Principles of Computerized Tomographic Imaging

Parallel CT, Fanbeam CT, Helical CT and Multislice CT

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Abstract

The total attenuation suffered by one beam of x-rays as it travels in a straight line through an object can be represented by a line integral. Combining a set of line integrals forms a projection. With the filtered back projection a three-dimensional reconstruction of the object can be made by using projections from different angles. The quality of a reconstructed image depends on the number of samples, the number of projections, the reconstruction grid, the photon energy, the x-ray source and the detector. Several techniques can be used to avoid or repair distortions. To improve the volume coverage speed performance of CT scans, new scanning methods, such as helical and multi-slice CT, are developed. These methods ask for new reconstruction algorithms.

1 Introduction

Tomography refers to the cross-sectional imaging of an object from either transmission or reflection data collected by illuminating the object from many different directions. The impact of this technique in diagnostic medicine has been revolutionary. Medical tomography has not only successfully been accomplished with x-rays, but also with radioisotopes, ultrasound and magnetic resonance.

In ‘Principles of Computerized Tomographic Imaging’ by A.C. Kak and M. Slaney the principles of medical tomography are explained. While the book focuses on computerized tomography with x-rays, variations of tomography are also discussed. This paper gives an overview of the mathematical principles and theory of imaging based on x-rays, and a description of how to apply the theory to actual problems in medical imaging. An introduction on signal processing fundamentals and variations on x-ray tomography are not included.

2 Algorithms for reconstruction with nondiffracting sources

The total attenuation suffered by one beam of x-rays as it travels in a straight line through an object, represented by a two-dimensional function f(x,y), can be represented by a line integral
\( P_\theta(t) \):

\[
P_\theta(t) = \int_{(\theta,t)\text{line}} f(x, y)ds
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\delta(x \cos \theta + y \sin \theta - t)dxdy
\]

With:

\[
t = x \cos \theta + y \sin \theta
\]

\[
s = -x \sin \theta + y \cos \theta
\]

A projection, \( S_\theta(w) \), is formed by combining a set of line integrals. The simplest projection, the parallel projection is a collection of parallel ray integrals. Such a projection can be measured by moving an x-ray and a detector along parallel lines on opposite sides of an object. A fanbeam projection is measured by using a single x-ray source in a fixed point and rotating it relative to a ring of detectors.

The Fourier Slice theorem states that the one-dimensional Fourier transform of a parallel projection of an image \( f(x,y) \) taken at angle \( \theta \) gives a slice of the two-dimensional transform \( F(u,v) \), subtending an angle \( \theta \) with the u-axis. This theorem indicates that, if an infinite number of projections, \( S_\theta(w) \), of an object are taken, \( F(u,v) \) is known at all points in the uv-plane. The object function \( f(x,y) \) can then be recovered using the two-dimensional inverse Fourier transform.

\[
F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)e^{-j2\pi(ux+vy)}dxdy
\]

\[
S_\theta(w) = \int_{-\infty}^{\infty} P_\theta(w)e^{-j2\pi wt}dt
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)e^{-j2\pi w(x \cos \theta + y \sin \theta)}dxdy
\]

Where \( w \) is called the spatial frequency and

\[
u = w \cos \theta \quad ; \quad v = w \sin \theta
\]

and thus:

\[
S_\theta(w) = F(u, v)
\]

Practical implementation is impossible with the Fourier Slice theorem only. The filtered backprojection algorithm is currently used in almost all applications of straight ray tomography. The filtered backprojection generates a (simple) reconstruction based on one projection. In order to do so, the values of Fourier transform of the projection are inserted on their proper place in the object’s two-dimensional Fourier domain, assuming the other projections to be zero. A summation of all reconstructions of the object from one single projection over all projections leads to the final reconstruction. A projection is in the Fourier domain represented by a single line with a certain height. During the summation, the height in the center is summated for every projection. A weighting function has to be applied before the summation in order to avoid over-estimation in the centre of the frequency domain.

The filtered backprojection algorithm is derived from the Fourier Slice theorem by rewriting the inverse Fourier transform in polar coordinates (eq. 6):

\[
f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v)e^{-j2\pi(ux+vy)}dudv
\]

\[
= \int_{0}^{2\pi} \int_{-\infty}^{\infty} F(w, \theta)e^{-j2\pi(x \cos \theta + y \sin \theta)}wdwd\theta
\]
The integral is split into two by considering $\theta$ from 0 to $\pi$ and from $\pi$ to $2\pi$ such that the following property can be used:

$$F(w, \theta + 180^\circ) = F(-w, \theta)$$  \hspace{1cm} (12)

Equation 11 can now be rewritten:

$$= \int_0^\pi \left[ \int_{-\infty}^{\infty} S_\theta(w) |w| e^{-j2\pi wt} dw \right] d\theta$$  \hspace{1cm} (13)

$$= \int_{-\infty}^{\infty} Q_\theta(w) d\theta$$  \hspace{1cm} (14)

$Q_\theta(w)$ is called the filtered projection, it is a filtering operation with filter $|w|$. In practice, only a limited number of Fourier components will be known. The spatial resolution in the reconstructed image is determined by the number of samples, N. The number of projections, K, is limited in practice as well. The expression for the filtered projection becomes in practice:

$$Q_\theta(t) \approx 2W_N \sum_{m=-N/2}^{N/2} S_\theta \left( m\frac{2W}{N} \right) \left| m\frac{2W}{N} \right| e^{j2\pi m(2W/N)t}$$  \hspace{1cm} (15)

Where W is a frequency higher than the highest frequency in each component.

The backprojection algorithm is relatively simple for parallel beam projection. However, these projections can not fastly be generated. Generation of fan beam projections is much faster. The backprojection algorithm of these projections is more complicated. A weighted backprojection algorithm was developed for both equiangular and equally spaced sampled fan projections. It is also possible to re-sort the fan beam projection data into equivalent parallel beam projection data, allowing the use of the simple backprojection algorithm.

During computer implementation of the algorithm problems occur, causing artifacts in the reconstruction of the image. A summary of all artifacts is shown in the appendix.

For parallel beams, an object is completely specified if the ray-integrals of the object are known for $\theta_0 \leq \theta \leq \theta_0 + 180^\circ$ and for $-t_{\text{max}} \leq t \leq t_{\text{max}}$ where $t_{\text{max}}$ is the widest projection of the object. For fan projections, $180^\circ$ is not enough. Some lines will be measured twice, from opposite directions, while other lines may not be measured at all.

## 3 X-ray tomography; single-slice CT

Projection data are a result of interaction between the radiation used for imaging and the substance of which the object is composed. For x-ray tomography, the measured photon energy of an x-ray beam reduces while going through an object. The mechanisms meanly causing this are the photoelectric effect (absorption) and the Compton effect (scattering). The photon loss rates of a narrow mono-energetic x-ray beam while going through a material is an indication for the linear attenuation coefficient of that material.

$$N(x) = N_0 e^{-\mu x}$$  \hspace{1cm} (16)

In practice x-ray sources do not produce mono-energetic beams. Filtering of the x-ray beam to get photons in a narrow energy-band would greatly reduce the number of photons available and is therefore not advisable. The linear attenuation coefficient is, in general a function of the photon energy. For most biological tissues, the linear attenuation coefficient decreases with energy. Low energy photons are preferentially absorbed in those tissues resulting in beam hardening, which may cause artefacts.
Detectors are assumed to be insensitive to scatter and to have a small aperture. Furthermore, the detector sensitivity ought to be constant over the energy range of interest. Three different types of detectors can be used: a photon counting type, a scintillation type and an ionisation type. The quantities measured by these types are proportional to the total number of incident photons, to the total energy of these photons or to the energy deposition per unit mass.

To reconstruct an object from a projection made with a polychromatic x-ray, it is necessary to know the effective energy of the CT scanner. This is defined as the monochromatic energy at which a given material will exhibit the same attenuation as measured by the scanner. The output of a CT scanner are given in Hounsfield units. Theoretically, the relation between the linear attenuation coefficient ($\mu$) and the corresponding Hounsfield unit ($H$) is:

$$H = \frac{\mu_{\text{material}} - \mu_{\text{water}}}{\mu_{\text{water}}} \times 1000$$  \hspace{1cm} (17)

The value of the Hounsfield unit varies from -1000 (for air) to 3000.

<table>
<thead>
<tr>
<th>Tissue</th>
<th>Range of Hounsfield units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>-1000</td>
</tr>
<tr>
<td>Lung</td>
<td>-500 to -200</td>
</tr>
<tr>
<td>Fat</td>
<td>-200 to -50</td>
</tr>
<tr>
<td>Water</td>
<td>0</td>
</tr>
<tr>
<td>Blood</td>
<td>25</td>
</tr>
<tr>
<td>Muscle</td>
<td>25 to 40</td>
</tr>
<tr>
<td>Bone</td>
<td>200 to 1000</td>
</tr>
</tbody>
</table>

The volume coverage speed performance refers to the capability of rapidly scanning a large longitudinal z volume with high longitudinal z-axis resolution and low image artifacts. The volume coverage speed performance is a deciding factor for the success of many medical CT applications which require a large volume scanning with high image quality and short time duration. Thus, one of the main themes in CT development is to improve its volume coverage speed performance.

Three-dimensional reconstructions are often formed by stacking two-dimensional slices. There are two modes for such a CT scan: step-and-shoot CT or helical or spiral CT. Another method is the cone beam algorithm, which reduces the data collection time.

### 3.1 Step-and-shoot CT

For step-and-shoot CT two alternate stages can be distinguished: data acquisition and patient positioning. During the data acquisition stage, the patient remains stationary and the x-ray tube rotates about the patient to acquire a complete set of projections at a prescribed scanning location. During the patient positioning stage, no data are acquired and the patient is transported to the next prescribed scanning location. The data acquisition stage typically takes one second or less while the patient positioning stage is around one second. Thus, the duty cycle of the step-and-shoot CT is 50% at best.

Clinically, two step-and-shoot scan configurations are used:

1. The fan beam rotational type; A fan beam of x-rays is used to illuminate a multidetector array. Both the source and the detector array are mounted on a yoke which rotates continuously around the patient over 360°.

2. The fixed detector ring with a rotating source type; A large number of detectors are mounted on a fixed ring. In- or outside this ring is an x-ray tube that continuously rotates around the patient.
The aspects of these CT scanners is summarized in the appendix.

3.2 Helical CT

Helical -or spiral- CT was introduced around 1990. The data are continuously acquired while the patient is simultaneously transported at a constant speed through the gantry. The patient translating distance per gantry rotation in helical scan is referred to as the table speed. Because the data are continuously collected without pausing, the duty cycle of the helical scan is improved to nearly 100% and the volume coverage speed performance can be substantially improved. Two commonly used helical reconstruction algorithms are the 360 and 180 linear interpolation LI.

The 360 LI algorithm explores the 360 periodicity in the projection data due to the fact that the projection data 360 apart would be identical in the absence of patient motion, noise variation and other errors. It uses two sets of helical CT projections 360 apart to estimate one set of projections at a prescribed location. On the other hand, the 180 LI algorithm also called the half-scan with interpolation or extrapolation algorithm, utilizes the 180 periodicity in the projection data due to the fact that the two measurements along the same path but in the opposite directions 180 apart would be the same in the absence of patient motion, noise variation and other errors. It uses two sets of helical CT projections 180 apart to estimate one set of projections at a prescribed location.

The table advancement per rotation of twice the x-ray beam collimation appears to be the limit of the volume coverage speed performance of a single slice CT, and further increase of the table translation would result in clinically unusable images.

4 Multi-slice Helical CT

The so-called multi-slice CT scanner seems to be a next step for a substantial improvement of the volume coverage speed performance. The multi-slice CT scanner refers to a special CT system equipped with a multiple-row detector array, as opposed to a single-row detector array. It allows for simultaneous scan of multiple slices at different z locations. Due to the distinct differences in scanner construction, the multi-slice CT scanner exhibits complex imaging characteristics and calls for new scan and reconstruction strategies.

The use of N detector rows enables us to divide the total x-ray beam into N subdivided beams (the detector row aperture is $1/N$ of the total x-ray beam collimation). In a multi-slice CT system, while the total x-ray collimation still indicates the volume coverage speed, the detector row collimation, rather than the total x-ray collimation, determines the z-axis resolution i.e., the slice thickness. In general, the larger the number of detector rows N, the better the volume coverage speed performance.

In the multi-slice CT the ray bundles not only fan out within the gantry plane but also diverge from the gantry plane. This imaging geometry is called the cone-beam imaging geometry, which calls for special cone-beam reconstruction algorithms. Because the scanner discussed has a relatively small number of detector rows and therefore relatively small cone-beam divergent effect, parallel fan-beam based reconstruction algorithms can be used to approximate the cone-beam geometry.

The pitch, $p$, of a helical scan refers to the ratio of the table translating distance per gantry rotation, $s$, to the thickness of the individual x-ray beam, $D$. The helical pitch of the multi-slice CT indicates roughly the number of contiguous slices that can be generated over the
table translating distance in one gantry rotation:

\[ p = \frac{N \cdot s(mm)}{D(mm)} \] (18)

4.1 Preferred pitch

Due to the use of multi-row detector array, projection data along a given path may be measured multiple times by different detector rows. One important consideration in multi-slice helical CT data acquisition is to select a preferred helical pitch to reduce the redundant measurements and therefore to improve the overall data z sampling efficiency.

In the single slice helical scan, the x-ray beam describes a spiral path i.e., a helix around the patient. Each point on the helix represents a set of fan-beam projection measurements. As said before, the projection data exhibit the 180 periodicity, thus two measurements along the same path in the opposite directions would be identical in the absence of patient motion, noise variation, and other errors. By exploring this 180 periodicity, actual fan-beam measurements can be regrouped to generate a set of complementary fan-beam projections. The z gap of 360 LI is equal to \( s \) or \( pD \) while the z gap of 180 LI the gap between the solid and dashed helices is equal to \( s/2 \) or \( (p/2)D \). This explains why 180 LI yields a better IQ than 360 LI. Furthermore, varying the table speed will stretch or compress both helices together but will not change this uniform helix pattern.

For a multi-slice helical scan a set of complementary fan-beam projections will be generated for each detector row. The preferred pitch is chosen such that the complementary projection of one row does not overlap the original or complementary projection of another row. Variation of the table speed will not change the uniform helix pattern of the separated detector rows, but it can change distance between the helix patterns of the different detector rows.

In general, one of the new challenges in multi-slice helical CT is to use a multi-row detector array efficiently, i.e., to achieve efficient z sampling. Only at these helical pitches, the volume coverage speed performance of the multi-slice scanner is substantially better than its single slice counterpart.

4.2 Reconstruction algorithms

the multi-slice helical reconstruction consists conceptually of the following two steps:

1. Estimation of a set of complete projection measurements at a prescribed slice location from the interlacing helical data.

2. Reconstruction of the slice from the estimated projection set using the step-and-shoot reconstruction algorithm.

4.2.1 Linear interpolation

In general, the estimation of the projection data along a given projection path is obtained by weighted averaging interpolating the contributions of those measurements from all detector rows that would be along the same projection path if the differences in z sampling positions due to the table translation and the displacement of multiple detector rows were ignored. The contribution the weighting factor of a measurement is larger as the z location of the measurement is closer to the slice location.
For efficient implementation it is important to know which portion of the data from each detector row is contributing to which slice reconstruction. Special helical interpolation algorithms that are suitable for efficient implementation and that correctly handle redundant measurements may be derived for a given number of detector rows and for a certain range of helical pitches.

4.2.2 Z-filtering (variable thickness) reconstruction

Reconstructing images with multiple slice thickness calls for a new type of helical reconstruction algorithm, referred to as the z-filtering or variable-thickness reconstruction algorithm, which contains z-axis resolution parameters in reconstruction to further control the tradeoff of the slice thickness versus image noise and artifacts.

The linear interpolation is extended to the z-filtering reconstruction algorithm by forming a composite slice by combining the slices reconstructed with the linear interpolation reconstruction algorithm. The z-filtering reconstruction enables users to generate from a single CT scan multiple image sets, representing different tradeoffs of the slice thickness, image noise and artifacts to suit for different application requirements.

5 Other CT applications

There are some other applications that use computed tomography to get information about the insight of a patient or an object:

**Gated CT** This technique is used to make image of periodically moving objects, such as the heart. It reconstructs only those projections that were made at a certain moment during this periodic movement.

**Angiographic CT** Projections are made of the same part of the object at different times in order to measure the flow of a injected contrast fluid in time. This technique is for example used to follow the bloodflow through a narrowed vein.

**Emission CT** Instead of measuring the linear attenuation coefficient of a tissue to get diagnostic information, emission CT uses the decay of administered radioactive isotopes to image the distribution of the isotope in the body as a function of time. Different kind of ECT can be distinguished of which Positron Emission Tomography (PET) is best known.

**Ultrasonic CT** With ultrasound it is possible to measure the exact pressure of a wave as a function of time. From the pressure waveform it is possible to determine not only the linear attenuation coefficient of the pressure field, but also the delay in the signal induced by the object. This technique can only be used for soft tissues.

6 Aliasing artefacts and noise in CT images

Some errors that occur in CT images are fundamental to the projection process and depend on the interaction of object inhomogeneities with the kind of energy (for example x-ray) that is used. Other reconstruction errors are caused by either insufficiency of data, or by the presence of noise in the measurements. Insufficiency in data may occur either by undersampling of projection data or by not recording enough projections. Both distortions are called aliasing distortions, that may also be caused by using a undersampled grid for displaying the reconstructed image.
In a real system neither the detector aperture nor the x-ray source are of zero size. This brings about a certain smoothing of the projections, unless these effects are taken into account. Convolving the ideal projection with the aperture function shows that at least two samples per detector should be taken to correct for the aperture size ($T_d$):

$$p_n = \delta(x - nT_s)[p(x) * a(x)]$$

$$a(x) = \begin{cases} 
1 & \text{for } |x| \leq \frac{T_d}{2} \\
0 & \text{elsewhere} 
\end{cases}$$

$$T_s < \frac{T_d}{2}$$

The effect of dimensions of the x-ray beam depends on the location of the object with respect to source and detectors, and can be determined similarly to the effect of the detector aperture.

To determine the effect of noise in the projection data, three types of noise have to be considered. The first, a continuously varying error due to electrical noise or roundoff errors. These errors can be modelled as Gaussian and uniform additive noise respectively. The second type, to which shot noise belongs to, is related to the number of photons that exit the object. Filtering of each projection with a filter that is chosen such that has an optimal trade-off between image distortion and noise variance, will reduce the effect of roundoff noise in the reconstructed image.

7 References


## A Aspects of the third and fourth generation CT scanners

<table>
<thead>
<tr>
<th>CT scan generation</th>
<th>Third</th>
<th>Fourth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scan configuration</td>
<td>Fan beam rotational</td>
<td>Fixed detector, rotating source</td>
</tr>
<tr>
<td>Name</td>
<td>Rotate-rotate</td>
<td>Rotate-fixed</td>
</tr>
<tr>
<td>Shown in</td>
<td>Figure 1a</td>
<td>Figure 1b</td>
</tr>
<tr>
<td>Data collection time</td>
<td>1-20 s. (over 1000 proj.)</td>
<td>ms (per projection)</td>
</tr>
<tr>
<td>Fan angle</td>
<td>30°-60°</td>
<td>-</td>
</tr>
<tr>
<td>Number of detectors</td>
<td>500-700</td>
<td>1088</td>
</tr>
<tr>
<td>X-ray detector type</td>
<td>Xenon ionization detector</td>
<td>Scintillation detector</td>
</tr>
<tr>
<td>Data limited by</td>
<td>Number of x-ray beams</td>
<td>Number of projections</td>
</tr>
<tr>
<td>Effect detector defect</td>
<td>Incorrect recording in every projection</td>
<td>One projection lost</td>
</tr>
<tr>
<td>Effect source instabilities</td>
<td>No great effect</td>
<td>Ring-artefacts</td>
</tr>
</tbody>
</table>

![Figure 1a](image1a.png)  ![Figure 1b](image1b.png)

Figure 1: (a) A third-generation fan beam x-ray tomography machine. (b) A fourth-generation fan beam x-ray tomography machine.
A Artifacts

A.1 Artifacts caused by computer implementation:

artifact Slight dishing and a dc shift (figure 2).

problem Projection not of finite order and finite bandwidth. W and N (eq. 15) are chosen too small, eq. 15 zero out for m=0, not only at w=0.

solution 1 Zero-padding of the projection data before sampling.

result 1 Dishing reduced, dc shift decreased (figure 2).

solution 2 Sample the projection data by using a filter that limits the bandwidth of the line integrals: \( H(w) = |w| \) for \( |w| < W \)

result 2 Elimination of both dishing and dc shift (figure 3).

Figure 2: (a) This reconstruction of the Shepp and Logan phantom shows the artifacts caused when the projection data are not adequately zero-padded and FFT’s are used to perform the filtering operation in the filtered backprojection algorithm. (b) A numerical comparison of the true and the reconstructed values on the \( y = -0.605 \) line. (c) Shown here are the reconstructed values.

Figure 3: (a) Reconstruction obtained after elimination of both dishing and dc shift. (b) A numerical comparison of the \( y = -0.605 \) line of the reconstruction in (a) with the true values.
A.2 Artifacts caused by beam hardening:

artifact White streaks and flares in the vicinity of thick bones and between bones (figure 5), higher CT numbers for tissues close to the skull (figure 4).

problem Usage of polychromatic x-rays. Due to a higher attenuation coefficient in biological material for low energy photons too many photons are photo-electrically absorbed.

solution 1 Preprocessing of the data. Beam hardening is taken into account by increasing the ray integral.

result 1 Good results for soft tissue cross sections, but solution fails when bone is present.

solution 2 Postprocessing of the reconstructed image. After the above mentioned prepro-cessing reconstruction bone areas are segmented from the image by thresholding. The contribution of bone to each ray integral is determined by forward projection of the thresholded image. A correction is made to the ray integrals.

result 2 Artefact can be eliminated.

solution 3 Dual energy techniques. The energy dependence of the linear attenuation coefficient is modelled. The beam hardening artifact can be removed from the ray integrals by using a second scan at a different tube voltage.

result 3 Theoretically this is the most elegant solution. The need for a second CT scan however is a great disadvantage.

Figure 4: This reconstruction shows the effect of polychromaticity artifacts in a simulated skull. (a) shows the reconstructed image, while (b) is the center line of the reconstruction for both the poly- and monochromatic cases.

A.3 Artifacts caused by scatter:

artifact Streaks in one direction (figure 6a)

problem A scattered photon does not travel parallel to the x-ray beam it belonged to. When the detector is not perfectly collimated, it can be detected by another detector than it would be detected by when no scattering had taken place.
Figure 5: Hard objects such as bones, can also cause streaks in the reconstructed image. (a) Reconstruction from polychromatic projection data of a phantom consisting of a skull with five circular bone and water inside. (b) Reconstruction of the same phantom with monochromatic x-rays.

**solution 1** Use perfectly collimated detectors, i.e. detectors that only detect photons coming from one direction, from the source.

**solution 2** Estimate the scatter intensity by mounting detectors slightly out of the image plane and assume a constant scatter intensity over the entire projection.

### A.4 Aliasing artifacts:

**artifact** Long streaks in a star-shaped pattern (figure 6b).

**problem** The number of samples per projection is too small. Aliasing errors in projection data because object and its projections are not bandlimited.

**solution** More samples per projection, such that the number of samples is almost equal to the number of projections.

**artifact** Thin streaks in a star-shaped pattern (figure 6b).

**problem** The number of projections is too small (figure 7).

**solution** More projections, such that the number of samples is almost equal to the number of samples per projection.

**artifact** A Moiré pattern. Lines that appear to be broken (figure 6b)

**problem** Display reconstruction grid is too small.

**solution** Tailor the bandwidth of the reconstruction filter to match the display resolution.
Figure 6: (a) Reconstructions from a x-ray phantom with 15-cm-diameter water and two 4-cm Teflon rods. (A) without correction (B) with polynomial beam hardening correction and (C) with 120-kp/80-kp dual energy reconstruction. (b) Sixteen reconstructions of an ellipse for different values of K, the number of projections and N the number of samples per projection.

Figure 7: The backprojection operation introduces a star-shaped pattern to the reconstruction.