THE DESIGN OF AN AUDIO FREQUENCY VACUUM TUBE AMPLIFIER

A thesis submitted in partial fulfillment

of the requirements for the degree of

Master of Science in Engineering

By

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY Brad Bryant ENTITLED The Design of an Audio Frequency Vacuum Tube Amplifier BE ACCEPTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Engineering

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Abstract

Bryant, Brad. M.S., Department of Electrical Engineering, Wright State University, 2000. The Design of an Audio Frequency Vacuum Tube Amplifier.

An in-depth procedure for the design of an audio-frequency vacuum tube amplifier is not readily available. Books that discuss vacuum tubes are also difficult to find due to the fact that the vacuum tube was replaced by the transistor in the 1960's as the active circuit element of choice for modern electronics. The older books are lacking in detail, leaving out the steps involved to arrive at the solution. Due to the persistence of the vacuum tube remaining the active element of choice for audio-frequency amplification, there have been new books published on the topic of vacuum tube amplifiers. However, they are not technical analysis. They skip over details, being targeted for the electrically illiterate. This thesis will use circuit analysis techniques to determine equations that will enable an amplifier to be designed to meet desired performance specifications. The derivation of these equations will be presented step by step. A graphical method of design will also be presented. This method can give insights into the design procedure that are unattainable by using only equations. Equations will be derived step by step to enable the design of an active load for the amplifier. The use of an active load will improve amplifier performance. This thesis presents the design of a vacuum tube amplifier with a level of detail and insight that does not exist elsewhere in a like form.

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Acknowledgements

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1 Introduction

1.1 Background Information

Why would one study the design of audio-frequency vacuum tube amplifiers? Vacuum tubes were discarded in favor of transistors over thirty years ago. In the amplification of electric musical instruments, vacuum tubes have not been discarded. High end musical instrument amplifiers are designed with vacuum tubes. Musicians prefer the sound of vacuum tube amplifiers over transistor amplifiers.

The debate over the difference between these two technologies has been around for thirty years. This debate will always be around. The debate is based upon individual opinion on which technology sounds better. The author of this thesis will not attempt to prove scientifically the reason for a perceived difference in audible sound quality. This would be pointless because there is nothing inherently scientific about human opinion. The author will study the design of vacuum tube amplifiers because they are the hot item in the musical amplifier industry.

1.2 What is a Vacuum Tube Amplifier?

An amplifier is a circuit that will take a small AC input signal of one volt or less, and amplify it. Ideally, the output of the amplifier is a mirror image of the input, except for having larger magnitude. A vacuum tube amplifier uses vacuum tubes as the amplifying element, as opposed to using transistors. The standard vacuum tube amplifier inverts the output with respect to the input. The type of vacuum tube to be used in the design of the amplifier is a 12AX7WA, also known as a 7025 or ECC83 tube. The 12AX7WA is a high-μ dual triode, capable of dissipating one watt. High-μ means the tube has an amplification factor from 50 to 100. The 12AX7WA consists of two triodes enclosed within the same vacuum tube, hence a dual triode. A triode consists of four elements. These elements are the control grid, the cathode, the heater, and the plate. The plate is also known as the anode. The control grid is located between the cathode and the plate. The heater is located in close proximity to the cathode.

The basic principle of operation of a triode is as follows. The circuit is biased with a DC current flowing from the plate to the cathode. The DC current is set to enable class A operation. The signal to be amplified is applied to the control grid. A change in the voltage on the control grid creates an AC current from the plate to the cathode. The plate is connected to a high DC voltage supply by a load resistor. When an AC current flows through the triode, it creates an AC voltage across the load resistor. This AC voltage is ideally identical in shape to the input signal, except for being amplified and inverted.

The heater function is to heat the cathode so it will readily give up electrons. The heater can be a source of noise in the amplifier due to its location near the cathode and the fact that it is generally heated by an AC voltage. In this thesis, the heater will be considered to behave ideally. This is a safe assumption given a tube is functioning properly. If the heater is introducing noise into the circuit, the solution is to replace the tube. A DC voltage supply can be used to operate the heater if the noise cannot be removed to a tolerable level.

1.3 Purpose

The purpose of this thesis is to design an audio-frequency amplifier with the popular technology. A technical understanding will be developed by deriving equations to aid in circuit design. This thesis will develop a detailed design procedure with every step of equation derivation included. Commercial vacuum tube audio-frequency amplifiers are not as good as they could be. The capitalist society that we live in creates in general a market place where you get what you pay for. Therefore, the author has great interest in vacuum tube amplifier design. A good amplifier can be built by an intelligent person for less money than you would pay for an inferior commercially produced amplifier.

2 The Common-Cathode Amplifier

2.1 The Common-Cathode Amplifier Circuit

The first step in designing an amplifier is to determine the circuit topology to be used. The most popular vacuum tube amplifier circuit used in small-signal amplification is the common-cathode arrangement. Figure 1 shows a common-cathode amplifier circuit.

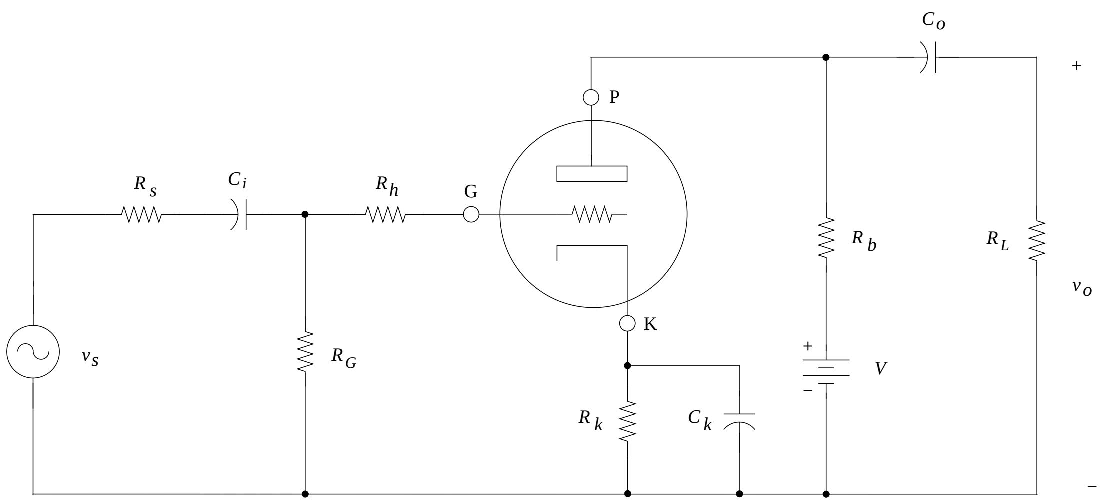

Figure 1: Common-cathode amplifier circuit.

In Fig. 1, v_s and R_s are the small-signal source voltage and resistance. The triode terminals are the grid, G, the cathode, K, and the plate, P. R_G is used to keep a potential from building up on the grid of the triode. R_h is used to limit the high-frequency response. Theoretically, no current flows into the grid of the triode. This fact means that R_h will have no effect on low or mid-frequencies. C_i is the input capacitor. C_i will keep a DC current from developing through the source. R_k is the cathode bias resistor. An advantage of using a cathode resistor for self-biasing is that the grid can be kept at ground, thus enabling the source to be DC coupled with the grid. However, the pole introduced by C_i will be shown to be virtually at DC, so it will cause no harm. C_k is used to bypass the AC signal away from R_k, thus enabling the bias point to remain constant. R_b is the plate load resistor, which is connected to V, the high voltage supply. C_o is the coupling capacitor that is used to remove the DC offset from the output of the triode. R_L is the output load resistance.

2.2 Low-Frequency Response

2.2.1 Low-Frequency Common-Cathode Amplifier Circuit

For AC analysis the V voltage supply is shorted. This creates the low-frequency circuit for AC analysis, shown in Fig. 2.

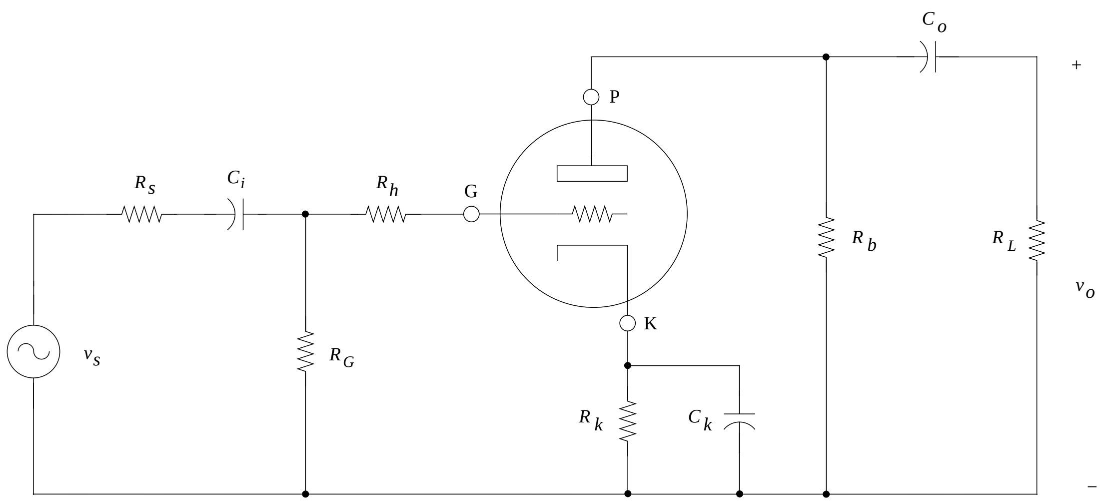

Figure 2: Low-frequency common-cathode amplifier circuit.

Next the triode is replaced with its small-signal model for AC analysis. First we will derive the T model from the small-signal current source model of the triode. The T model will prove useful in the analysis of the effects of C_k. Figure 3 shows the steps involved. Notice that r_p, the dynamic plate resistance, has been ommited for simplification. It can be added between the plate and the cathode.

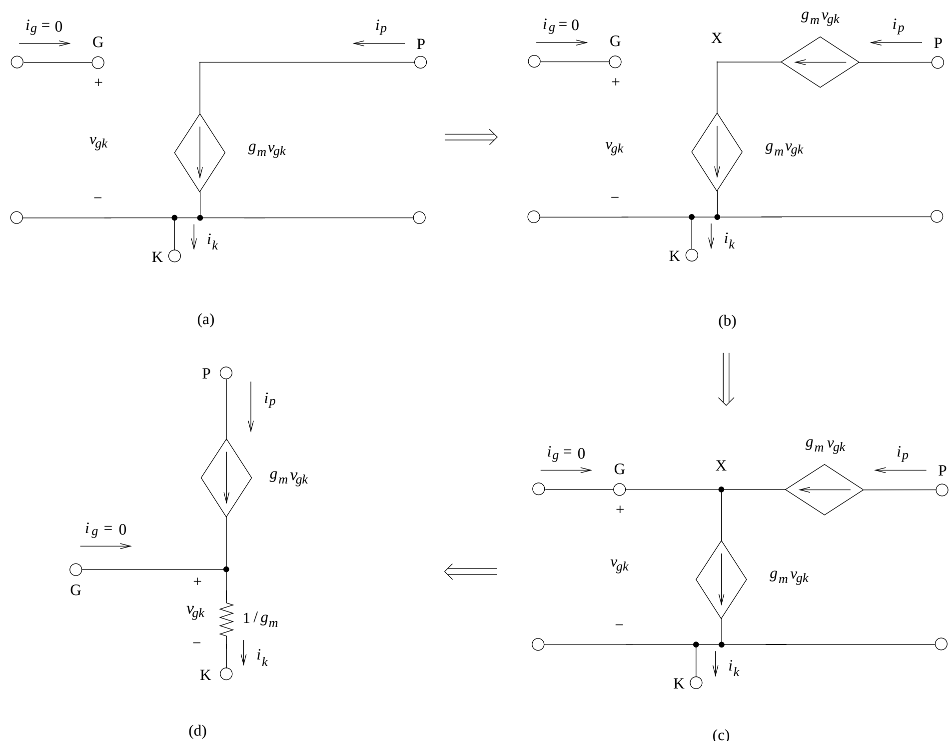

Figure 3: Derivation of the T model.

2.2.2 Effect of C_i on Low-Frequency Response

When the triode is replaced with its T model we get the circuit in Fig. 4. Now we find the transfer function for the voltage of the grid. We disregard R_h because no current flows through it. Therefore, the voltage at the grid is found by observing from Fig. 4 that the grid voltage is taken from a voltage divider connected to the input source. The voltage at the grid is given by the equation

$$v_g(s) = \frac{R_G}{R_s + \frac{1}{sC_i} + R_G} v_s(s) \tag{1}$$

We will multiply the right hand side of Eq. (1) by $\frac{s}{s}$, which results in

$$v_g(s) = \frac{R_G s}{s(R_s + R_G) + 1/C_i} v_s(s) \tag{2}$$

Now we manipulate Eq. (2) by dividing both sides of the equation by $v_s(s)$, and multiply the right hand side with $\frac{1/(R_s+R_G)}{1/(R_s+R_G)}$. This gives us the transfer function for

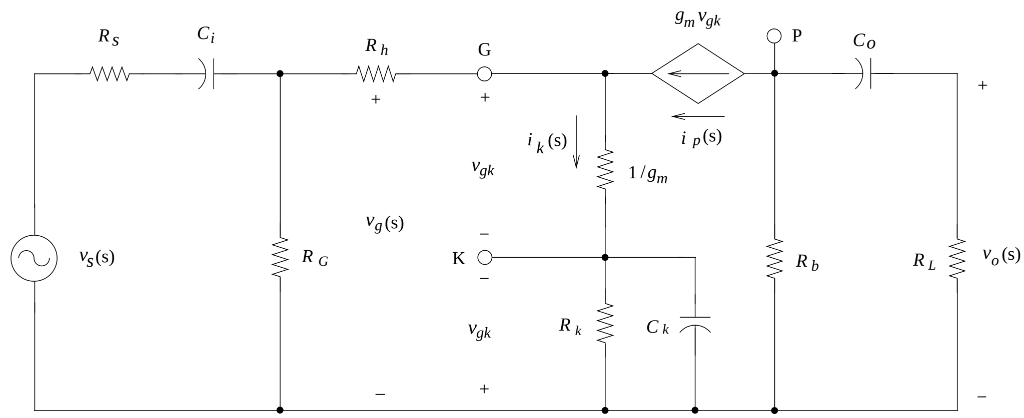

Figure 4: Low-frequency small-signal T model of common-cathode amplifier.

the grid voltage, which is

$$\frac{v_g(s)}{v_s(s)} = \left(\frac{R_G}{R_s + R_G}\right)\left(\frac{s}{s + \frac{1}{C_i(R_s+R_G)}}\right) \tag{3}$$

From Eq. (3), we can see that C_i introduces a zero at DC, and a pole ω_p at

$$\omega_p = \frac{1}{C_i(R_s + R_G)} \tag{4}$$

2.2.3 Effect of C_k on Low-Frequency Response

Next we find the plate current $i_p(s)$. From Fig. 4, we can see that the plate current is given by

$$i_p(s) = i_k(s) = \frac{v_g(s)}{1/g_m + Z_k} \tag{5}$$

If we manipulate the right hand side of Eq. (5) by multiplying both the numerator and denominator with $g_m Y_k$ after replacing Z_k with $1/Y_k$, we get

$$i_p(s) = g_m v_g(s) \frac{Y_k}{g_m + Y_k} \tag{6}$$

We now replace the admittance Y_k with its value

$$Y_k = 1/R_k + sC_k \tag{7}$$

This gives us

$$i_p(s) = g_m v_g(s) \frac{1/R_k + sC_k}{g_m + 1/R_k + sC_k} \tag{8}$$

After multiplying the numerator and denominator of the right hand side of Eq. (8) with $1/C_k$, we obtain

$$i_p(s) = g_m v_g(s) \frac{s + 1/C_k R_k}{s + (g_m + 1/R_k)/C_k} \tag{9}$$

This shows us that the cathode bypass capacitor C_k introduces a real zero and a real pole. The frequency of the real zero ω_z is

$$\omega_z = \frac{1}{C_k R_k} \tag{10}$$

The frequency of the real pole ω_{p1} is

$$\omega_{p1} = \frac{g_m + 1/R_k}{C_k} = \frac{1}{C_k(R_k \parallel 1/g_m)} \tag{11}$$

2.2.4 Effect of C_o on Low-Frequency Response

Now we will use the low-frequency small-signal model shown in Fig. 5 to find the effect of the coupling capacitor C_o. We will make an assumption for our analysis.

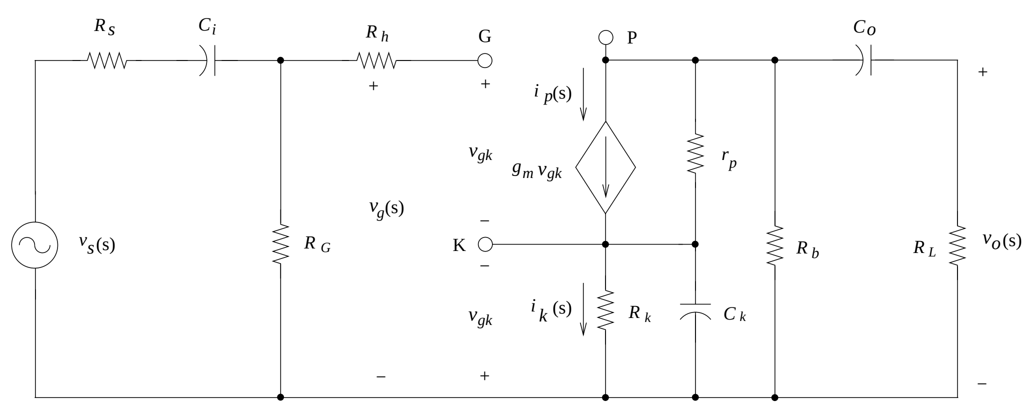

Figure 5: Low-frequency small-signal model of common-cathode amplifier.

This assumption is that the cathode will be at ground potential. For low-frequencies it is not truly at ground. This is a disadvantage of self-biasing with a cathode resistor and bypassing the AC signal to ground through a capacitor. However, the reactance of C_k will be small enough to assume the cathode is connected to ground. With this assumption we can manipulate Fig. 5 to get Fig. 6.

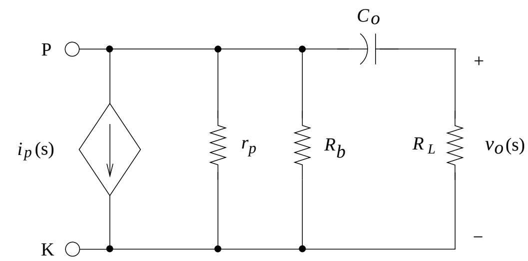

Figure 6: Low-frequency equivalent output circuit of common-cathode amplifier.

Next we combine R_b and r_p in Fig. 6 , and call the result R'. So R' is

$$R' = R_b \parallel r_p \tag{12}$$

We now manipulate Fig. 6 to get Fig. 7. From Fig. 7 we can see that the output $v_o(s)$

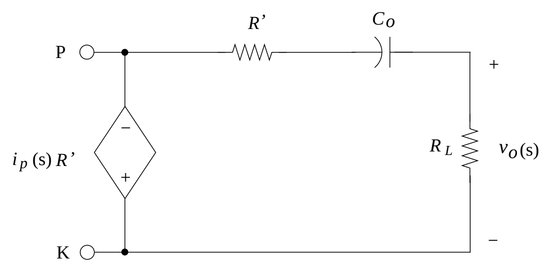

Figure 7: Simplified low-frequency equivalent output circuit of common-cathode amplifier.

is taken from a voltage divider connected to the plate. $v_o(s)$ is

$$v_o(s) = i_p(s)R'\frac{R_L}{R_L + R' + 1/sC_o} \tag{13}$$

With manipulation we get

$$v_o(s) = i_p(s)\frac{R'R_L(\frac{s}{R_L+R'})}{\frac{s(R_L+R')}{(R_L+R')} + \frac{s}{s(R_L+R')C_o}} \tag{14}$$

which reduces to

$$v_o(s) = i_p(s)(R' \parallel R_L)\frac{s}{s + \frac{1}{C_o(R_L+R')}} \tag{15}$$

After substituting for R', we get

$$v_o(s) = i_p(s)(r_p \parallel R_b \parallel R_L)\frac{s}{s + \frac{1}{C_o(R_L+r_p\|R_b)}} \tag{16}$$

From Eq. (16), we can see that C_o introduces a zero at DC, and a real pole at ω_{p2}

$$\omega_{p2} = \frac{1}{C_o(R_L + r_p \parallel R_b)} \tag{17}$$

We can now find the lower 3-dB frequency ω_L by using Eq. (18)

$$\omega_L \cong \sqrt{\omega_p^2 + \omega_{p1}^2 + \omega_{p2}^2 - 2(\omega_z^2)} \tag{18}$$

The frequencies ω_p, ω_{p1}, ω_{p2}, and ω_z are found from the Eqs. (4), (11), (17), and (10), respectively.

2.3 Mid-Frequency Response

We now will find A_m,the mid-frequency gain of the circuit. This is done with the help of the mid-frequency circuit, shown in Fig. 8. From Fig. 8, we can see that

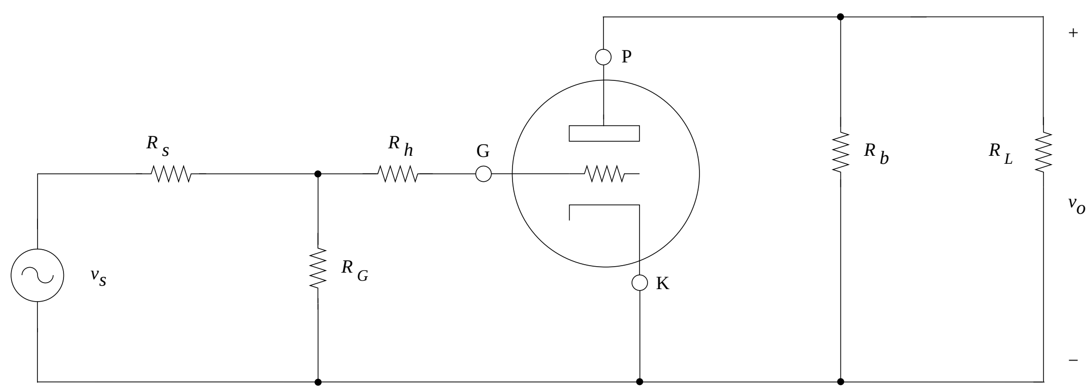

Figure 8: Mid-frequency circuit of common-cathode amplifier.

there are no capacitors in the circuit. The capacitors have no effect on the gain for mid-frequencies. To find A_m we replace the triode with its small-signal model to get Fig. 9. From Fig. 9 we can see that v_o is given as

$$v_o = -g_m v_{gk}(r_p \parallel R_b \parallel R_L) \tag{19}$$

A_m is the transfer function

$$A_m = \frac{v_o}{v_{gk}}\frac{v_g}{v_s} = -g_m(r_p \parallel R_b \parallel R_L)\frac{v_g}{v_s} \tag{20}$$

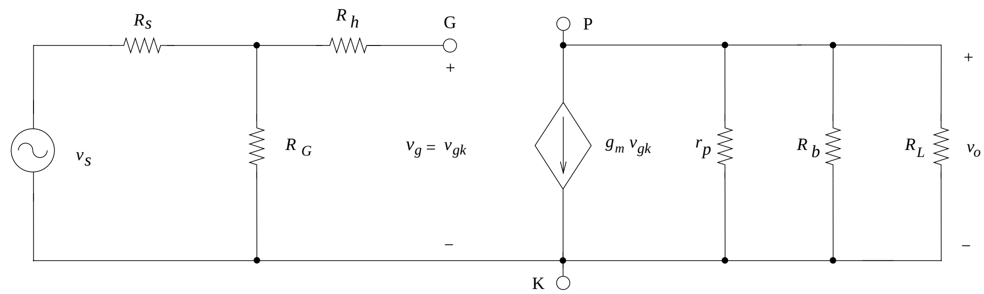

Figure 9: Mid-frequency small-signal model of common-cathode amplifier.

From Fig. 9 we can see that v_g is the output of a voltage divider connected to v_s. Equation (21) gives the value of v_g.

$$v_g = \frac{R_G}{R_s + R_G} v_s \tag{21}$$

This reduces to

$$\frac{v_g}{v_s} = \frac{R_G}{R_s + R_G} \tag{22}$$

We can see from Fig. 9 that $v_g = v_{gk}$. This lets us replace v_g/v_s in Eq. (20) with the value for v_g/v_s from Eq. (22). Therefore, A_m is

$$A_m = \frac{v_o}{v_s} = -g_m(r_p \parallel R_b \parallel R_L)\frac{R_G}{R_s + R_G} \tag{23}$$

2.4 High-Frequency Response

2.4.1 High-Frequency Common-Cathode Amplifier Circuit

The high-frequency response circuit is identical to the mid-frequency response circuit in Fig. 8. However, the interelectrode capacitances of the triode will effect the high-frequency response. This can be seen by observing the high-frequency small-signal model in Fig. 10.

To analyze the circuit in Fig. 10, we will use the help of Miller's theorem. With Miller's theorem Fig. 10 can be manipulated to get the circuit shown in Fig. 11. In Fig. 11, C_1 and C_2 are given by the equations

$$C_1 = C_{\mathrm{GP}}(1 - A_m) \tag{24}$$

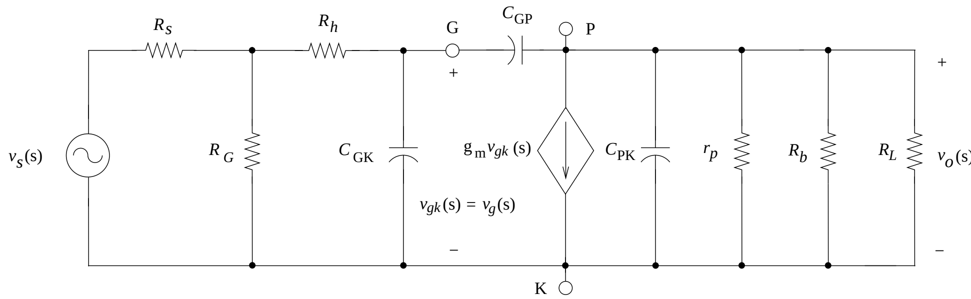

Figure 10: High-frequency small-signal model of common-cathode amplifier.

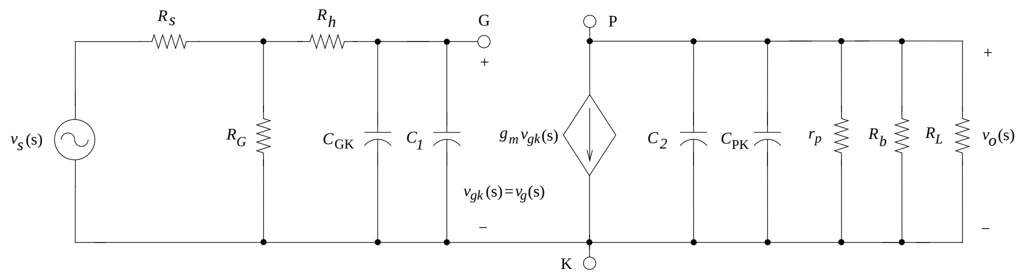

Figure 11: Modified high-frequency small-signal model of common-cathode amplifier.

and

$$C_2 = C_{\mathrm{GP}}(1 - 1/A_m) \tag{25}$$

A_m is found from Eq. (23). This shows us that the pole due to C_1 and C_{GK} will be the dominant pole.

2.4.2 Effect of the Triode Inter-Electrode Capacitances on the Input

Now we will find the dominant high-frequency pole due to the capacitances C_{GK} and C_1 by manipulating the input of Fig. 11. Figure 12 shows the manipulations.

From Fig. 12, we can see the output, $v_{gk}(s)$, is taken from our old friend the voltage divider. The equation for $v_{gk}(s)$ is

$$v_{gk}(s) = v_s(s)\frac{\frac{R''}{s(C_{\mathrm{GK}}+C_1)}}{R_s(R''+R_h)+\frac{R_s}{s(C_{\mathrm{GK}}+C_1)}} \tag{26}$$

where

$$R'' = R_s \parallel R_G \tag{27}$$

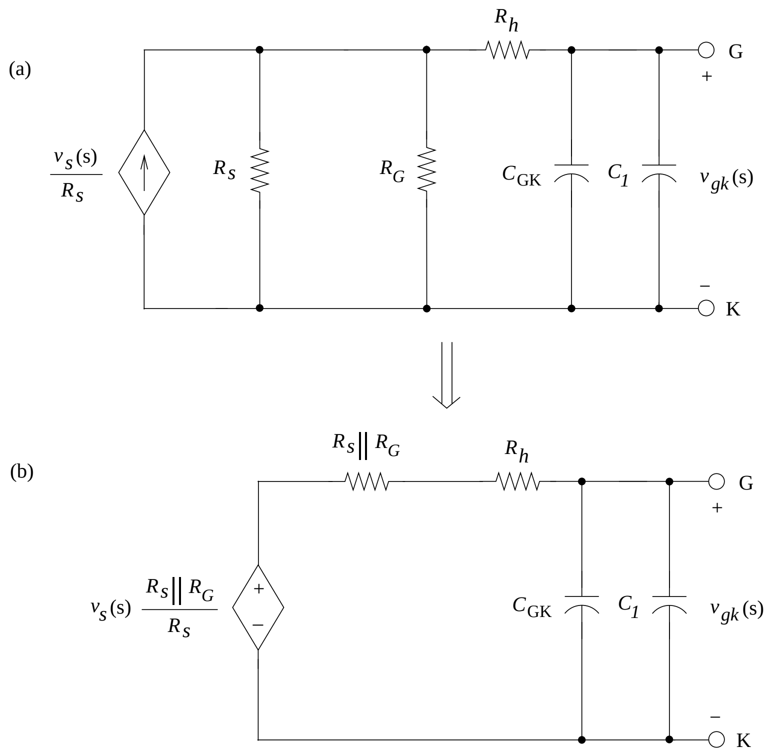

Figure 12: Circuit to determine the high-frequency input pole of common-cathode amplifier.

After dividing both sides of Eq. (26) by $v_s(s)$, and multiplying the numerator and denominator of the right hand side of Eq. (26) with $\frac{s}{R_s(R''+R_h)}$, we get

$$\frac{v_{gk}(s)}{v_s(s)} = \frac{\frac{R''}{R_s(C_{\mathrm{GK}}+C_1)(R''+R_h)}}{s + \frac{1}{(C_{\mathrm{GK}}+C_1)(R''+R_h)}} \tag{28}$$

By substituting in the values for R'' and C_1 in Eq. (28) we see that there is a pole, ω_{p3}, at

$$\omega_{p3} = \frac{1}{[C_{\mathrm{GK}} + C_{\mathrm{GP}}(1 - A_m)][(R_s \parallel R_G) + R_h]} \tag{29}$$

From Eq. (29) we see that R_h can be used to limit the high-frequency response.

2.4.3 Effect of the Triode Inter-Electrode Capacitances on the Output

Though it will be of little effect on high-frequency response, ω_{p4}, the pole due to C_2 and C_{PK}, will be derived for completeness. We can find ω_{p4} by inspection of the output circuit shown in Fig. 13.

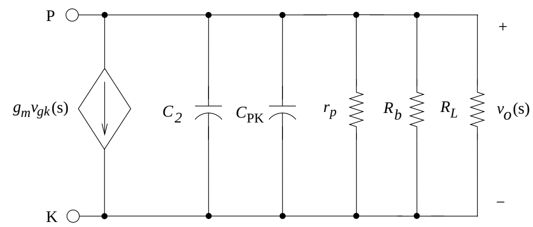

Figure 13: Circuit to determine the high-frequency output pole of common-cathode amplifier.

The equation for $v_o(s)$ is

$$v_o(s) = g_m v_{gk}(s)\left[\left(\frac{1}{s(C_2 + C_{\mathrm{PK}})}\right) \parallel (r_p \parallel R_b \parallel R_L)\right] \tag{30}$$

This reduces to

$$\frac{v_o(s)}{v_{gk}(s)} = g_m \frac{1}{s(C_2 + C_{\mathrm{PK}}) + \frac{1}{(r_p \| R_b \| R_L)}} \tag{31}$$

After further reduction we get

$$\frac{v_o(s)}{v_{gk}(s)} = \frac{\frac{g_m}{(C_2 + C_{\mathrm{PK}})}}{s + \frac{1}{(C_2 + C_{\mathrm{PK}})(r_p \| R_b \| R_L)}} \tag{32}$$

From Eq. (32) we see that the pole ω_{p4} is given by the equation

$$\omega_{p4} = \frac{1}{[C_{\mathrm{GP}}(1 + 1/A_m) + C_{\mathrm{PK}}](r_p \parallel R_b \parallel R_L)} \tag{33}$$

We can now find ω_H, the upper 3-dB frequency, by using Eq. (34)

$$\omega_H \cong \frac{1}{\sqrt{\frac{1}{\omega_{p3}^2} + \frac{1}{\omega_{p4}^2}}} \tag{34}$$

The frequencies ω_{p3} and ω_{p4} are found from Eqs. (29) and (33), respectively.

2.5 Graphical Design Procedure

2.5.1 Selection of Resistor Values

Now that we have the equations for the frequency response of the common-cathode amplifier, we need the numbers to evaluate them. Our first step is to choose

$V = 400$ V. Next we choose $R_G = 1$ MΩ. This will give us a large input resistance, which will limit the loading of v_s. R_s will be chosen as 8 kΩ. This is a typical resistance of the transducers used on electric musical instruments. We will choose $R_L = 1$ MΩ. This will limit the loading of the output of our amplifier. Also worth noting here, R_L is typically a potentiometer. This enables us to obtain an attenuated output if needed. Simply stated the potentiometer is a volume control. The best way to chose values for R_b and R_k is by using a graphical method. First we will use Eq. (23) to help us determine initially what direction to take in selecting the value of R_b. In Eq. (23) we will set $R_b \parallel R_L = R$. We can see the logic for this move by observing Fig. 9. We will also ignore the term $\frac{R_G}{R_s+R_G}$ in Eq. (23). We can do this because $\frac{R_G}{R_s+R_G} \approx 1$. After we do these steps Eq. (23) becomes

$$A_m = \frac{v_o}{v_s} = -g_m \left(r_p \parallel R\right) = -g_m \left(\frac{r_p \; R}{r_p + R}\right) \tag{35}$$

We will know take advantage of the fact that $g_m r_p = \mu$, the amplification factor of a vacuum tube. By using this fact Eq. (35) becomes

$$A_m = \frac{v_o}{v_s} = -\mu \left(\frac{R}{r_p + R}\right) = -\mu \left(\frac{1}{r_p/R + 1}\right) \tag{36}$$

From Eq. (36) we can see that to maximize the gain we want the ratio r_p/R to be as small as possible. It is more useful to think that we want to maximize the ratio R/r_p. Since $R = R_b \parallel R_L$, and $R_L = 1$ MΩ, we want to make R_b as large as possible. Fig. 14 shows the dependence of A_m on R_b/r_p. This shows us that R_b only needs to be a few times the magnitude of r_p to get appreciable gain. We will choose $R_b = 330$ kΩ. With this value of R_b we will have the DC load line shown in Fig. 15. Fig. 15 is a plot of the characteristic curves for $I_p$vs.V_p for an actual 12AX7 triode. The DC load line was created by using the fact that the plate voltage is given by

$$V_p = (V) - (I_p R_b) \tag{37}$$

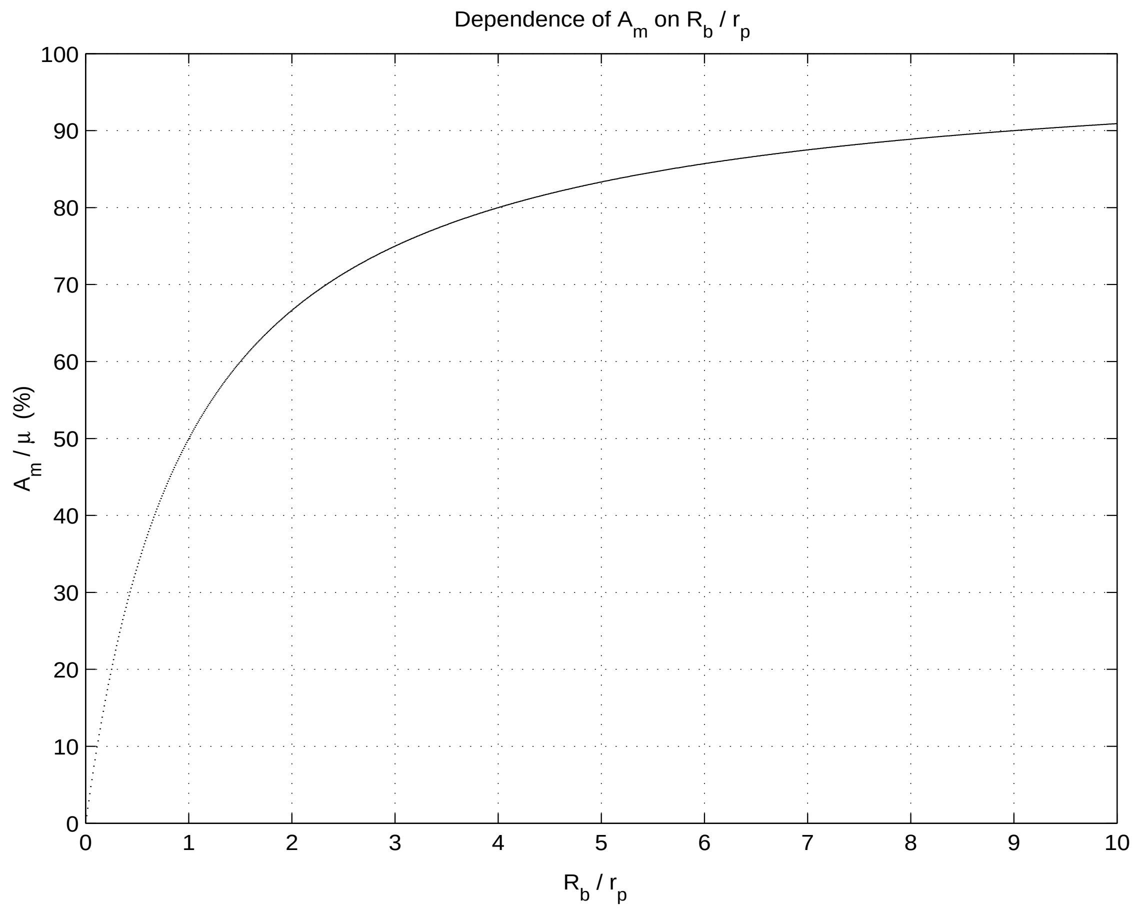

Figure 14: Dependence of A_m on R_b/r_p.

When $I_p = 0$ mA, $V_P = 400$ V. When $V_p = 0$ V, I_p will be

$$I_P = \frac{V}{R_b} = \frac{400 \text{ V}}{330 \text{ k}\Omega} = 1.21 \text{ mA} \tag{38}$$

A straight line is then drawn connecting these two points to create the DC load line.

We now pick a suitable quiescent operating point by observing Fig. 15. We could set our quiescent point of operation at $V_G = -2.5$ V. This will enable us to amplify an input signal of approximately 2.5 V without clipping. First, we need to find the value needed for R_k to achieve $V_G = -2.5$ V. We will use the fact that

$$V_G = -I_p R_k \tag{39}$$

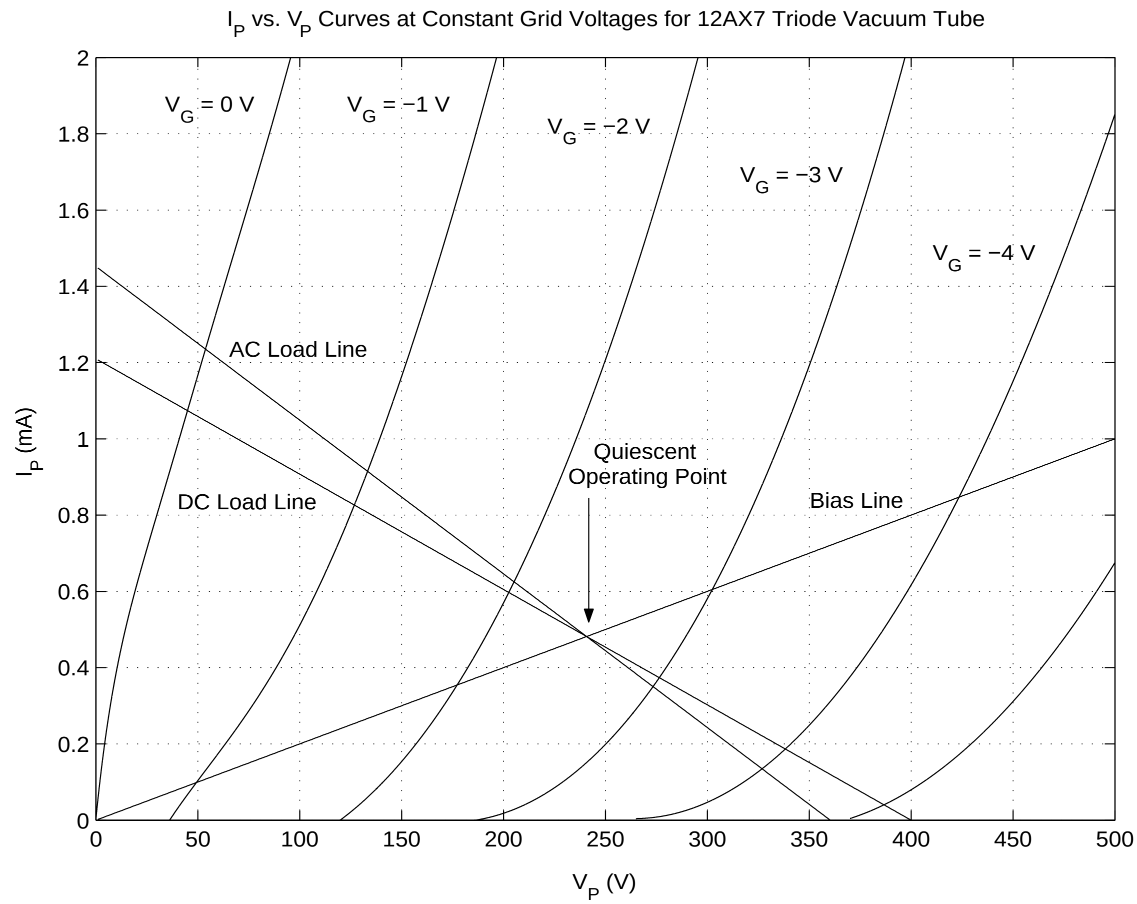

Figure 15: Design graph, I_P vs. V_P at constant grid voltages for 12AX7 triode vacuum tube.

From Fig. 15 we can see that at $V_G = -2.5$ V, $I_p = 0.48$ mA. We manipulate Eq. (39) to find

$$R_k = \frac{V_G}{-I_p} = \frac{-2.5}{-0.48 \text{ mA}} = 5.2 \text{ k}\Omega \tag{40}$$

We will choose $R_k = 4.7$ kΩ. With the value of R_k chosen we can draw the bias line. To do this we will use Eq. (39). When $I_p = 0$ mA, $V_G = 0$ V. This is one point for our bias line. We will find the next point by determining the value of I_p that corresponds to $V_G = -4$ V. We manipulate Eq. (39) to obtain

$$I_p = \frac{V_G}{-R_K} = \frac{-4 \text{ V}}{-4.7 \text{ k}\Omega} = 0.85 \text{ mA} \tag{41}$$

We draw a line connecting these two points to form our bias line. The point were the bias line intersects the DC load line is our quiescent point of operation. At our quiescent point of operation V_G will be

$$V_G = -I_P R_k = -0.48 \text{ mA} \times 4.7 \text{ k}\Omega = -2.26 \text{ V} \sim -2.5 \text{ V} \tag{42}$$

Obviously this is not a perfect match. However it definitely gets us in the ball park. The advantage of the graphical method is that it enables us to visualize the circuit operation.

Now we will draw the AC load line. To draw the AC load line we will take advantage of the fact that it must pass through our quiescent point of operation. Therefore, we start by using the current at our quiescent operating point, $I_Q = 0.48$ mA, and multiply it by the AC load. The AC load is

$$R_{AC} = R_b \parallel R_L = \frac{330 \times 1000}{330 + 1000} \text{ k}\Omega = 248.12 \text{ k}\Omega \tag{43}$$

Therefore, if our input signal drives the tube into cut-off, $I_p = 0$ mA, the change in V_P will be

$$\Delta V_P = I_Q R_{AC} = 0.48 \text{ mA} \times 248.12 \text{ k}\Omega = 119 \text{ V} \tag{44}$$

We add ΔV_P and V_{QP}, the plate voltage at our quiescent operating point, to get the plate voltage at cut-off, $V_{P_{cut-off}}$.

$$V_{P_{cut-off}} = \Delta V_P + V_{QP} = 119 \text{ V} + 240 \text{ V} = 359 \text{ V} \tag{45}$$

Now we have two points I_Q at V_{QP} and $I_P = 0$ mA at $V_{P_{cut-off}}$. We draw a line connecting these two points to get our AC load line. With the AC load line we can obtain the gain graphically. To do this we can start at $V_G = -2$ V and follow the AC load line to reach $V_G = -3$ V. The gain will be the change in plate voltage divided by the change in grid voltage. Since the change in grid voltage will be 1 V, the gain will simply be the change in plate voltage. From Fig. 15, we can see that the gain will be approximately -70 V/V, the negative sign is added because the common-cathode amplifier is an inverting amplifier.

2.5.2 Determining the Triode Parameters g_m, r_p, and μ

We will now determine the values of g_m, r_p, and μ graphically. We can do this by using the relationships

$$g_m = \left.\frac{\partial I_p}{\partial V_G}\right|_{V_P=\text{const.}} = \left.\frac{\Delta I_p}{\Delta V_G}\right|_{V_P=\text{const.}} \tag{46}$$

$$r_p = \left.\frac{\partial V_p}{\partial I_p}\right|_{V_G=\text{const.}} = \left.\frac{\Delta V_p}{\Delta I_p}\right|_{V_G=\text{const.}} \tag{47}$$

and

$$\mu = \left.\frac{\partial V_p}{\partial V_G}\right|_{I_P=\text{const.}} = \left.\frac{\Delta V_p}{\Delta V_G}\right|_{I_P=\text{const.}} \tag{48}$$

After we have determined these values for our quiescent point of operation, we can determine the values for the capacitors. Then we can find the frequency response of our amplifier. To find these values we will zoom in on Fig. 15. The magnified graph is shown in Fig. 16. We will find g_m first. In Fig. 16 there is a vertical line labeled ΔI_{P3} through the quiescent point of operation. This line traverses a change in grid voltage of 1 V. It is obvious from Fig. 16 that ΔI_P is greater from $V_G = -2.5$ V to -2 V than from $V_G = -3$ V to -2.5 V. Therefore, we will use $\Delta V_G = 1$ V to average this difference out. This gives g_m as

$$g_m = \left.\frac{\Delta I_{p3}}{\Delta V_G}\right|_{V_P=\text{const.}} = \frac{0.91 \text{ mA}}{1 \text{ V}} = 0.91 \; \frac{\text{mA}}{\text{V}} \tag{49}$$

Not so obvious is the difference in r_p. To find r_p we hold the grid voltage constant and determine ΔV_P for a given ΔI_P. We will do this for $V_G = -2$ V and $V_G = -3$ V. We will then take the average of these two values to obtain r_p at the quiescent point of operation. In Fig. 16 you can see a line labeled ΔI_{P1}, and a line labeled ΔV_{P1} along the curve for $V_G = -2$ V. These will give us our first value for r_p, call it r_{p1}.

$$r_{p1} = \left.\frac{\Delta V_{p1}}{\Delta I_{p1}}\right|_{V_G=\text{const.}} = \frac{63 \text{ V}}{0.6 \text{ mA}} = 105 \text{ k}\Omega \tag{50}$$

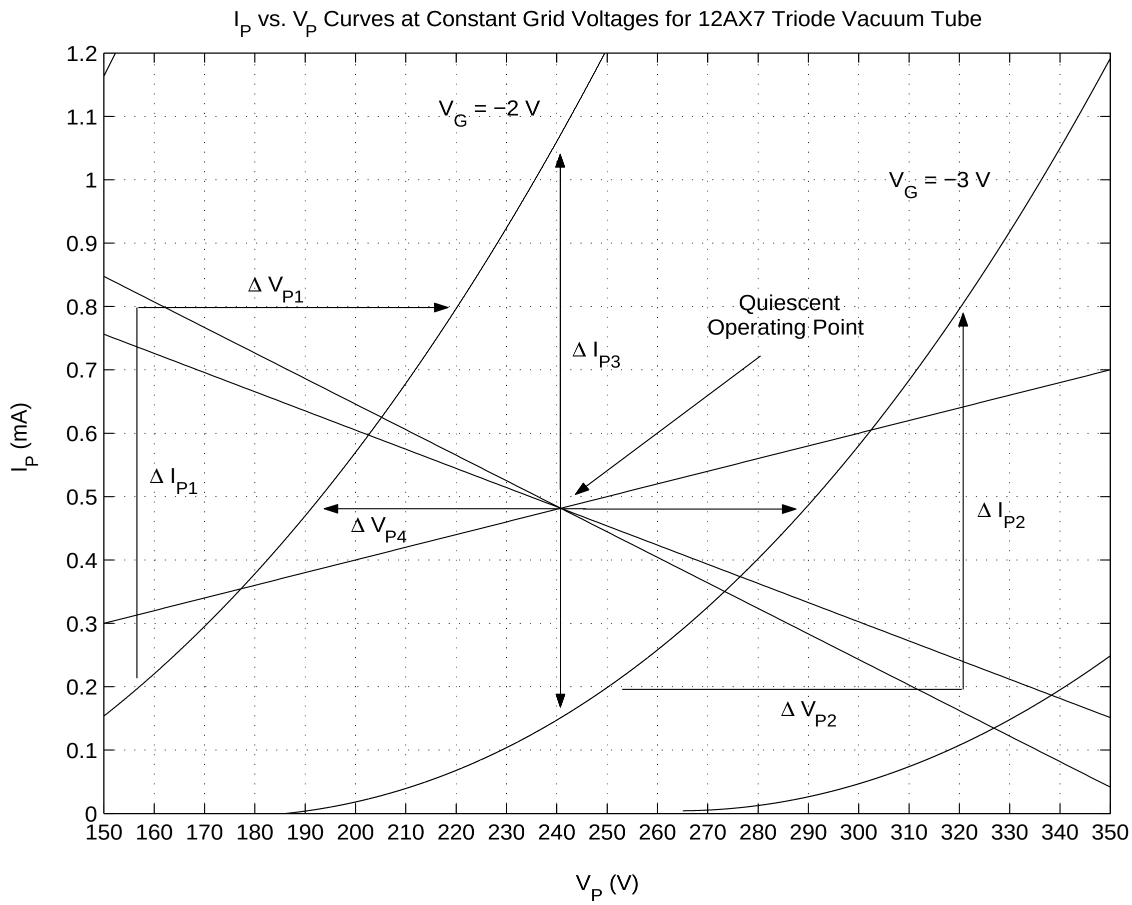

Figure 16: Plot for finding r_p, g_m, and μ at the chosen quiescent operating point.

In Fig. 16 you can see a line labeled ΔI_{P2}, and a line labeled ΔV_{P2} along the curve for $V_G = -3$ V. These will give us our second value for r_p, call it r_{p2}.

$$r_{p2} = \left.\frac{\Delta V_{p2}}{\Delta I_{p2}}\right|_{V_G=\text{const.}} = \frac{70 \text{ V}}{0.6 \text{ mA}} = 116.6 \text{ k}\Omega \tag{51}$$

This will gives us our value of r_p as

$$r_p = \frac{r_{p1} + r_{p2}}{2} = 110.8 \text{ k}\Omega \tag{52}$$

To find μ we observe the line labeled ΔV_{p4} in Fig. 16. This line traverses a change in V_G of 1 V. Therefore, μ is

$$\mu = \left.\frac{\Delta V_p}{\Delta V_G}\right|_{I_P=\text{const.}} = \frac{99 \text{ V}}{1 \text{ V}} = 99 \; \frac{\text{V}}{\text{V}} \tag{53}$$

We can check our values by using the fact that $\mu = r_p g_m$. The results are

$$\mu = r_p g_m = 110.8 \text{ k}\Omega \times 0.91 \, \frac{\text{mA}}{\text{V}} = 100.8 \, \frac{\text{V}}{\text{V}} \approx 99 \, \frac{\text{V}}{\text{V}} \tag{54}$$

2.5.3 Selection of Capacitor Values

Now that we have values for all of the resistors in our amplifier, as well as determined the dynamic triode parameters, we can select the capacitor values to obtain the frequency response desired. For audio frequencies, we would like $f_L \approx 20$ Hz, and $f_H \approx 20$ kHz. We first find the mid-frequency gain using Eq. (23).

$$\begin{aligned}
A_m &= -g_m(r_p \parallel R_b \parallel R_L)\frac{R_G}{R_s + R_G} \\
A_m &= -0.91 \, \frac{\text{mA}}{\text{V}} \frac{1}{(1/110.8 + 1/330 + 1/1000)} \text{ k}\Omega \frac{1000}{1000 + 8} \\
A_m &= -69.2 \, \frac{\text{V}}{\text{V}} \approx -70 \, \frac{\text{V}}{\text{V}}
\end{aligned}$$

This result shows the accuracy of the graphical method in predicting the gain of the amplifier.

Now we will find the value for C_i to place f_p at 0.16 Hz. To find the value of C_i we will manipulate Eq. (4).

$$\begin{aligned}
\omega_p &= \frac{1}{C_i(R_s + R_G)} \\
C_i &= \frac{1}{2\pi f_p(R_s + R_G)} \\
C_i &= \frac{1}{2\pi(0.16 \text{ Hz})(8 \text{ k}\Omega + 1 \text{ M}\Omega)} = 1 \ \mu\text{F}
\end{aligned}$$

The pole ω_p is chosen this low to keep the source at a virtual DC couple with the grid. We do not want this to be the dominant pole either.

We next select a value for C_o. We will place this pole around 7 Hz. We select C_o with the help of Eq. (17)

$$\omega_{p2} = \frac{1}{C_o(R_L + r_p \parallel R_b)}$$

$$\begin{aligned} C_o &= \frac{1}{2\pi f_p(R_L + r_p \parallel R_b)} \\ C_o &= \frac{1}{2\pi(7\text{ Hz})(1\text{ M}\Omega + \frac{110.8\times 330}{110.8+330}\text{ k}\Omega)} = 21\text{ nF} \end{aligned}$$

We will select $C_o = 20$ nF. This makes f_{p2} large enough to have an effect on f_L. However, its effect will be minimal.

We will now select C_k to place ω_{p1}, as well as ω_z. We will make $f_z = 3.5$ Hz. We will accomplish this with the help of Eq. (10).

$$\begin{aligned} \omega_z &= \frac{1}{C_k R_k} \\ C_k &= \frac{1}{2\pi f_p R_k} \\ C_i &= \frac{1}{2\pi(3.5\text{ Hz})(4.7\text{ k}\Omega)} = 9.675\ \mu\text{F} \end{aligned}$$

We will select $C_k = 10\ \mu$F. We now find f_{p1} with the help of Eq. (11).

$$\begin{aligned} \omega_{p1} &= \frac{g_m + 1/R_k}{C_k} \\ f_{p1} &= \frac{0.91\ \frac{\text{mA}}{\text{V}} + 1/4.7\text{ k}\Omega}{2\pi(10\ \mu\text{F})} \\ f_{p1} &= 17.85\text{ Hz} \end{aligned}$$

Since f_{p1} is semi-dominant, $f_L \approx f_{p1}$.

We know that f_{p3} will be the dominant pole effecting f_H. Therefore, we will select The value of R_h which will set $f_H = 20$ kHz. We will do this with the help of Eq. (29).

$$\begin{aligned} \omega_{p3} &= \frac{1}{[C_{GK} + C_{GP}(1 - A_m)][(R_s \parallel R_G) + R_h]} \\ R_h &= \frac{1}{2\pi f_H[C_{GK} + C_{GP}(1 - A_m)]} - (R_s \parallel R_G) \\ R_h &= \frac{1}{2\pi 20\text{ kHz}[2\text{ pF} + 2\text{ pF}(1 + 70\frac{\text{V}}{\text{V}})]} - \left(\frac{8 \times 1000}{8 + 1000}\right)\text{ k}\Omega \\ R_h &= 55.25\text{ k}\Omega \end{aligned}$$

We will choose $R_h = 47$ kΩ.

The pole f_{p4} is found with the help of Eq. (33).

$$
\begin{aligned}
\omega_{p4} &= \frac{1}{[C_{GP}(1-1/A_m)+C_{PK}](r_p \parallel R_b \parallel R_L)} \\
f_{P4} &= \frac{1}{2\pi[C_{GP}(1-1/A_m)+C_{PK}](r_p \parallel R_b \parallel R_L)} \\
f_{p4} &= \frac{1}{2\pi[2\text{ pF}(1+1/70\frac{\text{V}}{\text{V}})+2\text{ pF}]\left(\frac{1}{1/110.8+1/330+1/1000}\text{ k}\Omega\right)} \\
f_{p4} &= 515.8\text{ kHz}
\end{aligned}
$$

Therefore, f_{p3} is definitely the dominant high frequency pole.

2.5.4 Frequency Response of the Designed Common-Cathode Amplifier

The bode diagram for the designed amplifier is shown in Fig. 17. The asterisks mark the f_L and f_H frequencies.

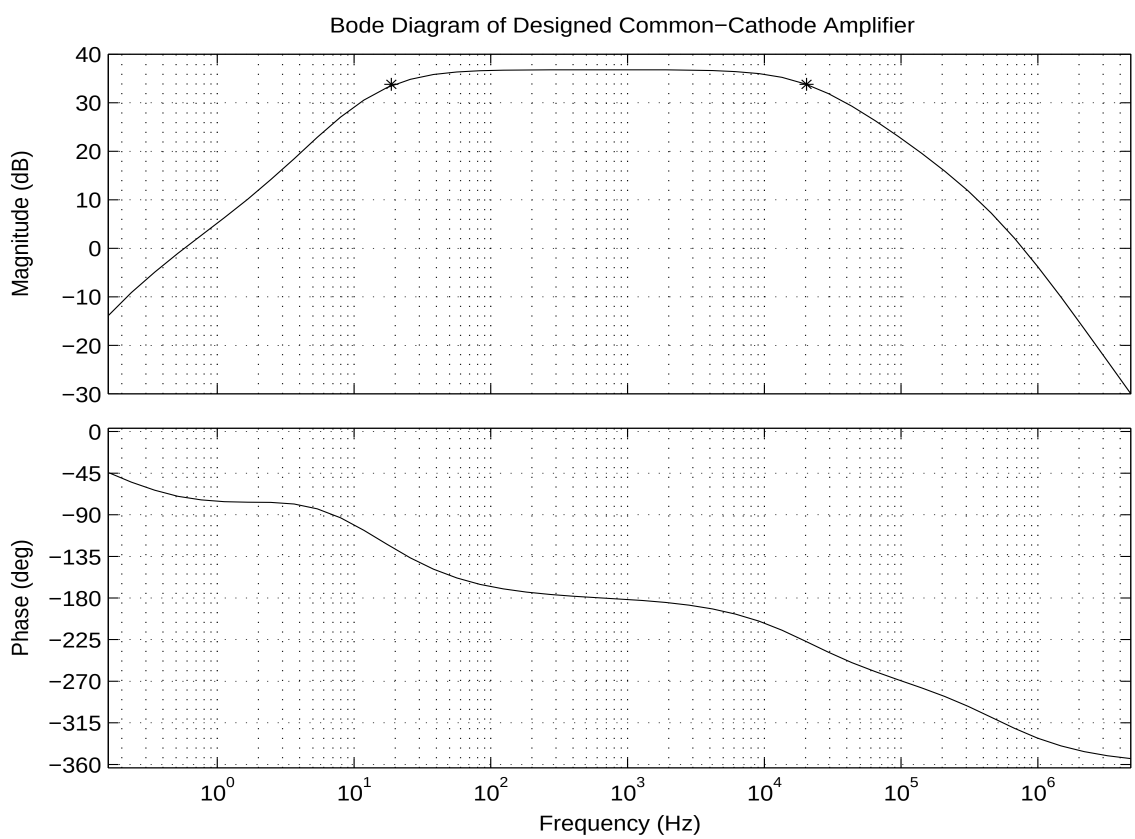

Figure 17: Bode diagram for the designed common-cathode amplifier.

2.6 Derivation of the Thevenin Equivalent Circuit for the Common-Cathode Amplifier

We will now find the Thevenin equivalent circuit for the common-cathode amplifier. The general topology for the Thevenin equivalent circuit of a voltage amplifier is shown in Fig. 18. Here A is the unloaded gain of the amplifier. This can be in-

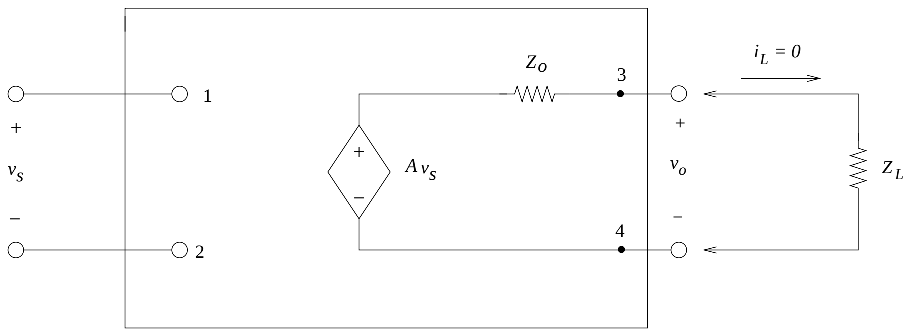

Figure 18: Basic thevenin equivalent circuit of a voltage amplifier.

terpreted as the gain when $Z_L = \infty$, or equivalently when $i_L = 0$. Z_o is the output impedance of the amplifier. From Fig. 18 we can see that v_o will be

$$v_o = Av_s \frac{Z_L}{Z_L + Z_o} \tag{55}$$

If we can get the equation for the output of our amplifier in this form, we will know our Thevenin equivalent circuit. Our amplifier gain from Eq. (36) is given as

$$\frac{v_o}{v_s} = -\mu \frac{R}{R + r_p}$$

Here R was our load resistance, which we are now calling Z_L. To find A, the unloaded gain we will set $R = \infty$. This results in

$$A = \frac{v_o}{v_s} = -\mu \frac{\infty}{\infty + r_p}$$
$$A = \frac{v_o}{v_s} = -\mu$$

Now that we know A we will manipulate Eq. (36) to put it in a form like Eq. (55).

This is done in the following manner

$$\frac{v_o}{v_s} = -\mu\frac{R}{R + r_p}$$
$$v_o = -\mu v_s\frac{R}{R + r_p}$$

Since $A = -\mu$ and $Z_L = R$, we have the correct equation form to obtain the Thevenin equivalent circuit. Recall that $R = R_b \parallel R_L$. Therefore, the Thevenin equivalent circuit for the common-cathode amplifier is as shown in Fig. 19. This shows that

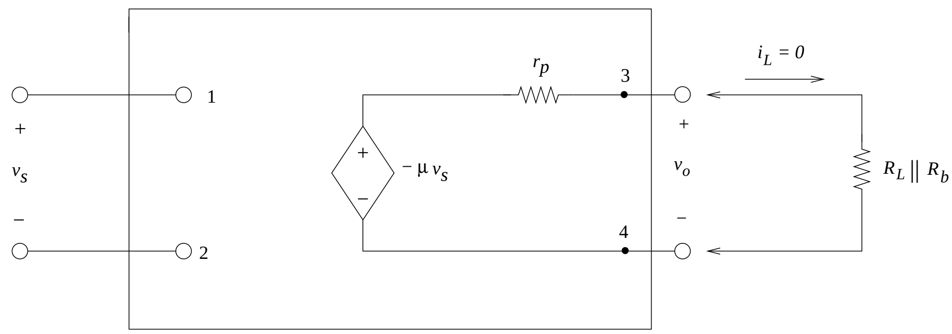

Figure 19: Thevenin equivalent circuit of the common-cathode amplifier.

$Z_o = r_p$. From Fig. 19 we can see that we want a very large load resistance, ideally infinite. As Z_L decreases, the amplifier is forced to deliver more current, i_L. This current must travel through r_p because it is in series with Z_L. Therefore, v_o will drop considerably with a small Z_L due to the large value of r_p for the 12AX7 triode. This kind of loss in v_o is known as loading the output of the amplifier.

3 The Cathode-Follower Amplifier

3.1 The Classical Cathode-Follower Amplifier Circuit

The cathode-follower amplifier is an important circuit, even though it will be shown to provide no voltage amplification. The circuit topology for the classical cathode-follower is shown in Fig. 20. We can see that the plate is connected directly

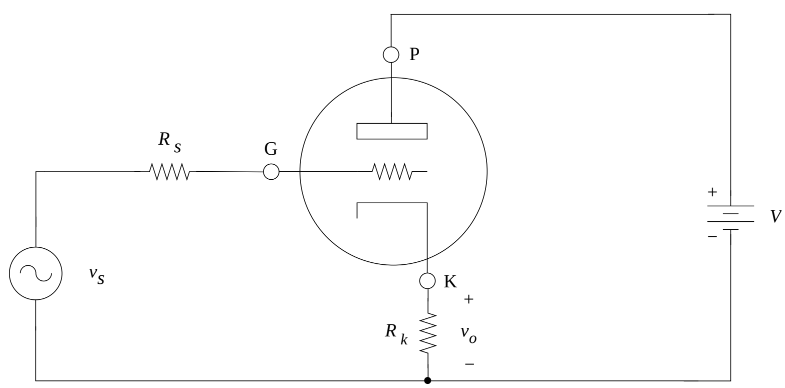

Figure 20: Classical cathode-follower amplifier circuit.

to V. This results in the alternate name, common-plate amplifier, to describe this circuit. There is no capacitor bypassing AC signals around R_k, the cathode bias resistor. This would render the amplifier useless. We are now taking the output from across R_k. Therefore, we do not want a constant DC voltage at the cathode.

The important aspect of the cathode-follower is its low output resistance, which enables it to drive low impedance loads. This makes it useful as a buffer between the high output resistance of the common-cathode amplifier, and a low impedance passive filter network. In sec. (2.2.1) we derived the T-model for a triode vacuum tube, shown in Fig. 3. In the T-model we found the impedance of the triode looking up into the cathode to be $1/g_m$. This is the theoretical cathode-follower output impedance. We will analize the classical cathode-follower circuit, and hopefully arrive at a result that is in agreement with the T-model.

3.2 Classical Cathode-Follower Analysis

The first step in analizing the cathode-follower is to replace the triode with its small-signal model. We will use the voltage source model. The result is shown in Fig. 21. We can see that v_o is given by

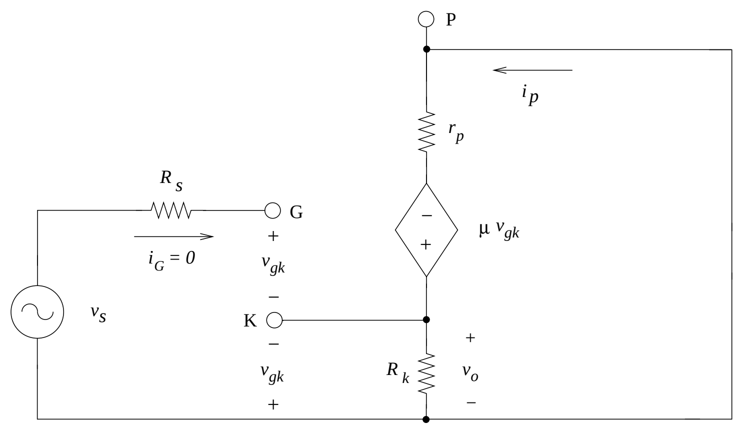

Figure 21: Small-signal model of cathode-follower amplifier circuit.

$$v_o = i_p R_k \tag{56}$$

Likewise, v_{gk} is given by

$$v_{gk} = v_s - v_o \tag{57}$$

We will now sum the voltages in the loop to obtain

$$i_p(r_p + R_k) - \mu v_{gk} = 0 \tag{58}$$

Now we perform the following steps on Eq. (58) to arrive at the transfer function v_o/v_s.

$$\begin{aligned} i_p r_p + i_p R_k &= \mu v_{gk} \\ i_p r_p + v_o &= \mu(v_s - v_o) \end{aligned}$$

$$\begin{aligned}\frac{v_o}{R_k} r_p + v_o &= \mu v_s - \mu v_o \\ v_o \left(\frac{r_p}{R_k} + 1 + \mu \right) &= \mu v_s\end{aligned}$$

From here we find G, the gain of the cathode-follower, as

$$G = \frac{v_o}{v_s} = \frac{\mu}{\mu + 1 + \frac{r_p}{R_k}} \tag{59}$$

From Eq. (59) we can see we want to minimize the ratio r_p/R_k. Thus, we want to maximize the ratio R_k/r_p. If we assume $R_k \gg r_p$, Eq. (59) reduces to

$$\frac{v_o}{v_s} = \frac{\mu}{\mu + 1 + \frac{r_p}{R_k}} \approx \frac{\mu}{\mu + 1}$$

Since for the 12AX7 triode $\mu = 100 \gg 1$, this reduces further to

$$\frac{v_o}{v_s} = \frac{\mu}{\mu + 1} \approx \frac{\mu}{\mu} = 1$$

Therefore,

$$\frac{v_o}{v_s} \approx 1$$

Thus the name cathode-follower. The output is in theory an exact replica of the input.

The one step we used in this simplification that makes the result questionable is our assumption that $R_k \gg r_p$. However, even if this is not true the result is still accurate. Fig. 22 shows the gain of the cathode-follower as a function of R_k/r_p. We can see that R_k only needs to be a few times larger than r_p to achieve a gain of approximately one.

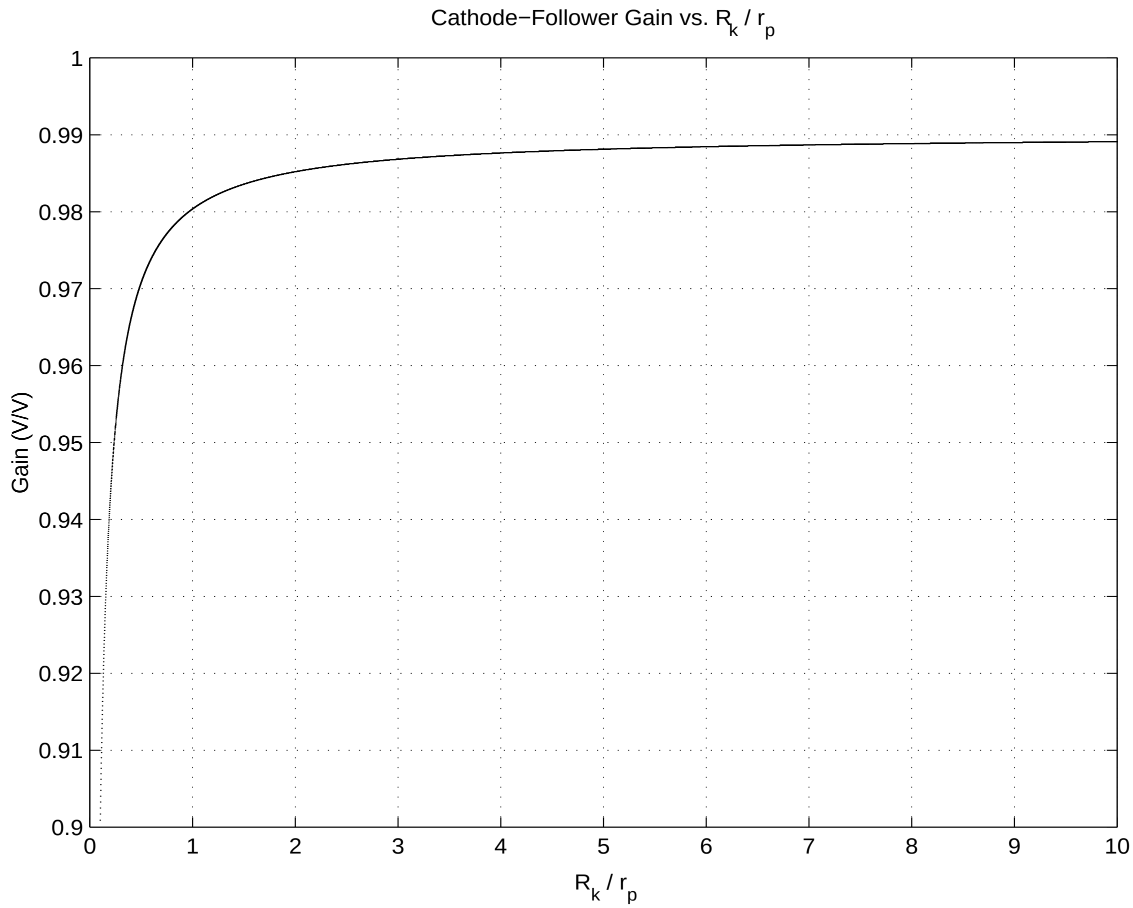

Figure 22: Cathode-follower gain vs. R_k/r_p .

3.3 Thevenin Equivalent Circuit for the Cathode-Follower Amplifier

We will use the same technique presented in sec. (2.6) to find the Thevenin equivalent circuit of the cathode-follower. We know that the load resistance for the cathode-follower is R_k, the cathode resistance. In sec. (3.2) we found the gain of the cathode-follower, given by Eq. (59). To find the unloaded gain we set $R_K = \infty$, resulting in

$$G = \frac{v_o}{v_s} = \frac{\mu}{\mu + 1 + \frac{r_p}{\infty}}$$
$$G = \frac{v_o}{v_s} = \frac{\mu}{\mu + 1}$$

Now that we know the unloaded gain we only need to rearrange Eq. (59). to put it in a form like Eq. (55). The steps are as follows

$$
\begin{aligned}
G &= \frac{v_o}{v_s} = \frac{\mu}{\mu + 1 + \frac{r_p}{R_k}} \\
G &= \frac{v_o}{v_s} = \frac{\mu R_k}{\mu R_k + R_k + r_p} \\
G &= \frac{v_o}{v_s} = \frac{\mu R_k}{R_k(\mu + 1) + r_p} \\
v_o &= v_s \frac{\mu R_k}{(\mu + 1)\left(R_k + \frac{r_p}{\mu+1}\right)} \\
v_o &= v_s \left(\frac{\mu}{\mu + 1}\right)\left(\frac{R_k}{R_k + \frac{r_p}{\mu+1}}\right)
\end{aligned}
$$

We now have the Thevenin equivalent for the cathode-follower, shown in Fig. 23.

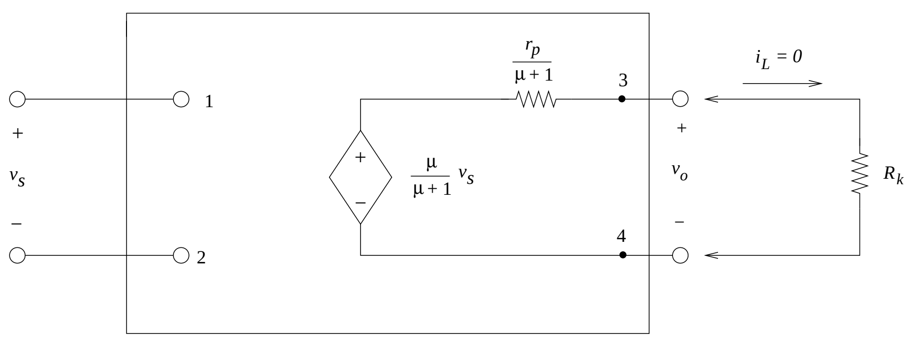

Figure 23: Thevenin equivalent for cathode-follower amplifier.

We have found the ouput resistance Z_o to be $r_p/(\mu + 1)$. For the 12AX7 triode $\mu \gg 1$. Therefore, by using the fact that $\mu = g_m r_p$ we can find Z_o as

$$
\begin{aligned}
Z_o &= \frac{r_p}{\mu + 1} \\
Z_o &\approx \frac{r_p}{\mu} \\
Z_o &= \frac{r_p}{g_m r_p} \\
Z_o &= \frac{1}{g_m}
\end{aligned}
$$

This is in agreement with the resistance looking into the cathode that we found by deriving the T-model for a triode vacuum tube in sec. (2.2.1). Recall from sec. (2.6) that the output resistance for the common-cathode amplifier is r_p. Therefore, the cathode-follower amplifier has an output resistance 100 times less than the ouput resistance of the common-cathode amplifier, when using a 12AX7 triode. This low Z_o will enable the cathode-follower to drive low impedance loads without significant signal loss.

3.4 Cathode-Follower as an Active Load for the Common-Cathode Amplifier

3.4.1 The Active Load Circuit Topology

A very useful application for the cathode-follower is as an active load for the common-cathode amplifier. A possible circuit topology is shown in Fig. 24. There are

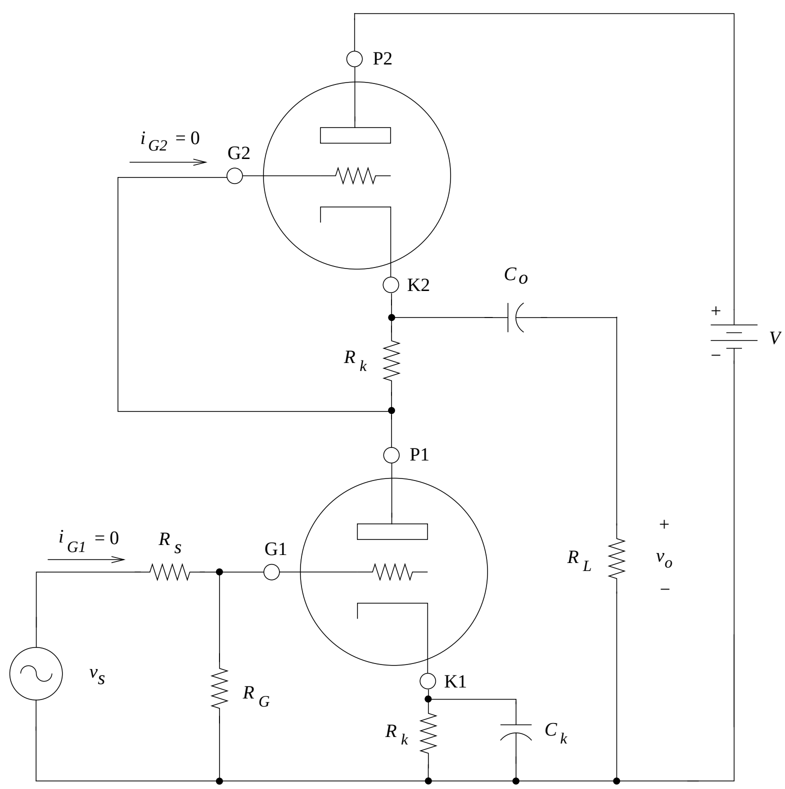

Figure 24: Cathode-follower as an active load for common-cathode amplifier.

two main reasons for this circuit being so useful. The first is a popular misconception. It is based on the fact that the cathode-follower has a gain of approximately 1. We can see in Fig. 24 that $v_{P1} = v_{G2}$. Therefore, $\Delta v_{P1} = \Delta v_{G2} \approx \Delta v_{K2}$. This means that v_{P1} and v_{K2} change at a one to one ratio. This leads to the cathode-follower behaving as a constant current source with respect to AC signals. Therefore, our AC load line will in theory be perfectly horizontal. This will lead to high gain, and low distortion. However, in reality we know that the cathode-follower is not perfect, which means $\Delta v_{K2} \neq \Delta v_{P1}$. Therefore, the cathode-follower is not a perfect constant current source with respect to AC signals. More importantly, if there is no AC component to i_p there can be no v_o. Therefore, even if the cathode-follower was ideal, R_L would physically force it to behave non-ideally. The second reason this circuit is so useful is that we can take the output directly from the cathode of the cathode-follower. This will give us the low output impedance we need for driving low impedance passive filter networks.

3.4.2 Analysis of the Cathode-Follower as an Active Load for the Common-Cathode Amplifier

DC Analysis

The first step we will perform in our analysis is to replace the triodes with there small-signal models. We will once again use the voltage source models. The resulting circuit is shown in Fig. 25. Now we sum the voltages around the loop. Notice that for DC, $v_{gk1} = v_{gk2}$. Therefore, we will refer to them both as v_{gk} for simplification. This results in

$$i_p(2r_p + 2R_k) - 2\mu v_{gk} - V = 0 \tag{60}$$

We now manipulate Eq. (60) in the following manner to obtain an expression for i_p.

$$\begin{aligned} i_p(r_p + R_k) &= \mu v_{gk} + \frac{V}{2} \\ i_p &= \frac{\mu v_{gk}}{r_p + R_k} + \frac{V}{2(r_p + R_k)} \end{aligned}$$

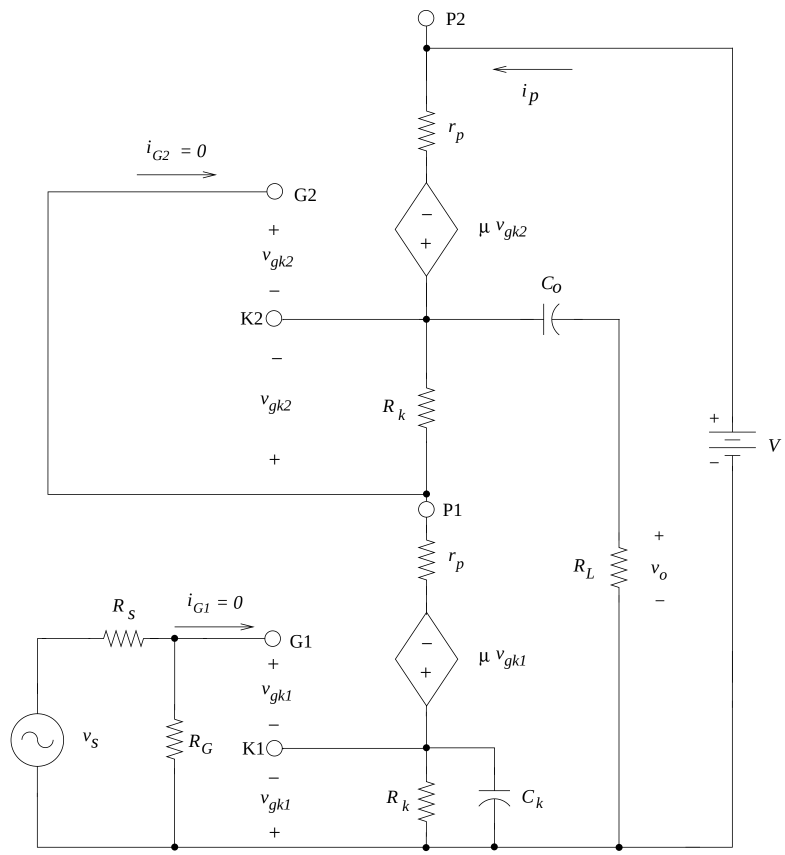

Figure 25: Small-signal model of cathode-follower as an active load for common-cathode amplifier.

Since $v_{gk} = -i_p R_k$, we get

$$\begin{aligned}
i_p &= \frac{-\mu i_p R_k}{r_p + R_k} + \frac{V}{2(r_p + R_k)} \\
i_p \left(1 + \frac{\mu R_k}{r_p + R_k}\right) &= \frac{V}{2(r_p + R_k)} \\
i_p &= \frac{V}{2(r_p + R_k)\left(1 + \frac{\mu R_k}{r_p + R_k}\right)} \\
i_p &= \frac{V}{2(r_p + R_k + \mu R_k)}
\end{aligned}$$

and finally,

$$i_p = \frac{V}{2(r_p + R_k(1 + \mu))} \tag{61}$$

We can now determine a design equation to enable us to select a value for R_k. To do this we manipulate Eq. (61) in the following manner,

$$
\begin{aligned}
r_p + R_k(1+\mu) &= \frac{V}{2i_p} \\
(1+\mu)R_k &= \frac{V}{2i_p} - r_p
\end{aligned}
$$

and finally,

$$R_k = \frac{V}{2(1+\mu)i_p} - \frac{r_p}{1+\mu} \tag{62}$$

Designing for a Quiescent Point of Operation

We will start by selecting the bias current value. We will choose a bias current of 1 mA. We can see from Fig. 25 that

$$
\begin{aligned}
v_{P1} &= (V)\left(\frac{r_p + R_k}{(r_p + R_k) + (r_p + R_k)}\right) \\
v_{P1} &= (V)\left(\frac{r_p + R_k}{2(r_p + R_k}\right)
\end{aligned}
$$

which gives v_{P1} as

$$v_{P1} = \frac{V}{2} \tag{63}$$

The V supply will be 400 V. Therefore, by Eq. (63)

$$
\begin{aligned}
v_{P1} &= \frac{400 \text{ V}}{2} \\
v_{P1} &= 200 \text{ V}
\end{aligned}
$$

This results in an initial quiescent point of operation at

$$v_{P1Q} = 200 \text{ V}$$

and,

$$i_{pQ} = 1 \text{ mA}$$

We will assume r_p to be 60 kΩ at this operating point. Therefore, by Eq. (62),

$$
\begin{aligned}
R_k &= \frac{400 \text{ V}}{2(1+100)(1 \text{ mA})} - \frac{60 \text{ k}\Omega}{1+100} \\
R_k &= 1.386 \text{ k}\Omega
\end{aligned}
$$

We will choose $R_k = 1.5$ kΩ. By using Eq. (61), this will result in a bias current of

$$i_p = \frac{400}{2(60 \text{ k}\Omega + 1.5 \text{ k}\Omega(1 + 100))}$$
$$i_p = 0.946 \text{ mA}$$

This results in a grid bias voltage V_G of

$$V_G = -i_p R_k$$
$$V_G = -0.946 \text{ mA} \times 1.5 \text{ k}\Omega$$
$$V_G = -1.42 \text{ V}$$

Therefore, our quiescent point of operation will be

$$i_{pQ} = 0.946 \text{ mA}$$
$$v_{P1Q} = 200 \text{ V}$$

and

$$V_{GQ} = -1.42 \text{ V}$$

AC Analysis

Our first step is to simplify Fig. 25 into an equivalent AC circuit. We do this by removing the capacitors, and shorting out anything in parallel with a capacitor. We also short out any DC voltage supplies. The result is shown in Fig. 26. We will now use a source transformation on the cathode-follower model. The result is shown in Fig. 27 Notice that $v_s = v_{gk1}$. We will now combine r_p and R_L in parallel before performing another source transformation on the cathode-follower model. After these steps we have a simplified circuit for performing AC analysis. The simplified circuit is shown in Fig. 28.

We will start our analysis by observing that

$$v_{gk2} = -i_p R_k$$

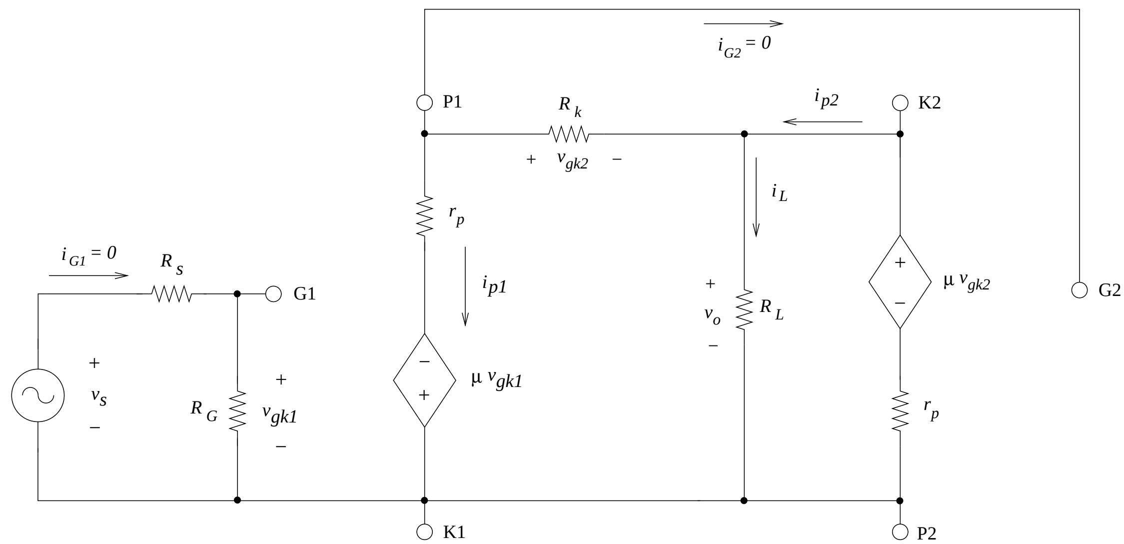

Figure 26: Small-signal AC circuit of common-cathode amplifier with active load.

We can also determine i_p as

$$i_p = \frac{v_o + \mu v_s}{r_p + R_k}$$

We will also simplify the cathode-follower dependent voltage source value as follows

$$\begin{aligned} \frac{\mu v_{gk2}}{r_p} R_L \parallel r_p &= \left(\frac{\mu v_{gk2}}{r_p}\right)\left(\frac{r_p R_L}{r_p + R_L}\right) \\ \frac{\mu v_{gk2}}{r_p} R_L \parallel r_p &= \mu v_{gk2}\left(\frac{R_L}{r_p + R_L}\right) \end{aligned}$$

We can now sum the voltages around the loop. This results in

$$\mu v_{gk2}\left(\frac{R_L}{r_p + R_L}\right) - i_p\left(r_p + R_k + \frac{R_L r_p}{R_L + r_p}\right) + \mu v_s = 0$$

We will substitute for v_{gk2} to get

$$\begin{aligned} -\mu i_p R_k\left(\frac{R_L}{r_p + R_L}\right) - i_p\left(r_p + R_k + \frac{R_L r_p}{R_L + r_p}\right) + \mu v_s &= 0 \\ i_p\left(\frac{-\mu R_k R_L}{r_p + R_L} - r_p - R_k - \frac{R_L r_p}{R_L + r_p}\right) &= -\mu v_s \\ i_p\left(-\mu R_k R_L - r_p(R_L + r_p) - R_k(R_L + r_p) - R_L r_p\right) &= -\mu v_s(R_L + r_p) \end{aligned}$$

We now substitute for i_p and get

$$(v_o + \mu v_s)(-\mu R_L R_k - r_p R_L - r_p^2 - R_k R_L - R_k r_p - R_L r_p) = -\mu v_s(R_L + r_p)(r_p + R_k)$$

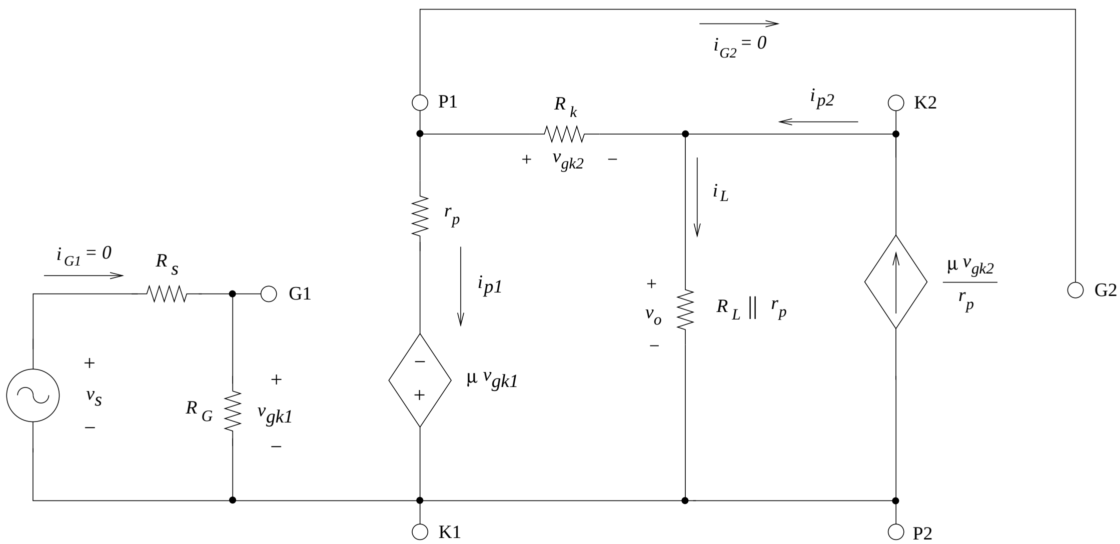

Figure 27: Modified small-signal AC circuit of common-cathode amplifier with active load.

We collect like terms to get

$$v_o(-\mu R_L R_k - 2R_L r_p - r_P^2 - R_k R_L - R_k r_p) \;=$$
$$-\mu v_s((R_L + r_p)(r_p + R_k) - \mu R_L R_k - 2R_L r_p - r_p^2 - R_k R_L - R_k r_p)$$

$$\frac{v_o}{v_s} = \frac{-\mu(R_L r_P + R_L R_k + r_p^2 + R_k r_p - 2R_L r_p - r_p^2 - R_L R_k(1+\mu) - R_k r_p)}{-R_L R_k(1+\mu) - 2R_L r_p - r_p^2 - r_p R_k}$$

After term cancellations we get

$$\frac{v_o}{v_s} = \frac{-\mu(-R_L)(R_k\mu + r_p)}{(-1)(R_L(2r_p + R_k(1+\mu)) + r_p(r_p + R_k))}$$
$$\frac{v_o}{v_s} = \frac{-\mu R_L(R_k\mu + r_p)}{R_L(2r_p + R_k(1+\mu)) + r_p(r_p + R_k)}$$

This leads to the equation for the gain of our active loaded common-cathode amplifier as

$$\frac{v_o}{v_s} = \frac{-\mu R_L}{R_L\left(\frac{2r_p + R_k(1+\mu)}{r_p + R_k\mu}\right) + \left(\frac{r_p(r_p + R_k)}{r_p + R_k\mu}\right)} \tag{64}$$

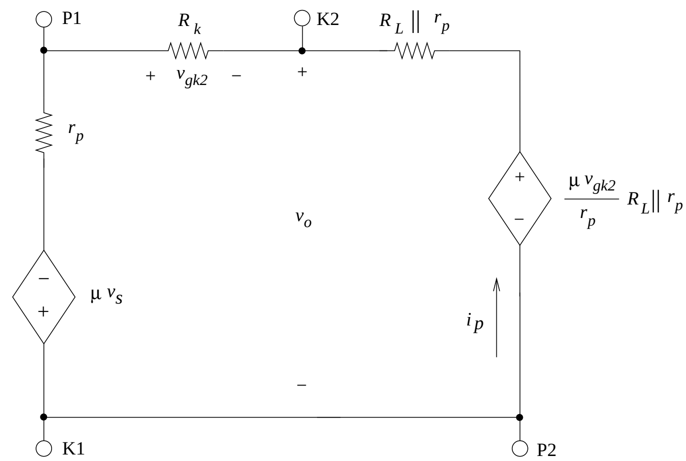

Figure 28: Simplified small-signal AC circuit of common-cathode amplifier with active load.

Comparison Between an Active Load Verse a Resistive Load

If we substitute our values

$$r_p = 60\ \text{k}\Omega$$
$$R_k = 1.5\ \text{k}\Omega$$

and

$$R_L = 1\ \text{M}\Omega$$

into Eq. (64) we get

$$\frac{v_o}{v_s} = \frac{-100 \times 1\ \text{M}\Omega}{1\ \text{M}\Omega\left(\frac{2\times 60\ \text{k}\Omega + 1.5\ \text{k}\Omega(1+100)}{60\ \text{k}\Omega + 100\times 1.5\ \text{k}\Omega}\right) + \left(\frac{60\ \text{k}\Omega(60\ \text{k}\Omega + 1.5\ \text{k}\Omega)}{60\ \text{k}\Omega + 100\times 1.5\ \text{k}\Omega}\right)}$$
$$\frac{v_o}{v_s} = -76.3\ \frac{\text{V}}{\text{V}}$$

Recall that the common-cathode amplifier designed in chapter 2 with a resistive load had a gain of -70 V/V. Therefore, the gain with an active load is only slightly higher. However, the quiescent bias current with the active load is twice the value of the quiescent bias current with a resistive load. This means that are quiescent point of operation with an active load is farther away from the highly non-linear cutt-off region for plate current than the quiescent point of operation with a resistive load. This will in theory result in lower amplitude distortion for the common-cathode amplifier that is designed with an active load. Therefore, the use of an active load over a resistive load for the common-cathode amplifier is a definite improvement in performance.

4 Conclusions

This thesis has presented the design of an audio-frequency vacuum tube amplifier. Equations have been derived that give the ability to choose circuit component values that will achieve the desired amplifier performance. The method of using a graph of the characteristic plate current curves at constant grid voltages to aid in amplifier design has been introduced. This method enables the triode dynamic parameters to be determined for a chosen quiescent point of operation. The use of an active load to improve the basic amplifier has been introduced. Equations have been derived that enable the selection of circuit components that will achieve a desired quiescent point of operation, as well as determine the gain when an active load is used in place of a resistive load. An active load will give higher gain than a resistive load, at a higher bias current. An increased bias current will decrease amplitude distortion introduced by the highly non-linear cutt-off region for plate current. This result has been verified with the use of PSPICE to simulate circuit operation. PSPICE also calculates lower harmonic distortion with the use of an active load. This thesis has shown the common-cathode amplifier to have one weakness, high output impedance. The cathode-follower is useful due to it's low output impedance, even though it offers no gain. Therefore, using the cathode-follower as an active load for the common-cathode amplifier offers a considerable improvement in performance.

Further work on this topic could include the discovery of an acceptable form of feedback to decrease distortion introduced by the amplifier. We have designed a voltage amplifier. Therefore, we would like to increase input resistance, so as to not load the source. We would also like to decrease output resistance, which will lighten the loading effect on the output. Therefore, we need to apply series-shunt feedback, as this feedback topology has both of these characteristics. The use of pre-distortion, or pre-correction technique to linearize the amplifier could be studied. This would enable us to improve amplifier performance without sacrificing gain, which is the case when applying negative feedback. Also worthy of consideration is the effect of having a large potential difference between the heater and the cathode. Vacuum tubes have a recommended maximum potential difference between the heater and cathode. When used as a cathode-follower this potential difference often exceeds the recommended value for a 12AX7 triode. This could have adverse effects on performance that need to be considered in the design. The analysis of the effect of grid current on circuit operation would be useful. This thesis used the assumption that no grid current was present. The flow of grid current may need to be considered when amplifying large signals. A large input signal may drive the grid voltage positive, where grid current will definitely flow.

Appendix A

Matlab code to plot the design graph. Values are from a RCA data sheet for a 12AX7 vacuum tube.

```
%********************************************************************

n0 = 8; n1 = 5; n2 = 4; n3 = 3; n4 = 2; n5 = 2; % POLYNOMIAL FIT ORDERS

%********************************************************************
% PLATE CURRENT AT GRID = 0 V
%********************************************************************

x0 = [0 10 20 30 40 50 60 70 80 90 100 110 120 130 150 160 170 180];
y0 = [0 .4 .6 .8 1 1.16 1.35 1.52 1.7 1.9 2.09 2.3 2.5 2.72 2.95 3.18 3.42 3.7 4];
p0 = polyfit(x0, y0, n0);
x0i = linspace(0, 180, 10000);
z0 = polyval(p0, x0i);
plot(x0i, z0)%(x, y,' -o', xi, z,' :')

%********************************************************************
grid; hold; xlabel('V_P(V)'); ylabel('I_P(mA)'); axis([0, 500, 0, 2])
title('I_Pvs.V_PCurvesatConstantGridVoltagesfor12AX7TriodeVacuumTube')
%********************************************************************

%********************************************************************
% PLATE CURRENT AT GRID = -1 V
%*******************************************************************

x1 = [50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200 210 220 230 240];
y1 = [.1 .18 .25 .33 .41 .5 .61 .75 .89 1 1.17 1.31 1.5 1.68 1.86 2.07 2.3 2.5 2.72 3];
p1 = polyfit(x1, y1, n1);
x1i = linspace(30, 340, 10000);
z1 = polyval(p1, x1i);
plot(x1i, z1)%(x1, y1,' -o', x1i, z1,' :')

%********************************************************************
% PLATE CURRENT AT GRID = -2 V
%********************************************************************

x2 = [140 150 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310];
y2 = [.1 .15 .21 .3 .38 .47 .57 .69 .8 .91 1.05 1.2 1.39 1.52 1.7 1.9 2.09 2.3];
p2 = polyfit(x2, y2, n2);
x2i = linspace(100, 500, 10000);
z2 = polyval(p2, x2i);
plot(x2i, z2)%(x2, y2,' -o', x2i, z2,' :')
```

```
%*********************************************************************
% PLATE CURRENT AT GRID = -3 V
%*********************************************************************

x3 = [230 240 250 260 270 280 290 300 310 320 330 340 350 360 370];
y3 = [.1 .15 .2 .27 .31 .4 .49 .58 .69 .8 .9 1.05 1.2 1.35 1.5];
p3 = polyfit(x3, y3, n3);
x3i = linspace(180, 500, 10000);
z3 = polyval(p3, x3i);
plot(x3i, z3)%(x3, y3,'-o', x3i, z3,':')

%*********************************************************************
% PLATE CURRENT AT GRID = -4 V
%*********************************************************************

x4 = [320 330 340 350 360 370 380 390 400 410 420];
y4 = [.1 .15 .2 .25 .31 .39 .45 .51 .61 .71 .82];
p4 = polyfit(x4, y4, n4);
x4i = linspace(265, 500, 10000);
z4 = polyval(p4, x4i);
plot(x4i, z4)%(x4, y4,'-o', x4i, z4,':')

%*********************************************************************
% PLATE CURRENT AT GRID = -5 V
%*********************************************************************

x5 = [405 420 430 440 450 460 465];
y5 = [.1 .15 .2 .26 .31 .38 .4];
p5 = polyfit(x5, y5, n5);
x5i = linspace(370, 500, 10000);
z5 = polyval(p5, x5i);
plot(x5i, z5)%(x5, y5,'-o', x5i, z5,':')

%*********************************************************************
% DC LOAD LINE
%*********************************************************************

x = [1 : 1 : 500];
i = 1 : 500;
y(i) = -0.003025 * x(i) + 1.21;
plot(i, y(i))

%*********************************************************************
% AC LOAD LINE
%*********************************************************************

z(i) = -0.00403 * x(i) + 1.452;
plot(i, z(i))
```

```
%***********************************************************************
% BIAS LINE
%***********************************************************************

b(i) = 0.002 * x(i);
plot(i, b(i))
```

Appendix B

Matlab code to obtain the common-cathode amplifier frequency response.

```
%*********************************************************************
% Brad Bryant Oct. 5, 2000
% Common-Cathode Design
%*********************************************************************

clear

%*********************************************************************
% Prompt User for Component Values
%*********************************************************************

gm=input(' Enter the Transconductance gm in mA ');
rp=input(' Enter the Dynamic Plate Resistance rp in K-Ohms ');
Rs=input(' Enter the Source Resistance Rs in K-Ohms ');
RG=input(' Enter the Grid Leak Resistance RG in K-Ohms ');
Rb=input(' Enter the Plate Load Resistance Rb in K-Ohms ');
RL=input(' Enter the Output Load Resistance RL in K-Ohms ');
Rh=input(' Enter the High Frequncy Limiting Resistance Rh in K-Ohms ');
Rk=input(' Enter the Cathode Bias Resistance Rk in K-Ohms ');
Ck=input(' Enter the Cathode Bypass Capacitance Ck in Farads ');
Ci=input(' Enter the Input Capacitance Ci in Farads ');
Co=input(' Enter the Output Capacitance Co in Farads ');
Cgp=2e-12; %input(' Enter the Grid to Plate Capacitance ');
Cgk=2e-12; %input(' Enter the Grid to Cathode Capacitance ');
Cpk=2e-12; %input(' Enter the Plate to Cathode Capacitance ');


%*********************************************************************
% Calculate mid-frequecy gain
%*********************************************************************

Am = -gm * (RG/(RG + Rs)) * (1/(1/rp + 1/Rb + 1/RL));

%*********************************************************************
% Calculate pole and zero due to Ci
%*********************************************************************

wp = .001/(Ci * (Rs + RG));
wz = 0;
```

```
%**********************************************************************
% Calculate pole and zero due to Ck
%**********************************************************************

wp1 = .001/(Ck*(1/(1/Rk+gm)));
wz1 = .001/(Ck*Rk);

%**********************************************************************
% Calculate pole and zero due to Co
%**********************************************************************

wp2 = .001/(Co*(RL+(rp*Rb/(rp+Rb))));
wz2 = 0;

%**********************************************************************
% Calculate Miller capacitances
%**********************************************************************

C1 = Cgp*(1-Am);
C2 = Cgp*(1-1/Am);

%**********************************************************************
% Calculate pole due to Cgk and C1
%**********************************************************************

wp3 = .001/((Cgk+C1)*((Rs*RG/(Rs+RG))+Rh));

%**********************************************************************
% Calculate pole due to Cpk and C2
%**********************************************************************

wp4 = .001/((Cpk+C2)*(1/(1/rp+1/Rb+1/RL)));

%**********************************************************************
% Calculate the 3-dB frequencies, bandwidth, and gain-bandwidth product
%**********************************************************************

wL = sqrt(wp^2+wp1^2+wp2^2-2*(wz1^2));
wH = 1/sqrt(1/(wp3^2)+1/(wp4^2));
fL = wL/(2*pi);
fH = wH/(2*pi);
BW = fH-fL;
GBWP = -Am*BW;
Rin = 1000*RG;
Rout = 1000/(1/rp+1/Rb);
```

```
%*********************************************************************
% Calculate the transfer function
%*********************************************************************

num = Am * conv([1wz], [1wz1]);
num = conv(num, [1wz2]);

den = conv([1wp], [1wp1]);
den = conv(den, [1wp2]);
den = conv(den, [1/wp31]);
den = conv(den, [1/wp41]);

sys = tf(num, den);

%*********************************************************************
% Bode Plot
%*********************************************************************

r = 20 * log10(-Am);
q = r - 3;

bode(sys,1,30000000);
hold;
plot(fL,q,'*')
plot(fH,q,'*')

%*********************************************************************
% Print values to command window
%*********************************************************************

Am
fL
fH
BW
GBWP
Rin
Rout
sys
```

Appendix C

Matlab code to obtain common-cathode amplifier gain dependency on R_b/r_p.

```
%*********************************************************
% Brad Bryant October 24, 2000
% Dependance of Am on Rb/rp
%*********************************************************

for r = 1 : 1000
rr = r/100;
y = 100 * (rr/(1 + rr));
plot(rr, y)
if r == 1
hold;
title(' Dependence of A_m on R_b/r_p ');
xlabel(' R_b/r_p ');
ylabel(' A_m/μ (%) ');
grid;
end
end
```

Appendix D

Matlab code to obtain cathode-follower gain dependency on R_k/r_p.

```
%*********************************************************
% Brad Bryant October 24, 2000
% Dependance of G on Rk/rp
%*********************************************************

mu = 100;

for r = 100 : 10000
x = r/1000;
y = (mu * x)/(1 + x * (mu + 1));
plot(x, y)
if r == 100
hold;
end
end
grid;
title(' Cathode - Follower Gain vs. R_k/r_p ')
xlabel(' R_k/r_p ')
ylabel(' Gain(V/V) ')
axis([0 10 .9 1])
```

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