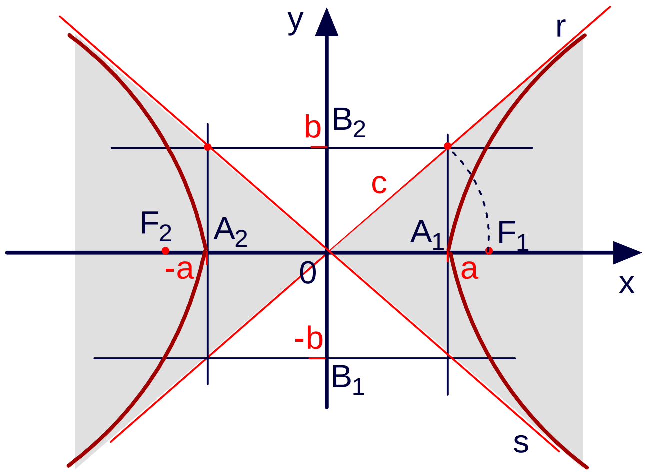
Iperbole

Iperbole riferita al centro ed agli assi: Fuochi sull'asse delle x



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$\overline{A_1A_2}=2\cdot a$ (asse trasverso)

$\overline{B_1B_2}=2\cdot b$ (asse non trasverso)

$\overline{F_1F_2}=2\cdot c$ (distanza focale)

$A_1(a\ ;\ 0)\quad A_2(-a\ ;\ 0)$ (vertici)

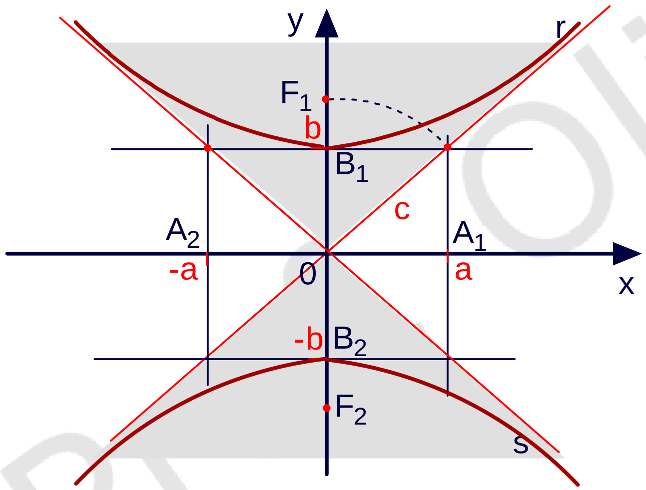
$F_1(c\ ;\ 0)\quad F_2(-c\ ;\ 0)$ (fuochi)

$c^2=a^2+b^2$

$y=\pm\frac{b}{a}\cdot x$ (asintoti)

$e=\frac{c}{a}$ $\left(\frac{\text{distanza focale}}{\text{asse transverso}}\right)$

Iperbole riferita al centro ed agli assi: Fuochi sull'asse delle y



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$$

$\overline{A_1A_2}=2\cdot a$ (asse non trasverso)

$\overline{B_1B_2}=2\cdot b$ (asse trasverso)

$\overline{F_1F_2}=2\cdot c$ (distanza focale)

$B_1(0\ ;\ b)\quad B_2(0\ ;\ -b)$ (vertici)

$F_1(0\ ;\ c)\quad F_2(0\ ;\ -c)$ (fuochi)

$c^2=a^2+b^2$

$y=\pm\frac{b}{a}\cdot x$ (asintoti)

$e=\frac{c}{b}$ $\left(\frac{\text{distanza focale}}{\text{asse transverso}}\right)$

Sdoppiamento nell'iperbole riferita al centro ed agli assi:

$\mathfrak{I}: \quad \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad ; \quad A(x_0 : y_0) \quad A \in \mathfrak{I} \quad ;$ fascio in $A: \quad y-y_0=m\cdot(x-x_0)$ (aggiungendo ad entrambi i membri $2\cdot y_0$)

$$y+y_0=m\cdot(x-x_0)+2\cdot y_0$$

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad \Rightarrow \quad b^2\cdot x^2-a^2\cdot y^2=1$ Passaggio per A: $\quad b^2\cdot x_0{}^2-a^2\cdot y_0{}^2=1$

Sottraendo:

$$\Rightarrow \quad b^2\cdot\left(x^2-x_0{}^2\right)-a^2\cdot\left(y^2-y_0{}^2\right)=0 \quad \Rightarrow \quad b^2\cdot(x-x_0)\cdot\left(x+x_0{}^2\right)-a^2\cdot(y-y_0)(y+y_0)=0 \quad \Rightarrow$$

$$\Rightarrow \quad b^2\cdot(x-x_0)\cdot(x+x_0)-a^2\cdot m\cdot(x-x_0)\cdot\left[m\cdot(x-x_0)+2\cdot y_0\right]=0 \quad \Rightarrow$$

$$\Rightarrow \quad b^2\cdot(x-x_0)\cdot(x+x_0)-a^2\cdot m^2\cdot(x-x_0)^2-2\cdot a^2\cdot m\cdot(x-x_0)\cdot y_0=0 \quad \Rightarrow \quad (x-x_0)\cdot\left[b^2\cdot(x+x_0)a^2\cdot m^2\cdot(x-x_0)-2\cdot a^2\cdot m\cdot y_0\right]=0$$

$\Downarrow$

- $x-x_0=0 \quad \Rightarrow \quad x=x_0$
- $b^2\cdot\left(x+x_0{}^2\right)-a^2\cdot m^2\cdot(x-x_0)-2\cdot a^2\cdot m\cdot y_0=0$

Imponendo il passaggio per A:

$$b^2\cdot(x_0+x_0)-a^2\cdot m^2\cdot(x_0-x_0)-2\cdot a^2\cdot m\cdot y_0=0 \quad \Rightarrow \quad 2\cdot b^2\cdot x_0-2\cdot a^2\cdot m\cdot y_0=0 \quad \Rightarrow \quad m=\frac{b^2\cdot x_0}{a^2\cdot y_0}$$

Sostituisco il coefficiente angolare trovato nel fascio in A:

$$y-y_0=m\cdot(x-x_0) \quad \Rightarrow \quad y-y_0=\frac{b^2\cdot x_0}{a^2\cdot y_0}\cdot(x-x_0) \quad \Rightarrow \quad a^2\cdot y_0\cdot y-a^2\cdot y_0{}^2=b^2\cdot x_0\cdot x-b^2\cdot x_0{}^2 \quad \Rightarrow$$

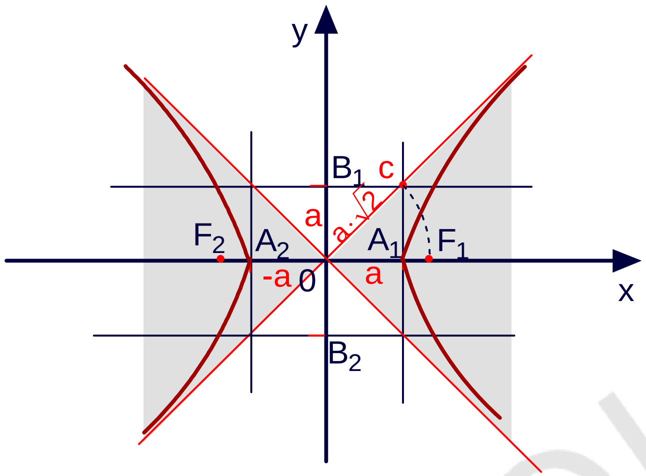
$$\Rightarrow \quad b^2\cdot x_0{}^2-a^2\cdot y_0{}^2=b^2\cdot x_0\cdot x-a^2\cdot y_0\cdot y \quad \Rightarrow \quad a^2\cdot b^2=b^2\cdot x_0\cdot x-a^2\cdot y_0\cdot y \quad \Rightarrow \quad \frac{b^2\cdot x_0\cdot x-a^2\cdot y_0\cdot y}{a^2\cdot b^2}=\frac{a^2\cdot b^2}{a^2\cdot b^2} \quad \Rightarrow \quad \frac{x_0\cdot x}{a^2}-\frac{y_0\cdot y}{b^2}=1$$

Iperbole

Iperbole equilatera:

Se in una iperbole: $a = b \quad \Rightarrow \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ diventa $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \quad \Rightarrow \quad x^2 - y^2 = a^2$

$$x^2 - y^2 = a^2$$



$\overline{OA_1} = \overline{OB_1} = a$

$\overline{A_1A_2} = 2 \cdot a$ (asse trasverso)

$\overline{B_1B_2} = 2 \cdot a$ (asse non trasverso)

$c^2 = a^2 + a^2 = 2 \cdot a^2 \quad \Rightarrow \quad c = \pm a \cdot \sqrt{2}$

$\overline{OC} = \overline{OF_1} = a \cdot \sqrt{2}$

$\overline{F_1F_2} = 2 \cdot a \cdot \sqrt{2}$ (distanza focale)

$A_1(0\,;\ a) \quad A_2(0\,;\ -a)$ (vertici)

$F_1\left(0\,;\ +a \cdot \sqrt{2}\right) \quad F_2\left(0\,;\ -a \cdot \sqrt{2}\right)$ (fuochi)

$y = \pm \frac{b}{a} \cdot x \Rightarrow y = \pm \frac{a}{a} \cdot x \Rightarrow y = \pm x$ (asintoti)

$e = \frac{c}{a}$ $\left(\frac{\text{distanza focale}}{\text{asse transverso}}\right)$