corresponding complementary slackness conditions for the four constraints. As in Appendix A, we also conjecture that the *ex-post* incentive constraint does not bind, in that the Lagrangian multiplier λ_2 associated with the constraint below is zero:

$$\left(\frac{P_0}{P_2}\right)^{\gamma}w_2 \geq \left(\frac{P_0}{P_1}\right)^{\gamma}(w_1-\Delta\theta)\,. \qquad \text{(C.2)}$$

We will verify this conjecture for each region.

The first-order conditions with respect to w_1 and w_2 imply

$$0 = (\lambda_1+\lambda_3)\left(\frac{P_0}{P_1}\right)^{\gamma}-\left(\frac{P_0}{P_1}\right)^{\beta}\,, \qquad \text{(C.3)}$$

$$0 = -(\lambda_1+\lambda_3)\left(\frac{P_0}{P_2}\right)^{\gamma}-\frac{1-q_H}{q_H}\left(\frac{P_0}{P_2}\right)^{\beta}+\lambda_4\,. \qquad \text{(C.4)}$$

Using (C.3) to simplify (C.4) gives $\lambda_4>0$. The complementary slackness condition implies that $w_2=0$. The first-order conditions with respect to P_1 and P_2 are given by

$$P_1 = \frac{\beta}{\beta-1}\left[K-\theta_1+\left(1-\frac{\gamma}{\beta}\right)w_1\right]\,, \qquad \text{(C.5)}$$

$$P_2 = \frac{\beta}{\beta-1}\left[K-\theta_2+\lambda_1\frac{q_H}{1-q_H}\frac{\gamma}{\beta}\left(\frac{P_0}{P_2}\right)^{\gamma-\beta}\Delta\theta\right]. \qquad \text{(C.6)}$$

Therefore, it must be the case that at least one of (53) and (54) binds (Similar to Proposition 5 of Section 2). Depending on the cost benefit ratio $\xi/\Delta q$, we have three disjoint regions to be analyzed below, similar to the analyses in Section 3.

C.1 The Hidden Information Only Region

Suppose that the constraint (54) does not bind and thus $\lambda_3=0$. Then, $\lambda_1=(P_0/P_1)^{\beta-\gamma}>1$. A binding *ex-ante* incentive constraint (53) implies that the wage payment is $w_1 =$

$(P_1/P_2)^{\gamma}\,\Delta\theta$. The first-order conditions (C.5) and (C.6) give the following coupled equations:

$$P_1 = \frac{\beta}{\beta-1}\left[K-\theta_1+\left(1-\frac{\gamma}{\beta}\right)\left(\frac{P_1}{P_2}\right)^{\gamma}\Delta\theta\right], \tag{C.7}$$

$$P_2 = \frac{\beta}{\beta-1}\left[K-\theta_2+\frac{q_H}{1-q_H}\frac{\gamma}{\beta}\left(\frac{P_1}{P_2}\right)^{\gamma-\beta}\Delta\theta\right]. \tag{C.8}$$

Note that with $\gamma > \beta$, we immediately have $P_1 < P_1^*$ and $P_2 > P_2^*$. Therefore, $w_1 < \Delta\theta$, as conjectured, confirming that (C.2) does not bind and $\lambda_2 = 0$.

Define the ratio $x = P_1/P_2$. The coupled equations (C.7) and (C.8) allow us to first solve for ratio x^*, in that

$$G(x^*) = 0, \tag{C.9}$$

where

$$G(x) = x\left[K-\theta_2+\frac{\gamma}{\beta}\left(\frac{q_H}{1-q_H}\right)x^{\gamma-\beta}\Delta\theta\right]-\left[K-\theta_1+\left(1-\frac{\gamma}{\beta}\right)x^{\gamma}\Delta\theta\right]=0. \tag{C.10}$$

First, note that $G(0) = -(K-\theta_1) < 0$ and $G(1) = \gamma\Delta\theta/(\beta(1-q_H)) > 0$. Second,

$$G'(x) = K-\theta_2+\frac{\gamma+\gamma(\gamma-\beta)}{\beta}\frac{q_H}{1-q_H}x^{\gamma-\beta}\Delta\theta+\gamma x^{\gamma-1}\left(\frac{\gamma}{\beta}-1\right)\Delta\theta > 0\,,$$

for $\gamma > \beta$. Therefore, there exist a unique $x^* \in (0,1)$ solving (C.9).

Therefore, for the region defined by $\xi/\Delta q < (P/\hat{P}_3^*)^{\gamma}\Delta\theta$, where

$$\hat{P}_3^* = \frac{\beta}{\beta-1}\left[K-\theta_2+\frac{q_H}{1-q_H}\frac{\gamma}{\beta}(x^*)^{\gamma-\beta}\Delta\theta\right], \tag{C.11}$$

the optimal contract can be written as:

$$P_1 = \frac{\beta}{\beta-1}\left[K-\theta_1+\left(1-\frac{\gamma}{\beta}\right)(x^*)^{\gamma}\,\Delta\theta\right], \tag{C.12}$$

$$P_2 = \hat{P}_3^*\,, \tag{C.13}$$

$$w_1 = \left(\frac{P_1}{P_2}\right)^{\gamma}\Delta\theta, \tag{C.14}$$

$$w_2 = 0. \tag{C.15}$$

Finally, if $\xi/\Delta q < (P_0/\hat{P}_3^*)^{\gamma}\Delta\theta$, constraint (54) is indeed not binding, consistent with our conjecture.

C.2 The Joint Hidden Information/Hidden Action Region

We derive the optimal contract in this region by conjecturing that both (53) and (54) bind. Solving these two equality constraints gives (56) and (57). Plugging (56) and (57) into the first-order condition (C.5) gives

$$P_1 = \frac{\beta}{\beta-1}\left[K-\theta_1+\left(1-\frac{\gamma}{\beta}\right)\left(\frac{P_1}{\hat{P}_J}\right)^{\gamma}\Delta\theta\right]. \tag{C.16}$$

The solution for P_1 is $\hat{P}_1$, the same solution for P_1 as in the hidden action region. In Section 5 we proved that a unique $\hat{P}_1$ exists, where $\hat{P}_1 \in (0, P_1^*)$. Naturally, we have $w_1 = \left(P_1/\hat{P}_J\right)^{\gamma}\Delta\theta$. As before, we have verified that (C.2) does not bind in this region, because $\hat{P}_1 < \hat{P}_J$ implies that $w_1 < \Delta\theta$.

We know that the only possible regions in which both (53) and (54) bind is $(P_0/\hat{P}_3^*)^{\gamma}\Delta\theta \leq \xi/\Delta q \leq (P_0/P_2^*)^{\gamma}\Delta\theta$, since we have already shown that in the other regions only one of these constraints binds.[31] Equivalently stated in terms of $\hat{P}_J$, this region is characterized by $P_2^* < \hat{P}_J < \hat{P}_3^*$. We now verify that the above solutions are indeed optimal for this entire region. Recall that $\lambda_1+\lambda_3 = \left(P_0/\hat{P}_1\right)^{\beta-\gamma}$. Therefore, if we show that λ_1 lies within the range defined by

$$0 < \lambda_1 < \left(\frac{P_0}{\hat{P}_1}\right)^{\beta-\gamma}, \tag{C.17}$$

then we have shown both (53) and (54) bind ($\lambda_1,\ \lambda_3 \neq 0$).

The first-order condition with respect to P_2 implies that

$$\lambda_1 = \left[\frac{\beta}{\beta-1}\frac{q_H}{1-q_H}\frac{\gamma}{\beta}\left(\frac{P_0}{\hat{P}_J}\right)^{\gamma-\beta}\Delta\theta\right]^{-1}\left(\hat{P}_J-P_2^*\right). \tag{C.18}$$

Since $\hat{P}_J > P_2^*$, we have confirmed that $\lambda_1 > 0$. We next prove that $\lambda_1 < \left(P_0/\hat{P}_1\right)^{\beta-\gamma}$.

Expressing λ_1 as a function of $\hat{P}_J$, we can rewrite (C.18) as:

$$\lambda_1(\hat{P}_J) = \left(\frac{P_0}{\hat{P}_1(\hat{P}_J)}\right)^{\beta-\gamma}\left(\frac{\hat{P}_1(\hat{P}_J)}{\hat{P}_J}\right)^{\beta-\gamma}\left[\frac{q_H}{1-q_H}\frac{\gamma}{\beta}(P_2^*-P_1^*)\right]^{-1}\left(\hat{P}_J-P_2^*\right). \tag{C.19}$$

Note that from (C.16), $\hat{P}_1$ is a function of $\hat{P}_J$; we make this functional dependence explicit in the above equation. Proving that $\lambda_1 < \left(P_0/\hat{P}_1\right)^{\beta-\gamma}$ over the region $\hat{P}_J \in (P_2^*,\ \hat{P}_3^*)$ is equivalent to showing that

$$N(x) > 0, \quad \text{for} \quad x \in (P_2^*,\ \hat{P}_3^*),$$

[31] Note that in the region $\xi/\Delta q > (P_0/\hat{P}_1)^{\gamma}\Delta\theta$, it can be shown that effort cannot be induced. This result is available upon request.

where $N(x)$ is defined by

$$N(x) = P_2^* + \left(\frac{\hat{P}_1(x)}{x}\right)^{\gamma-\beta} \frac{q_H}{1-q_H}\frac{\gamma}{\beta}(P_2^* - P_1^*) - x. \tag{C.20}$$

Using implicit differentiation in (C.16), we can write:

$$\frac{d\hat{P}_1(x)}{dx} = \frac{\hat{P}_1(x)}{x} \frac{\gamma(\hat{P}_1(x) - P_1^*)}{\gamma(\hat{P}_1(x) - P_1^*) - \hat{P}_1(x)} > 0, \tag{C.21}$$

because $\hat{P}_1(x) < P_1^*$ in this region. Therefore,

$$\frac{dL(x)}{dx} = (\gamma - \beta)\frac{q_H}{1-q_H}\frac{\gamma}{\beta}(P_2^* - P_1^*)\left(\frac{\hat{P}_1(x)}{x}\right)^{\gamma-\beta-1} \frac{1}{x^2}\left(x\frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x)\right) - 1\,. \tag{C.22}$$

From (C.21),

$$x\frac{d\hat{P}_1(x)}{dx} - \hat{P}_1(x) = \frac{\left(\hat{P}_1(x)\right)^2}{\gamma(\hat{P}_1(x) - P_1^*) - \hat{P}_1(x)} < 0, \tag{C.23}$$

because $\hat{P}_1(x) < P_1^*$ in this region. Therefore, $N'(x) < 0$, for $x \in (P_2^*,\, \hat{P}_3^*)$. Since $N(\hat{P}_3^*) = 0$, we thus have $N(x) > 0$ for $x \in (P_2^*, \hat{P}_3^*)$. This confirms that $\lambda_1, \lambda_3 > 0$ in this entire region, and therefore both (53) and (54) bind.

C.3 The Hidden Action Only Region

Suppose that (54) binds, while (53) does not. Thus, $\lambda_1 = 0$ and $\lambda_3 = (P_0/P_1)^{\beta-\gamma} > 1$. With $\lambda_1 = 0$, equation (C.6) implies that $P_2 = P_2^*$.

A binding (54) implies that the wage payment is

$$w_1 = \left(\frac{P_1}{P_0}\right)^{\gamma} \frac{\xi}{\Delta q} = \left(\frac{P_1}{\hat{P}_J}\right)^{\gamma} \Delta\theta, \tag{C.24}$$

where $\hat{P}_J$ is given in (56). Substituting (C.24) into (C.5) gives $P_1 = \hat{P}_1$, the root of the expression given in (59). Section 5 proves that a unique $\hat{P}_1$ exists, where $\hat{P}_1 \in (0, P_1^*)$.

To ensure that our conjecture that (C.2) is not binding, (C.24) implies that we need to check if $\hat{P}_1 < \hat{P}_J$ holds. This inequality can be written as $\xi/\Delta q < (P_0/\hat{P}_1)^{\gamma}\Delta\theta$, which is assured to hold in this region. In order to be consistent with the fact that (53) does not bind, we need $(P_0/P_2^*)^{\gamma}\Delta\theta > \xi/\Delta q$, which again holds in this region.

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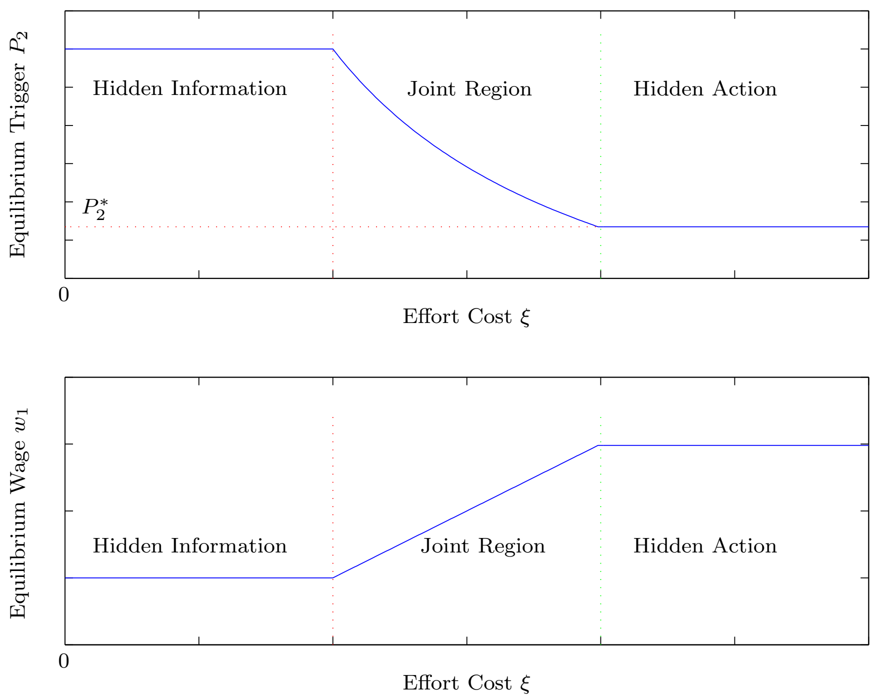

Figure 1: Optimal incentive contracts across the three parameter regions. The upper and lower graphs plot the equilibrium trigger strategy P_2 and wage payment w_1 in terms of effort cost ξ, respectively. As the cost of effort increases, the hidden action problem becomes more pronounced. The upper graph demonstrates that as the cost of effort increases, the equilibrium trigger strategy P_2 decreases, as it approaches the first best trigger P_2^*. The lower graph demonstrates that as the cost of effort increases, the wage payment must increase in order to induce effort from the manager. In summary, as the cost of inducing hidden effort increases, the timing of investment becomes more efficient while the value of the compensation package increases.

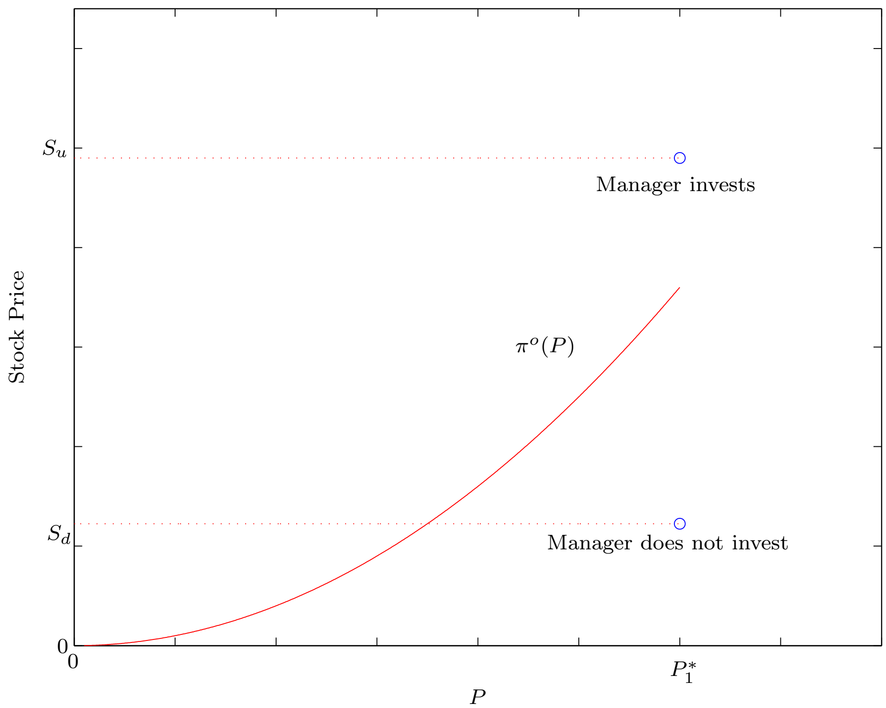

Figure 2: Stock price reaction to investment. This graph plots the stock price as a function of P, the present value of the observed component of cash flows. Whenever the level of P is below the lower investment trigger P_1^*, the market does not know the true value of θ, the present value of the unobserved component of cash flows. Thus, for all P below P_1^*, the stock price equals the value of the owner's option given in (31). At the moment the process P hits the trigger P_1^*, the true value of θ is revealed through the manager's action: if the manager invests, then the value of θ is the higher value θ_1; if the manager does not invest, then the value of θ is the lower value θ_2. Thus, the stock price is discontinuous at P_1^*. Investment signals good news and the stock price jumps to S_u, while failure to invest signals bad news and the stock price drops to S_d, where S_u and S_d are given in (37) and (38), respectively.