

Legality of Positions of Simple Chess Endgames

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1 Introduction

Complete analyses of simple chess endgames have been done for about 25 years. Probably the first were made by Ströhnlein in 1970 [Ströhnlein 1970]. Endgames with up to six pieces have been analyzed [Herik, Herschberg, Nakad 1987] [Stiller 1989] and very extensive data has been collected in recent works [Edwards 1996] [Edwards, Editors 1995]. Yet the very basic issue of the legality of positions in the analyzed endgames has never been looked into satisfactorily. This legality is not – as one could think – given by the rules of chess. Those rules only determine the legality of moves. Thus a lot of rather different definitions of the legality of a position have been used so far; in many papers the particular definition used is not even mentioned.

This article takes a closer look at the issue of the legality of chess positions. It is an extract from a paper (in German language) on the analysis of simple chess endgames which is available in the WWW together with the computer programs used and this article at the URL <http://www.wagr.informatik.uni-kl.de/~lippold/>.

At the beginning the problem of defining the legality of a position will be clarified with examples, and previous methods and general ways of obtaining definitions will be described. Conditions for the legality of a position and a method for proving it will be given. Then one complete and one correct definition will be given for endgames with three or four pieces. Finally ways of obtaining corresponding definitions for endgames with more pieces and ways of improving them will be explained.

2 Description of the Problem and of Possible Solutions

The definition of the legality of a position depends for a given algorithm on the exact way in which it models positions. Usually algorithms developed for a given number of pieces on the board do this by assigning to each piece the square it occupies. Other possible methods are to assign up to one piece to each square and to create a list consisting of pairs made up of a piece and a square.

The usual models allow for two pieces on one square. On the other hand it is impossible to have a piece without a square assigned to it. Therefore, when two pieces are on the same square (ie. the same square is assigned to both), one of them is regarded as being captured by the other. Which of the two pieces is

captured usually depends on the colour playing the next move. Thus as a rule one states that the piece of the colour which played the last move has captured the other one.

The following position of a position like this cannot be defined, because in the following position the relation of capture of the two pieces changes with the colour to move and thus the piece staying on the board. Likewise positions with a Pawn of the colour playing the next move on its promotion rank can't be modeled because the Pawn would have to be replaced by another piece. Therefore positions with two captured pieces or a Pawn of the colour to move on its promotion rank are considered to be illegal.

Apart from these restrictions on the legality of a position due to the model used one has to determine what to consider as a legal position on the board. For this purpose one can use the following definition which is the one the authors of most analyses seemingly meant to use:

A position is legal if it can be set up from the starting position by legal moves.

This definition is called *ideal definition*. *Starting position* denotes the position at the beginning of a game. The place a piece occupies in this position is called its *starting place*.

The problem of checking the legality of a position is then building a sequence of legal moves which leads from the starting position to the position present. Because such a move sequence can be very long, especially because all pieces are involved, and because the proof that a sequence of legal moves does not exist might bring about the need to build all move sequences (or at least all without redundancy with regard to the repetition of positions) leading to the position present a direct proof does for the general case not seem feasible.

Yet the illegality of some positions can be seen immediately, eg. positions with a white Pawn on the first rank or two pieces of the same colour on the same square. These positions can, regardless of the places of the other pieces, not be set up. They can, like the positions which are illegal due to the model used, be called *initially illegal*.

In most previous works a similar definition of the illegality of a position is used. Van den Herik and Herschberg proposed three rules for this in their analysis of the endgame of King, Bishop and Knight against Knight with White to move [Herik, Herschberg 1985a]. For the endgame of King, Rook and Pawn on a2 against King, Bishop confined to the black squares and Pawn on a3 with White to move van

den Herik, Herschberg and Nakad proposed seven rules [Herik, Herschberg, Nakad 1987]. The rules for 3-, 4- and some 5-pieces endgames were quite thoroughly described by Edwards in nine rules [Edwards 1996].

In 1977 Clarke proposed the following rules for the legality of a position in an analysis of the endgame of King and Pawn against King [Clarke 1977]:

1. Two pieces may not occupy the same square.
2. The Kings may not be immediate neighbours.
3. The Black King may not be checked when White is to move.

Shapiro and Niblett remarked [Shapiro, Niblett 1982] that hence positions are regarded as legal even if there is no previous position. Because of this they defined in their paper on positions with Black to move that those with a white Pawn on the second rank threatening the black King and those with the black King on c1 or c2 depriving the white King on a1 behind its Pawn on a2 of possible origin squares (cf. figure 1) are to be regarded as illegal, too.

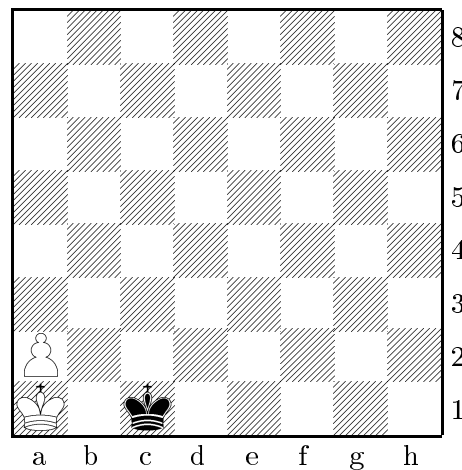


Figure 1: Position with Black to move which is illegal because the white King has no origin square

Positions without a previous position can be called *derivedly illegal*. First attempts to describe these positions were made by van den Herik and Herschberg [Herik, Herschberg 1985b] [Herschberg, Herik 1987].

The position in figure 2 shows that it is not sufficient to look only at the positions of the analyzed endgame in checking whether it is derivedly illegal.

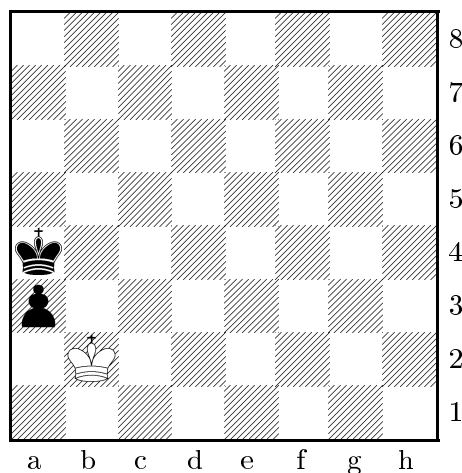


Figure 2: Positions with White to move in which the checking Pawn has no origin square in the same endgame

Due to the Pawn checking there is no previous position in the analyzed endgame with three pieces, yet there is one in the endgame with four pieces with a white piece on a3 which was captured by the Pawn coming from b4. If the black King was on a4 instead of b4 the position would be illegal in the 4-pieces endgame instead.

The Pawn does not only lack a origin square when it is blocked by another piece but as well when checking or having captured a piece while being in its starting place as the following position with a captured Queen makes clear (figure 3):

White: ♔h3, ♕b7 Black: ♔h6, b7 White to move

Figures 4 and 5, both of which lack a previous position in their endgame, show that it is not too easy to determine whether a position has a previous position.

While there could be a previous position for the position in figure 4, eg. with the Rook having come from h1 and captured a piece of e1, the position in figure 5 does, even in possible previous endgames, not have a previous position and is therefore illegal in any case.

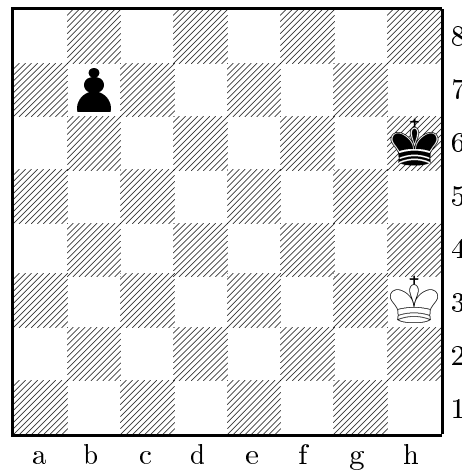


Figure 3: Illegal position of a 4-pieces endgame (with captured White Queen on b7) with White to move

In order to recognize one position as illegal (according to the ideal definition) and the other as legal, the existence of a previous position in a previous endgame will have to be checked. In turn for some of these the existence of a previous position in a previous endgame of these might have to be checked. Due to these recursive dependencies the cost of checking can increase almost without limits.

In any case the existence of a previous position is only a necessary but not a sufficient condition. Figure 6 shows a position which is neither initially nor derivedly illegal but illegal according to the ideal definition because the white King could not have moved in front of its Pawns. Figure 7 shows a position in the same endgame which is legal according to the ideal definition.

Thus there are sets of positions that are not initially illegal and which have a previous position in the analyzed endgame but which cannot be reached from the starting position and are therefore illegal according to the ideal definition. These positions can be called *isolatedly illegal*.

In general it is possible to prove the legality of a position (according to the ideal definition) either positively or negatively. The positive proof states conditions in which a positions can be reached from the starting position in any case, the negative proof states conditions in which it cannot be reached in any case.

In the following paragraph a definition of the illegality of a position using negative

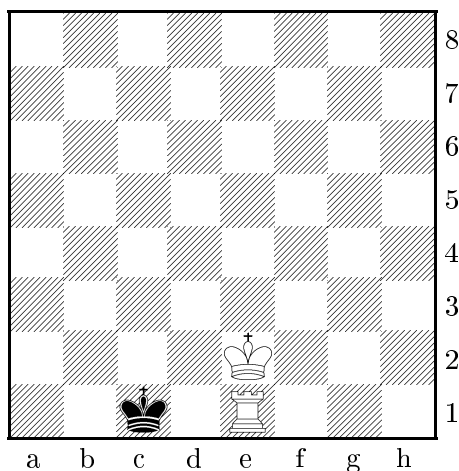


Figure 4: With Black to move legal according to the ideal definition

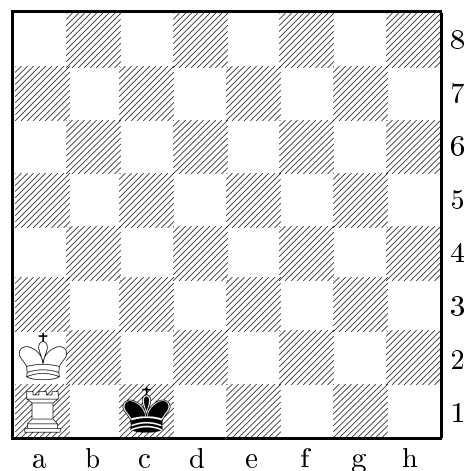


Figure 5: With Black to move illegal according to the ideal definition

conditions will be given. Following this, a method for positively proving the legality of a position will be described below.

3 Definitions

Pawn in the following means any Pawn, ie. one which is not restricted to a certain file (eg. like an a-Pawn).

The *promotion rank* of a *Pawn* is the 8th rank for a White Pawn and the 1st rank for a Black Pawn.

A **legal position** is defined as follows:

1. The starting position is legal. All positions not initially or derivedly or isolatedly illegal are legal, too.
2. A position is *initially illegal* in the following cases:
 - (a) The place of a piece is according to the rules of chess immediately illegal if

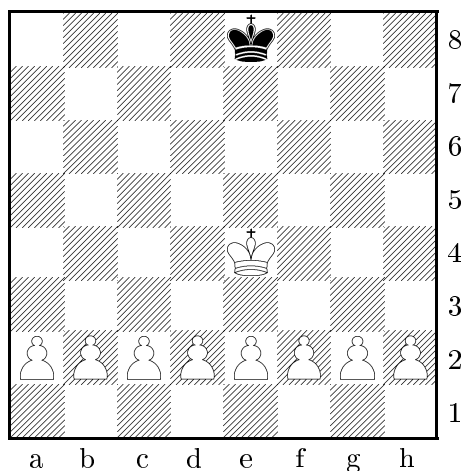


Figure 6: An unachievable and therefore according to the ideal definition illegal position

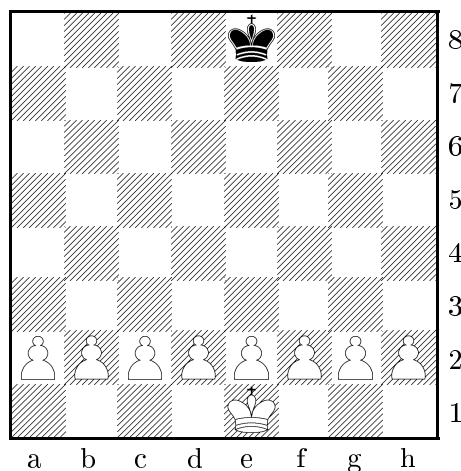


Figure 7: In the same endgame an achievable and therefore according to the ideal definition legal position

- i. there is a White Pawn on the 1st or a Black Pawn on the 8th rank,
- ii. the two Kings are on the same square or immediate neighbours,
- iii. two pieces of the same colour are on the same square.

(b) The representation of a position is impossible if

- i. the King of the colour to move is on the same square as a piece of the other colour,
- ii. a Pawn of the colour to move is on its promotion rank,
- iii. there are two different squares on each of which there is a piece captured or a Pawn on its promotion rank.

(c) A piece of the colour to move threatens the other colour's King.

A position which is not initially illegal is called *initially legal*.

3. A position is *derivedly illegal* if it is initially legal but simply or n times derived illegal.

It is *simply (once) derived illegal* if there is no initially legal position from which it can be reached with one legal move.

It is *n times derived illegal* if every position, from which it can be reached with one legal move, is initially or m times derived illegal with an arbitrary

m smaller than n and furthermore at least one position is $(n - 1)$ times and no position is n times derived illegal.

4. A position is *isolatedly illegal* if it is neither initially nor derivedly illegal but there is no sequence of legal moves that reaches it from the starting position.

A **legal move** is defined as follows:

A legal move is the movement of a piece in a way allowed by the rules of chess (eg. Rook straight, Bishop diagonal) from a *origin square* to a *target square* where both origin position and target position are initially legal.

In addition the following rules are necessary for legal moves:

- In the origin position no piece may be captured, in particular the moving piece must occupy a square on its own.
- If a Pawn moves diagonally (captures) the target square must be occupied by a piece (of the other colour).
- If a Pawn moves straight the target square must not be occupied by a piece.

The following remarks to these definitions seem appropriate here:

- It is immediately clear that a position is legal according to the above definition if and only if it is legal according to the ideal definition.
- The definition of derived illegality depends on that of a legal move (uses it), which in turn depends on that of initial illegality.
- The definition of initial illegality can be derived from the position itself. The way of determining whether a position is derivedly illegal will be described.
- Usually in isolatedly illegal positions a piece will be shut off (isolated) from a part of the chess board by Pawns (for an example cf. figure 6).
- If the algorithm for the analysis of the endgame makes it possible to restrict an existing Bishop to the squares of one colour an according rule will have to be filed under item 2a.

To find out whether a position is derivedly illegal one has to determine first whether it is initially illegal. For one which is not initially illegal one has to test whether it has a initially legal previous position. Positions without such a previous position are to be regarded as (simply) derived illegal. Since they may have been the only previous positions of other positions their illegality can bring about the derived illegality of yet other positions. Therefore as a next step one has to check if the following positions of the newly found illegal positions have at least one other previous position which is neither initially nor derivedly illegal. This iteration ends when no more derivedly illegal positions can be derived from the illegality of the positions of a step.

The positions in figures 8 and 9 may serve to clarify this. The piece on b1 in the position in figure 9 is shown as a Pawn in spite of the fact that after its promotion it represents another piece.

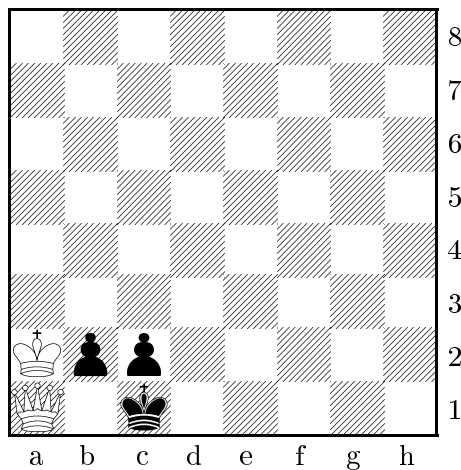


Figure 8: With Black to move a simply derived illegal position

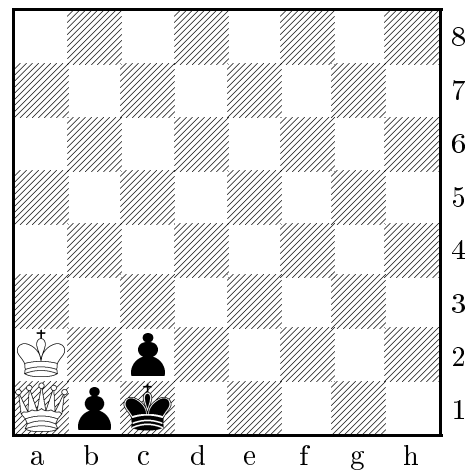


Figure 9: With White to move a two times derived illegal position

The position shown in figure 8 is derivedly illegal because the checking White Queen has no square from which she could have come, ie. there is no prior position in which she would not have threatened the King. Therefore the only previous position for the position in figure 9 is invalid and it is recognized as illegal, too.

4 Derivability of Positions

After the above definition has led to some negative conditions for legal positions a method of positively proving the legality of positions shall be described.

The proof is done by searching for the analyzed endgame for a set of positions, called *initial positions* for which the following conditions are valid:

- The positions are legal according to the ideal definition.
- As many positions as possible of the endgame can be reached (derived) from them, ie. there is a sequence of legal moves from one of the initial positions for each of them.

A position which can be derived from a legal position shall be called *derivable* from this position.

To ease the search for a set of initial positions for an endgame it is sensible to reflect on some basics.

The relation transforming a position into a following position by a legal move be called *derivative relation*. It creates equivalence classes on the set of initially legal positions. A *endgame* is here, as everywhere in this article, defined by the type and number of pieces, a *position* by the pieces' places and the colour to move. Furthermore the derivative relation creates a partial ordering on the equivalence classes by defining an equivalence class as being larger than another if a position of the larger class can be derived from one of a smaller class by legal moves (the uniqueness of this order can be seen easily). A Pawn move always leads to a position of a larger class.

The smallest equivalence classes of an endgame be called *basic classes* of the endgame. The positions displayed in figures 6 and 7 belong, for example, to two different basic classes of the endgame with eight White Pawns on the eight files.

All the positions of an equivalence class are rated equal according to derived and isolated illegality and therefore also equal according to the ideal definition where equivalence classes of derivedly illegal positions consist only of a single position anyway. Thus the concept of derived illegality, isolated illegality and legality in general can be transferred to equivalence classes. Furthermore all classes larger than a legal class are legal.

In order to derive in the ideal case all the positions in an endgame from initial positions every legal basic class must contain at least one initial position. Yet this

condition is only necessary but not sufficient. There are legal positions without a legal basic class as figure 10 makes clear. In this position the Pawn could have reached a4 by starting on the c-file and capturing two pieces. In its basic class the Pawn occupies a3 (cf. figure 11) which it can't have reached.

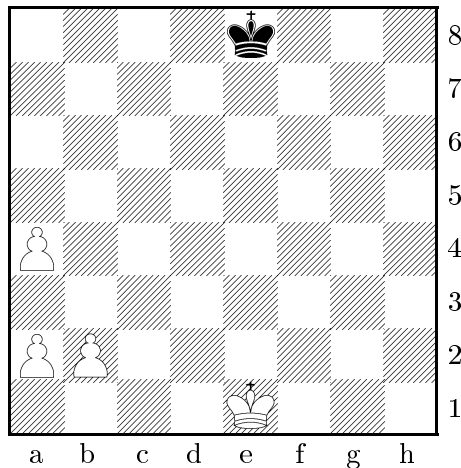


Figure 10: Legal position with an isolatedly illegal basic class

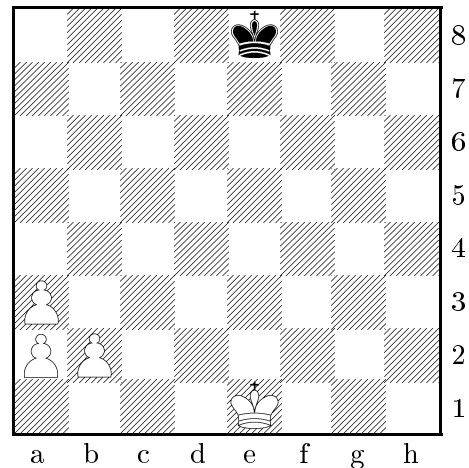


Figure 11: Position of the isolatedly illegal basic class of the position in the neighbouring figure

Finding a set of initial positions that fulfills the two conditions stated above can be difficult in a specific endgame. This is especially, as the positions in figures 10 and 11 suggest, the case for endgames with more than one Pawn on the same file.

Yet for a large number of endgames there is a canonical set of initial positions, called *canonical initial positions*. As follows from the remarks made in the basic class of an endgame without double Pawns all Pawns must be in their starting places. The canonical initial positions are found by applying this principle to all the pieces (ie. in them the White King must be on e1 and a White Rook on a1 or h1).

The canonical initial positions now can in general be derived easily from the starting position. This can, for example, be done by capturing all taken pieces with Knights and the Knights in turn by Kings or other still existent pieces. Before this the c- and g-Pawns would advance in order to avoid check and a possible checkmate.

An algorithm deriving the positions of an endgame from the initial positions is in principle equal to an algorithm for the analysis of an endgame. Contrary to the analysis, though, not the previous positions of a position but its following positions are created. A position can be rated with the minimal number of plies needed to derive it from an initial position. The algorithm does other than that for analysis not need any data from other endgames (which arise from capturing pieces or promoting Pawns).

5 Results for 3- and 4-pieces endgames

There are two main difficulties in determining whether a given position is legal according to the ideal definition.

- To find out whether a position is derivedly illegal it may be necessary to evaluate previous positions in previous endgames as well. For these in turn it may again be necessary to check the existence of previous positions in their previous endgames.
- Positions that are neither initially nor derivedly illegal have to be checked for isolated illegality.

There is no general method for either of those. Therefore a narrow and a broad definition for the legality of a position were used for the analysis of 3- and 4-pieces endgames.

According to the *narrow definition* a position is illegal if

- it is initially illegal
- or there is no previous position *in the analyzed endgame*.

Figures 12 (simply derived illegal position) and 13 (two times derived illegal position) show illegal positions following this definition.

Following the *broad definition* a position is only illegal if it is initially illegal.

For 4-pieces endgames both definitions have to be extended by a condition concerning isolated illegality according to which a position is illegal if a Bishop is in a corner with a Pawn of the same colour diagonally in front of it in its starting

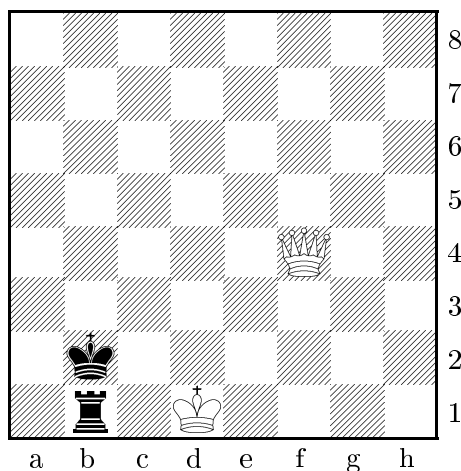


Figure 12: With White to move a simply derived illegal position following the narrow definition

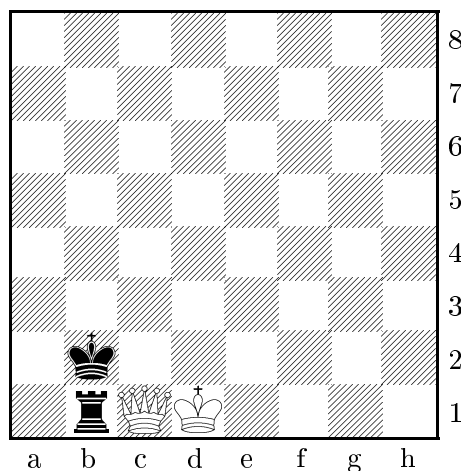


Figure 13: With Black to move a two times derived illegal position following the narrow definition

place (cf. figure 14). In a position like this the Bishop can't have reached its square.

The idea behind these two definitions is to make the narrow definition more restrictive than the ideal definition, the broad definition less so. Thus the legal positions according to the narrow definition should form a subset of the according to the ideal definition legal positions, the legal ones according to the broad definition a superset. So the narrow definition could be called *correct* and the broad *complete*.

To positively prove that this is indeed the case the following two conditions have to be proved:

- Every legal position according to the ideal definition is legal according to the complete definition.
- Every legal position according to the correct definition is legal according to the ideal definition.

The equivalent conditions for a negative proof are:

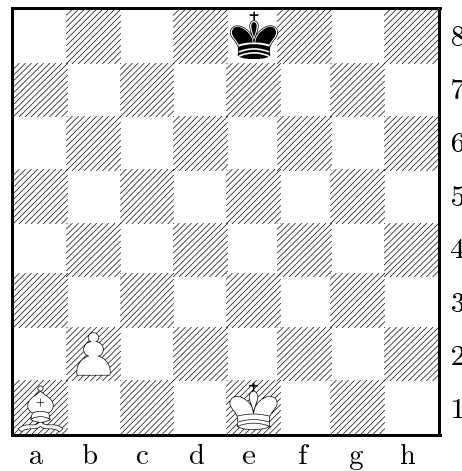


Figure 14: Position which is isolatedly illegal according to either definition of the legality of a position in a 4-pieces endgame

- Every illegal position according to the complete definition is illegal according to the ideal definition.
- Every illegal position according to the ideal definition is illegal according to the correct definition.

The (negative) condition for the broad (complete) definition follows from the fact that this definition contains apart from the conditions for initial illegality only the additional condition for isolated illegality which obviously describes only positions that can't be reached from the starting position.

The (positive) condition for the narrow (correct) definition arises from the investigation of the derivability of the positions which are legal according to this definition.

Using a computer program the derivability of all according to the narrow definition legal positions in all the analyzed 3- and 4-pieces endgames apart from those with two Pawns of different colours (see below) could be proved. In most cases the canonical initial positions were used as initial positions. Yet in some endgames with four pieces there are no canonical initial positions because of pieces that have come into play by means of promotions or captures and that do not exist in the starting position (endgame with two Queens of the same colour, two Bishops of the same colour on squares of the same colour or two Pawns of the same

colour on the same file). In those endgames the initial positions were chosen to be the positions in which a Queen, a Bishop or a Pawn is on the third or the sixth rank and the same piece on the first (White Queen or White Bishop) or the second rank (White Pawn) resp. the seventh (Black Pawn) or the eighth rank (Black Queen or Black Bishop). The positions with a second Queen or a second Bishop can then be derived eg. from a canonical initial position of a previous endgame in which there is instead of the piece on the third resp. sixth rank a Pawn of the same colour in its starting place on the a- resp. h-file. This Pawn can then be promoted to a Queen resp. a Bishop.

Yet in the endgame with Bishop and Pawn of different colours as well as with Rooks of the same colour there are positions that can't be derived from the initial positions. Eg. in the endgame with the two Rooks these are – apart from symmetric positions – exactly the following:

White: ♔a4, ♖b1, ♖b3 Black: ♚a1 Black to move

White: ♔a4, ♖b2, ♖b3 Black: ♚a2 Black to move

White: ♔b4, ♖b2, ♖b3 Black: ♚a2 Black to move

White: ♔b4, ♖b1, ♖b3 Black: ♚a1 Black to move

White: ♔b4, ♖b1, ♖b2 Black: ♚a2 Black to move

The first position is also displayed in figure 15.

The positions with White to move follow immediately from these through the only move of the Black King, where the last position hasn't got a following position because it is checkmate.

As can be seen the above positions except the last one form an own basic class because they can all be transformed one into another but do not have another previous position in this endgame. Accordingly their following positions are only positions with Black stalemate or in which the King has to capture one of the Rooks and from which therefore none of the initial positions is derivable.

Yet these positions can be derived from a position of a previous endgame, eg. with a captured Rook occupying b1 in the position in figure 15. Because this previous position can in turn be derived from a canonical initial position of its

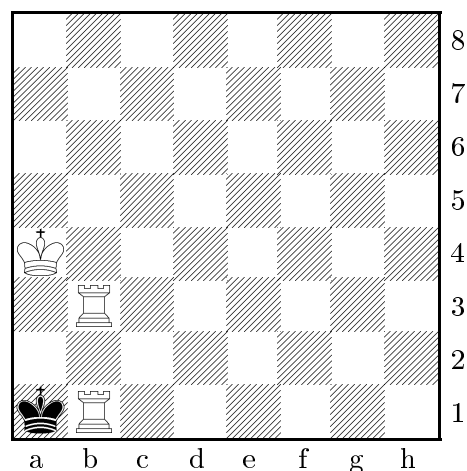


Figure 15: Position with Black to move that can't be derived from a canonical initial position in the endgame with two White Rooks

own endgame the above positions of an endgame with two Rooks of the same colour are legal.

Likewise the in the first order underivable positions of the endgame with Bishop and Pawn (example cf. figure 16) and those underivable in the second order (example cf. figure 17) can be derived from a previous position of a previous endgame through the promotion of a Pawn to a Bishop.

In the endgame with two Pawns of different colours there are a considerable number of not derivable positions. These are mainly the ones where the two Pawns are on the same file and swapped with regard to their direction of movement and the starting place, ie. the ones with the Black Pawn being nearer to the first file than the White one. For this endgame the correctness of the narrow definition could not be proved (even though it is likely to be true).

Tables 1 to 3 show the number of legal positions following from the correct and the complete definition for the most interesting 3- and 4-pieces endgames. The endgames are labeled with the colours and types of the pieces (with N for kNight) except the Kings.

Table 1 refers to the endgames without Pawns. In table 2 positions with Pawns on their promotion rank are counted once only, in table 3 four times (once for each possible promotion).

Endspiel	# lcorW ^a	# lcomW ^b	# lcorB ^c	# lcomB ^d
wQ	148120	148120	223944	223944
wR	178780	178780	223896	223944
wR wR	4337296	4337760	6510920	6830292
wB wB	5273596	5275312	6630400	6830292
wB wN	11271724	11274284	13466524	13660584
wQ - bQ	9325900	9400496	9325900	9400496
wQ - bR	9343816	9400496	11153868	11228616
wQ - bB	9311188	9400496	12205128	12280352
wQ - bN	9381472	9400496	12908468	12983144
wR - bR	11169880	11228616	11169880	11228616
wR - bB	11127536	11228616	12218848	12280352
wR - bN	11209448	11228616	12922776	12983144

^a# lcorW = Number of legal positions with White to move according to the correct definition

^b# lcomW = Number of legal positions with White to move according to the complete definition

^c# lcorB = Number of legal positions with Black to move according to the correct definition

^d# lcomB = Number of legal positions with Black to move according to the complete definition

Table 1: Number of legal positions in some interesting endgames without Pawns

Endspiel	# lcorW ^a	# lcomW ^b	# lcorB ^c	# lcomB ^d
wB	165988	166004	193426	195984
wQ - bP	8055204	8211280	10241776	10296944
wR - bP	9622576	9804392	10249518	10296944
wB - bP	10528086	10725920	10209380	10296944
wN - bP	11129972	11335902	10282148	10296944

^a# lcorW = Number of legal positions with White to move according to the correct definition

^b# lcomW = Number of legal positions with White to move according to the complete definition

^c# lcorB = Number of legal positions with Black to move according to the correct definition

^d# lcomB = Number of legal positions with Black to move according to the complete definition

Table 2: Number of legal positions in some interesting endgames with one Pawn where positions with a Pawn on its promotion rank are counted only once

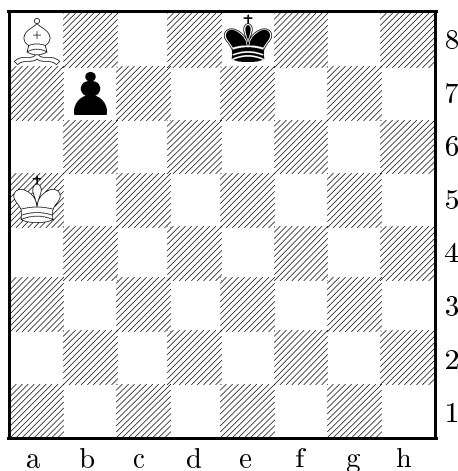


Figure 16: First order position that is not derivable from the canonical initial positions

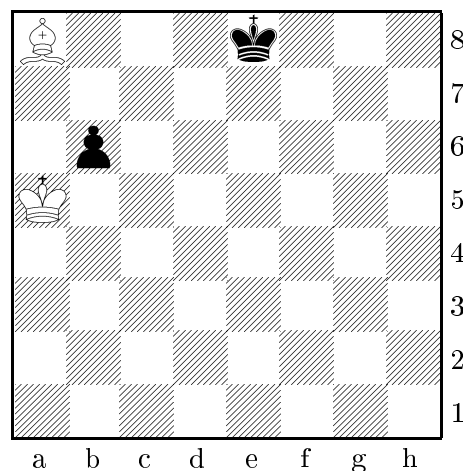


Figure 17: Second order position with White to move that is not derivable from the canonical initial positions

It should be noted that the numbers for the correct definition were found without regard to the castle rule. It is the same as if a castle was not possible any more. With this rule taken into consideration the number of legal positions should only change very little because there will be but a few positions in the analyzed endgames that can only be reached by a castle.

With regard to the above negative condition for the correct definition one can conclude that there are no isolatedly illegal positions in the 3- and 4-pieces endgames but those which are included in the above condition for isolated illegality.

6 Possible Improvements

After investigating endgames with three and four pieces one of the questions arising is how, based on the previous considerations, correct or complete definitions could look for endgames with more than four pieces. Another question is how all the definitions could be improved.

Due to the difficulties encountered above the possibility of finding a definition that is correct *and* complete for any endgame and thus equivalent to the ideal definition seems unlikely. By improving the definitions though it is possible to

Endspiel	# lcorW ^a	# lcomW ^b	# lcorB ^c	# lcomB ^d
wB	165988	166004	272182	279864
wQ - bP	11200584	11611168	10241776	10296944
wR - bP	13418572	13909304	10249518	10296944
wB - bP	14683476	15221456	10209380	10296944
wN - bP	15543548	16109868	10282148	10296944

^a# lcorW = Number of legal positions with White to move according to the correct definition

^b# lcomW = Number of legal positions with White to move according to the complete definition

^c# lcorB = Number of legal positions with Black to move according to the correct definition

^d# lcomB = Number of legal positions with Black to move according to the complete definition

Table 3: Number of legal positions in some interesting endgames with one Pawn where positions with a Pawn on its promotion rank are counted four times

reduce the number of positions that are evaluated differently according to the complete and the correct definition. For some endgames it is probably possible to get the same evaluation for all the positions and thereby arrive at the ideal definition in this endgame.

The complete definition given above for 3- and 4-pieces endgames can be transferred to endgames with more than four pieces without changes.

With the correct definition the problem of having to exclude the isolatedly illegal positions arises. Therefore it seems more adequate to postulate a correct definition by giving (for any endgame) a set of initial positions and regard all positions as legal according to the correct definition that can be derived from this set. In this case the definition would still be correct even if the set of initial positions was too small.

An improvement of the complete definition for endgames with more than four pieces can be achieved through rules for identifying isolatedly illegal positions (like the rule for positions with a Bishop in a corner and a Pawn diagonally in front of it in 4-pieces endgames).

Another chance for improvements (for the complete definition in 3- and 4-pieces endgames as well) is to check the possibility of the existence of a previous position in a previous endgame independent of its type. One would try to negatively prove that in the last move of a (eg. checking) piece no other piece could have been captured. If this could be proved for a (derivedly illegal) position without a previous position in the analyzed endgame or for all positions of an (isolatedly illegal) basic class, these positions could be regarded as illegal according to the

complete definition as well. In that way the position in figure 5, for example, would be recognized as illegal because the checking Rook can't have captured a piece.

Both definitions could be improved for positions without a previous position in the analyzed endgame or those belonging to a basic class without an initial position by an analysis of the possible previous positions in the possible previous endgames. If at least one of these previous positions was legal according to the correct definition the positions in the analyzed endgame would be legal, too (as in the five positions of the separate basic class in the endgame with two Rooks of the same colour given above). If all previous positions were illegal according to the complete definition the positions would also be illegal.

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